

SC READY Mathematics Grades 6–8

Data Review Report 2022

In fall 2022, the South Carolina Department of Education convened a panel of experts to review item data on the SC READY Grades 6 – 8 mathematics tests. The panel looked at items and data from spring 2022 assessments. The discussions of this year’s panel yielded the recommendations that follow. The panel recognizes the hard work of SC educators, particularly during the past three years, and offers these suggestions as an addendum to those from previous years.

Grade 6:

Sixth grade teachers should appreciate that the math in this grade is foundational for students. While many of the NS standards build upon previous ideas, students are introduced to negative numbers, rational numbers which includes a discussion of the number system, and inequality symbols. The RP standards introduce ratios and rates which are the foundation for grade 7. Students are first introduced to the Order of Operations and solving one-step equations. The DS standards introduce measure of center and spread their use and interpretation, shape, and creating three different data displays.

Number System (NS)

- When teaching least common multiple (LCM) work beyond giving two numbers and finding the LCM. An item may give the LCM of two numbers and give only one of the numbers and then ask for the missing number. For instance, “the LCM of 4 and x is 20.” Many students focused on key words only and answered, “4 and x is 20” as a single variable equation.
- Students must be continually exposed to the concept of absolute value and its meaning as distance on the number line. Students should see a number in absolute value symbols as an operation. When ordering numbers provide students with a variety of numbers in the list, fractions, decimals, positives, negatives, numbers in absolute value, etc.
- Sixth grade begins the discussion of the hierarchy of numbers. With the inclusion of all rational numbers, students will benefit from an understanding of what it means to be a rational number and how that differs from natural, whole, and integer numbers. Focus on the properties of the numbers that lead to this categorization. Venn diagrams are useful tools, but they do not tell the entire story.
- When teaching 6.NS.8c be sure to include decimals and fractions. The panel felt there is too much focus on whole numbers with this standard. Make sure to work on the concept of absolute value through this standard. Practice writing the expression(s) to find the distance rather than focusing solely on the answer. Students at this grade level do not operate on negative numbers.

Ratios and Proportional Reasoning (RP)

- Use a variety of real-world situations when discussing rates and how they relate to ratios. Students should be able to recognize the rate from the context of the item and the units provided.

- Unit rates that are integers are often not written as fractions. We say we are driving 25 miles per hour, not $25/1$ miles per hour. Students chose the option that was a fraction rather than an integer. Emphasize that sometimes unit rates are written as integers.
- When giving students tables include tables that do not start at one. When comparing unit rates give students two tables that do not start with the same number. In addition, have students write sentences that correctly describe the comparison of the unit rates.

Expressions, Equations, and Inequalities (EEI)

- The panel felt that some of the translation items on SC READY were more involved than what teachers are using in classroom instruction. Translations should include grouping symbols and grouping in general. The panel felt that students need a lot of practice on the grouping part of translation and the different ways items be written to indicate grouping. In addition, students need time to work on “less than” and the relationship to “minus.”
- Students need more practice relating verbal expressions to number lines. An item may ask students about a “more than” or “less than” a certain number and have students produce the corresponding number line.
- While the standard does not include the word “finite” the panel thought that teachers should include using the word, “finite” when discussing that inequalities have infinitely many solutions. The panel pointed out that real-world situations will have finite numbers of solutions and this should be discussed with 6.EE.8b. Teachers should include more real-world items and discuss limitations on the solution set that are not included in purely mathematical situations. In addition, there is no requirement of a number line for this standard.

Geometry and Measurement (GM)

- Students may need to unfold their own three-dimensional shapes to form nets to find surface area. Caution students to identify if the item is asking for volume or surface area. Many students rushed to find volume on a surface area item.
- When providing the net for students have them find missing values occasionally. Students were given a net for a box with the length, width, and total horizontal distance from the leftmost unfolded side to the rightmost unfolded side. Students had to use the information provided to calculate surface area. Many students thought the three numbers given were the length, width, and height.

Data Analysis and Statistics (DS)

- The panel felt that more time must be spent on the Data Analysis and Statistics standards. Consider teaching them through other standards, in particular the NS standards. The standards in 6th grade lay the foundation for the Data Analysis, Statistics, and Probability standards in both 7th and 8th grades.
- Measures of center and spread are first presented in 6th grade. Students need more time finding and interpreting mean, median, mode, range, interquartile range, and mean absolute deviation.
- Using shape to describe a distribution is first presented in 6th grade. Students need more time and examples of skewed and symmetrical data. Students require more practice describing skewed and symmetric distributions with the appropriate measures of center and spread.

- While outliers are not specifically mentioned in the 6th grade standards, they are part of creating a box plot (6.DS.4). Be sure to include a discussion of outliers and how to determine if a data point is an outlier. When describing the impact of inserting or deleting a point, 6.DS.5, the impact is greatest on the mean if the point is an outlier.
- Students need to know what center and spread mean in terms of statistical items. Students should know that measures of center are mean, median, and mode while measures of spread (or variability) include range, interquartile range, and mean absolute deviation.
- Students need more practice calculating measures of center and spread. Students did not know how to calculate the mean absolute deviation in an item with four data points on a number line.

