EOCEP Algebra 1
2019 Data Review Report

In October 2019 the South Carolina Department of Education convened a panel of experts to review item data on the End-of-Course Examination Program Algebra 1 test. The panel looked at items with a high percentage of students answering correctly and items with a low percentage of students answering correctly. The discussions of that panel yielded the recommendations that follow. Teachers on this year's panel felt that last year's suggestions were still extremely relevant and that teachers should be reminded to look at last year's Data Review Report (2018). The panel recognizes the hard work of SC educators and offers these suggestions as an addendum to those from last year.

Algebra 1 General Suggestions:

- The panel wanted to include the General Suggestions from previous Data Review Reports. However, they then produced this list of general suggestions. Listed below are highlights of previous suggestions with a strong recommendation to read previous Data Review Reports.
    - Look at the Sample Items on "Quick Links for Teachers."
    - Use correct mathematical vocabulary. There is no substitute for using academic vocabulary in academic classes. Treat mathematics as a foreign language for students.
    - Have mathematical discussions between and with your students. They need to correct each other, and you need to hear their misconceptions.
- The panel continued to suggest that teachers do less and have students do more. Have students define variables given a context. Give students a graph of a linear, quadratic, and exponential function at the beginning of the course. Then, have students tell each other, and the teacher, how they are alike and different. Give students difficult items and let them work. The panel expressed many times that teachers are "scaffolding the meat off the bones." Good items have multiple entry points, so allow students time to find their own. In particular, the panel warned against always telling the students to "Solve using [insert method here]." Many items simply ask for "an equivalent form of the equation" or just "Solve." Items rarely direct students in which thought process they are to use. Allow students to use what they have been taught.
- The panel also offered that Algebra 1 teachers are hindered by a perceived need to teach middle school skills. Addition and subtraction of integers is not an Algebra 1 standard. The panel suggested starting with solving multi-step equations and teaching prior skills as they are needed. They further suggested moving away from outdated resources tied to the old standards. Avoid using worksheets that drill the same skill with different items. Instead, begin a unit with a good story item and a graph. This gives your unit an anchor that can be referenced throughout. Lastly, they pointed out that this is not a course meant to be taught one standard at a time.
- The panel felt strongly that even when using 21st century resources every item gets worked on paper. Make sure students have room for their computer and scratch paper when testing.

- Students that did well on the test overall seem to answer the free response items more successfully than some multiple choice items that tested the same concept. Be sure to expose all students to multiple choice items. All students need exposure to the types of items they will see.
- Some items have sentences as answer choices. Give students multiple choice items that test conceptual understanding by having students pick out the correct explanation for why something is correct or incorrect.
- Ensure that students know they have to read in math. The panel suggested occasionally giving an item where the answer is actually part of the item, no work required. However, set it up in a way that it looks like there will be work. Hopefully, this will encourage students to read math items.
- Students need more exposure to real-world items and moreover real-world situations. The panel recommended incorporating formative assessment lessons as classroom challenges for students in order to develop and extend mathematical thinking.
- Students need to synthesize the ideas in the standards. Some of the standards contain ideas that are deceptively simple, and students are missing items aligned to them. For instance, A1.FLQE.3 is fundamentally easy to understand. However, students are missing items testing this standard. Similarly, A1.FIF.1c and A1.AREI.10 both have "easy" concepts that students are missing at high rates.
- The panel wanted to remind Algebra 1 teachers of the following:
    - Students in Algebra 1 should be able to read and use interval notation.
    - Students need to be continually exposed to function notation and its relevant meaning.
    - Many times, the panel noted that teachers should be using variables other than x and y.
- Practice the Online Tools Training (OTT). The OTT is there to minimize test day stress. The OTT familiarizes students with the mechanics of the test and the tools that are there to help them.

Standard Specific Suggestions:

Arithmetic with Polynomials and Rational Expressions (AAPR)

- Students had trouble correctly identifying if a given function was a polynomial. Ensure students are able to correctly identify a polynomial. Teach students the characteristics that make an equation a polynomial. The panel thought that giving students graphs of polynomial functions and functions that are not polynomials could help with understanding of the equations.
- Students are having difficulty explaining or identifying if polynomials are closed under a given operation. Remember to teach that polynomials are closed under addition, subtraction, and multiplication.

Creating Equations (ACE)

- The panel suggested working on solving literal equations throughout the year. The panel saw this as a good place to have students begin solving for variables other than x or y. Further, the panel saw that literal equations that involved more than a few simple steps proved extremely difficult. Be sure to have students solve many types of literal equations.

Reasoning with Equations and Inequalities (AREI)

- The panel wanted to stress that AREI.3 includes the phrase, "including equations with coefficients represented by letters." These items go beyond traditional literal equations. Items include types where one coefficient is a number and the other is a variable.
    - Teachers can make them by taking an item that would typically be used when solving for "the variable on both sides of the equation" and replacing one of the coefficients with a variable. Then, have students solve for the same variable as before.
    - The panel suggested revisiting GCF at this point. They considered factoring "3x – 6" as simple (factor out a 3) and "3x – ax" as complex (factor out an x). The panel suggested that teachers re-teach literal equations after factoring or embed factoring concepts earlier.
- The panel wanted to stress that teachers need to teach completing the square including items where $a \neq 1$. In addition, items may start set equal to zero or equal to an integer, and may start in any form. Items may not ask explicitly for students to "complete the square." Items may only ask for an equivalent form of the given equation where all the answer options are in vertex form. Remind students that when reporting the values of "h" and "k" they must mind their signs.
- Items that ask students to critique another student's work may include the 'between steps.' This is particularly important in completing the square. Items may also ask for just "the next step" in a process. Students should understand the process as well as the outcome.

- The panel stresses that students need to know the vertex form of a quadratic function. More generally, students need to understand that functions can be rewritten in many different ways, and all those ways represent the same graph. Students need to move flexibly between the different quadratic forms in the same way they move flexibly between linear forms.
- The panel recommended card sorts and matching games to work on quadratic equation equivalences. This could also be used to identify zeros, graphs, or any other key feature included in A1.FIF.4. The panel suggested ultimately mixing it all together in one card sort to get at a deep understanding of the math.
- The panel suggested giving students only one representation: the equation, the situation, the graph, or the function table. Then have students create the other three.
- Students need to know the term "perfect square trinomial." Items are written using correct academic vocabulary. It is important to use the same academic vocabulary in your classroom.
- Teachers in Algebra 1 must include some discussion of complex solutions. Students are still asked to recognize <u>when</u> a quadratic gives complex solutions even if they are not asked to solve for them. The panel recommended using the discriminant as a way to connect the graph of the quadratic and what it means for a quadratic to have real roots.
- The panel wanted to warn of the vocabulary in AREI.10. Teachers should be cautious when using the word "solution" to exclusively mean "zero" or "root." A quadratic equation given in one variable (typically set equal to zero) has at most two solutions. However, a function given in two variables has an infinite set of solutions. That is what AREI.10 is driving at. Teachers should be wary of using academic vocabulary casually in class. Additionally, students need to understand how to represent the solution of a function using function notation. This includes using numbers and variables.
- Students answered that a quadratic equation was fully graphed when shown two x-intercepts and a vertex plotted on a graph. They picked out this same answer when given it verbally as justification. The panel stressed teachers need to make sure students understand that the full parabola contains all the solutions when there is an unrestricted domain.
- The panel wanted to remind teachers that two variable inequalities are important. Students need to graph from standard form as well as slope-intercept form. Students should be able to move fluently between those two inequality forms as well. Though, it is possible to graph from standard form as well. Include items where students are given the graph and have to write the inequality in both slope-intercept and standard forms.
- The panel recommended teachers give students items that include fill in the blank responses. For instance, when graphing a linear inequality, give students the sentence, "The graph of the inequality has a ___ line with a y-intercept of ____ and a solution set that _____ (3, 5)."

Structure and Expressions (ASE)

- The panel recommended that teachers build conceptual understanding early by exhibiting three graphs and their corresponding functions, one linear, one quadratic, and one exponential. Use the graphs as a catalyst for discussing the structure of the associated equations.
- The panel also recommended giving a vivid real world example for students to use as a reference for all exponential equations. Then refer to that example throughout the year as an exemplar when needed.
- Students need to understand the parts of an exponential function and how they relate to one another. The panel recommended using terms such as "growth factor" when discussing exponential functions.
- The panel suggested teachers give a rich context and provide an equation (linear, quadratic, or exponential). Then have students define the variables used in the equation based on the context.
- The panel noted that students need a conceptual understanding of factoring. Some items do not specifically ask students to "factor the quadratic." Some items ask for "an equivalent form" and all the options are in factored form. Other items describe a real-world situation using a quadratic and then ask a question that forces students to factor the quadratic in order to find an answer.
- Students were asked to identify which fully factored quadratic had only "one real zero." Some teachers on the panel thought this was a vocabulary issue in that students do not understand what is truly meant by the phrase "one real zero" and read over it.
- The panel wanted to remind teachers to pay attention to ASE.3 and ASE.3a. They felt some of these connections are being missed by students. The panel suggested that understanding this standard will allow students to work backwards from the graph to the linear factors.
- The panel wanted to stress the need for students to know the ASE standards for exponential equations as well. They suggested giving students a rich story item accompanied by a function or expression such as $45(0.80)^x$ and have students explain if that represents growth or decay and by what percent each year. Have students either write an equivalent form or pick from a list of quality distractors.