Grade 7:

Seventh grade teachers are encouraged to look at the seventh-grade standards through the lens of ratios, rates, and proportions. This is true even in the geometry standards, especially with circles. The NS standards introduce all rational numbers and operations with which students must be fluent by the end of the grade. While students in 6th grade have operated with exponents in items involving the Order of Operations they have not yet been exposed the rules of how exponents operate. This foundational concept should be built conceptually with care and not rushed to the rules. The committee commented that there is not a lot of graphing in 7th grade. Consider supplementing plotting points in all four quadrants to reinforce this skill for 8th grade. Many of the probability standards can be taught together and made into a project of portfolio for the students that is engaging.

Number System (NS)

- The committee thought that teachers needed to focus on items that require multiple properties to reach the answer. An item may begin with an expression and ask which given expression has the same value. Students may be asked to place grouping symbols to create an equivalent expression. There may be multiple properties applied in rewriting the expression.
- The committee saw two similar simplification items where students performed poorly; one of the items had students use a calculator to find the answer. Students should know how to use technology to add and subtract rational numbers. Students should know the properties used and be able to apply the properties. Distributing negative numbers is one area where students need growth.
- Students must know that reciprocals and multiplicative inverses are the same thing and use them interchangeably. The committee suggested that for multiplicative inverse students often think the sign changes too. Stress this is not the case, and that the product must equal 1.
- The committee felt that too much time is being spent on integers. More time needs to be spent explaining all the rational numbers.
- The hierarchy of numbers must be stressed in this grade. While 6th grade may have introduced the hierarchy of numbers, it is in 7th grade that students are asked to categorize them and discuss what properties of numbers separate them into distinct groups. 7.NS.2e begins this conversation.

- The committee discussed showing all different forms of rational numbers. Students were unable to identify why a number with two digits repeating after the decimal point was rational. Expose students to fractions, repeating decimals with different numbers of repeating decimals, and decimals that terminate after a few decimal digits. Include numbers with nonzero values in the ones, tens, and hundreds places as well (e.g. $13.\overline{55}$).
- The committee observed that one common misconception is that any number that repeats is irrational.
- Encourage students to use common denominators to compare relative values of rational numbers. In addition, remind students that negative numbers with larger absolute values are further from zero on the number line than numbers with smaller absolute values. In addition, provide real-world examples for students. For example, relate this to having the ground level be zero and discuss the relative depths of different holes in the ground.

Ratios and Proportional Relationships (RP)

- Students were presented an item where they had to find the unit rate in an item with complex fractions. Both the initial numbers in the item were fractions. Students were expected to divide the fractions to find the unit rate. Most students either added or subtracted the fractions. The panel suspected this meant that students did not know how to divide them. This was a particular issue when not given a calculator. The committee suggested showing students how to simplify fractions before multiplying, especially without access to a calculator, to keep the numbers small.
- The panel suggested that at this grade level students should be working with fractions daily. Some suggestions were to give students their grades as fractions to pique their interest.
- Provide students with opportunities to explain how to find the unit rate. Students were presented with an item without numbers, and they were asked to explain how the unit rate should be calculated. Students were not successful.

Expressions, Equations, and Inequalities (EEI)

- When presenting students with items to simplify vary the expressions. Keep this as a practice all year. For instance, place the number to be distributed after the quantity into which it will be distributed, for example use $(2x + 3)(-3)$ rather than $-3(2x + 3)$. Watch for common mistakes such as distributing only to “like terms” or treating the first expression as a regular subtraction item and ignoring the parentheses.
- Present students with items where they may have to distribute to more than two terms. In addition, challenge students with similar items, where if they combine like terms inside the parentheses the number of terms inside is reduced.
- When solving equations or simplifying expressions do not always place the variable as the first term. The committee noticed that items with forms that vary from the very traditional classroom form (variable term always first, number to be distributed always in front, etc.) of an equation were challenging for students.
- When solving inequalities be sure to have students graph the solution set. Students should select a test point from the number line to double check that they have correctly graphed the inequality.
- Students should understand there is often more than one algebraically correct way to solve an equation. Incorporate the Process Standards by giving students a worked item

and asking what the student did at each step to solve. Show multiple paths to the solution. For instance, in an item such as $2(x + 5) = 16$ show students that one way to solve is using the distributive property and another way to solve is to divide both sides by two.

- Students need more work on exponents and exponent properties. The committee suggested that more time should be spent on this topic. Include numerical expressions where there are two different numerical bases being raised to the exponent.
- Students need more conceptual work on exponents. Given an item where the same numerical base is in the numerator and denominator with different exponents students were unable identify an equivalent simplified expression. The panel suggested that if students had applied the definition of exponents and then simplified each pair of numbers in the numerator and denominator, they would have been successful.

Geometry and Measurement (GM)

- The committee felt that teachers rush to circle formulas too quickly. The committee thought that an exploration through ratios to find pi would give students a better understanding of circles. One suggestion was to collect circular lids of different sizes to explore the connections. The committee felt that there is a disconnect between teacher understanding and the standard 7.GM.4a-d as written.

Data Analysis, Statistics and Probability (DSP)

- The committee felt that students need more exposure to probability especially the students who are in the advanced track. The panel suggested using more real-world examples such as predicting the weather based off the prior week.
- Given a completed tree diagram students were challenged to determine the likelihood of a particular outcome. Students were not comfortable with tree diagrams and their use. Students need work finding the probability of “at least one” or “none” in addition selecting certain numbers of items.
- The committee felt that most items do not include replacement. For example, there are two marbles in a bag, one black and one garnet. A student selects one marble from the bag at a time and puts the marble back after each selection. Have students create a tree diagram to illustrate four selections. Then ask them probability questions.
- The committee wanted to make sure that teachers use the language of “dice” and “number cubes” interchangeably. State assessments use “number cubes” rather than dice when describing probability situations. Students should be familiar with both descriptions.

Grade 8:

Eighth grade teachers have a great deal of content to cover. The committee encourages teachers to look for ways to combine teaching the standards. Eighth grade teachers expand upon the hierarchy of numbers to include real numbers and their properties. Functions and the nature of possible solutions make an entrance in 8th grade. The foundational knowledge of functions and their multiple representations will serve students through high school. Translational geometry lays the foundation for translating functions later in high school. The statistics standards at this grade level reinforce the functional understanding and will aide students in Algebra 1.

Number System (NS)

- Students need more exposure to and discussion around the hierarchy of numbers. Students should know more than just how to correctly label a Venn diagram. When teaching the hierarchy of numbers focus on the attributes of each group in addition to the types of numbers that represent those attributes. Students had trouble when asked questions about real numbers and their equivalent decimal expansions. They had similar trouble surrounding the connections between integers and rational numbers.
- The hierarchy of numbers was a focus for grades 6-8 for this committee. The committee hoped that eighth grade teachers could offer guidance to the earlier grades on this topic where necessary. They felt that more foundational understanding could help students in a variety of ways through high school.
- Continue having students convert among fractions, decimals, and percentages. Connect these conversions to the hierarchy of numbers.
- The committee suggested having students identify rational numbers that are not integers from a list. To make this more difficult include irrational numbers on the list. Teachers may consider card sorts, matching games, and other activities that allow students to categorize numbers in the hierarchy presented in different forms.

Functions (F)

- Students need more practice in identifying nonlinear functions. The committee thought that the students practice identifying functions and identifying nonlinear graphs separately. Give students many different graphs and have them select the nonlinear functions.
- Additionally, students need practice describing the attributes of parts of functions. Give students a compound inequality and ask them to describe attributes of that part of the graph.

Expressions, Equations, and Inequalities (EEI)

- Students at this grade still experienced issues with exponent rules and simplifying numerical expressions using them. Give students practice with multiple bases raised to exponents and then raise the whole quantity to an exponent.
- The committee thought that students were unable to identify that the square root of one is one. Students did not see one as a perfect square. While one is not an interesting perfect square students need to know that it is a perfect square.
- When selecting the appropriate units for an answer in scientific notation in a real-world item, these items may include a metric unit conversion.
- Eighth grade students need to be reminded that proportional relationships written in slope-intercept form have a y-intercept of zero. Students were given an equation in slope-intercept form and asked how it needed to be changed to make the relationship proportional. Students were not successful.

Geometry and Measurement (GM)

- Students need to be mindful that just because a translated figure ends up in the correct position it may not be the image. Students were not mindful of the names of the vertices when performing translations.

- Teachers should be aware that translations may use only one prime on the image. Teachers on the committee stated that some instructional materials use one prime for each step in the translation. When given the pre-image and image on a graph we tend to use only one prime regardless of the number of individual translations involved in the translation.
- Students need more practice on the Pythagorean Theorem. The standards ask students to use models to demonstrate a proof of the Pythagorean Theorem as well. Students need to interpret the models of the proof in addition to identifying a model of the Pythagorean Theorem.
- When teaching how to find the distance between two points on a number line it is ok to derive the distance formula for students, but that is an extension at this grade. Students need to understand how to use the Pythagorean Theorem to find distances on the coordinate plane.

Data Analysis, Statistics, and Probability (DSP)

- The committee wanted to remind teachers to supplement lessons on matrices. Students had trouble identifying a matrix of a given size.
- When looking at data in a two-way table students need more work finding relative frequencies. Work with students to find patterns of possible association between categorical variables as well. Students were asked to find a relative frequency from a table.