Interpreting Functions (FIF)

- The panel again stressed that FIF.1c is a deceptively easy standard to teach because students seem perplexed by it. Students are missing the connection between an ordered pair being on a graph of f(x) and that same ordered pair being a solution of y = f(x). The panel suggested giving students a list of ordered pairs and telling students that they are all on the graph of f(x). Later, ask students how many of those ordered pairs are a solution to y = f(x).
- The panel suggested that students need substantially more practice with function notation. In particular, expose students to functions with variables other than x and y. For instance, the panel suggested that if students are told that (c, d) is a solution on the graph of f(x), then students should be able to say f(c) = d. In addition, students need to understand what variables represent, test function values, and interpret values in a real-world context.

- The panel suggested practicing domain restrictions based on a context. Refrain from always giving students the graph in the first quadrant. Ask students to define the theoretical domain and to define the domain based on the context of the item. Allow them time to find the restrictions on the domain and range and then communicate them mathematically to the class.
- Teachers need to stress that graphs are read from the left to the right. This is especially important when discussing increasing/decreasing. Remind students to only report domain values when discussing where a function is exhibiting a key feature. The panel suggested questioning students about "where is it increasing (or any other key feature)?" vs "what are the values where it is increasing?" The first question encourages students to look and point at the function itself. The second question asks, "What are the x-values that describe that portion of the domain?" The second question asks students to communicate their answer mathematically without pointing at the function.
- Students need more understanding of increasing and decreasing functions. In particular, work with students on this concept with quadratic and exponential graphs. Students need exposure to the graph and also in using the domain values when reporting these intervals. Students should also report domain intervals using a variety of notations to include interval notation.
- When working with domain and range, ask students as x increases what happens to y? The panel wanted to encourage many different types of graphs here as well. Encourage students to draw graphs on paper if they are on a computer screen and use their fingers to trace them from left to right.
- Students are expected to answer domain and range questions from graphs and equations, including vertex form. Many students seem unfamiliar with this particular form of the quadratic equation. Students answered that the y-intercept was the most easily determined key feature from vertex form when presented with a function in vertex form. Ask students what information can be easily read from vertex form? Standard form? Factored form? Then continue to work on fluidity between the forms.
- The panel wanted to remind teachers to find the "average rate of change" when looking at quadratic and exponential functions too. Average rate of change is not rate of change. The panel suggested initially highlighting the entire phrase in items or directions for students. We can find the average rate of change for any polynomial function between two points. Students are answering that curves cannot have average rates of change. Further, students should know what it means for a graph to have an average rate of change of zero. Students need exposure to average rate of change items in verbal, symbolic, and tabular forms.
- Students need exposure to average rate of change items in a context. Beyond simply calculating average rate of change, students will be asked to interpret the meaning in a context. The panel encouraged flexibility with vocabulary and concepts. Teachers should be having mathematical discussions that cover the average rate of change vs rate of change, total decrease/increase, and percent increase/decrease.
- When working with domain and range, remind students that "greater than" and "greater than or equal to" are different concepts. For instance, the range for the parent graph of the exponential function is greater than zero; it is not greater than or equal to zero. More generally, remind students that an open circle represents not being equal to a number. This is important when sketching a graph given a real-world context.

- The panel suggested that teachers practice more items where students critique the reasoning of another person. Have students write a sentence explaining the parameters and their meaning in the context of the problem. Ask questions about what each part means to identify parts of the function instead of just asking for the answer. Students need time to compare characteristics of functions given in various representations of linear, quadratic, and exponential.
- The panel wanted to point out that the standard FIF.2 applies to linear, quadratic, and exponential functions.

Linear, Quadratic, and Exponential (FLQE)

- Beware of stressing common terms used in story items without encouraging thinking about how those terms are being used. Percent does not mean exponential. Constant does not mean linear. For instance, consider the phrases "a home's value increased at a constant percentage" vs. "a home's value increased at a constant rate."
- The panel recommended that teachers give students, or have them create, different situational real-world items for linear, quadratic, and exponential functions. Encourage students to express linear and exponential growth in different ways.
- Have students use first and second differences when looking at the growth of functions. In addition, students need to understand that exponential functions grow faster than quadratic functions. The panel emphasized that increasing and decreasing and end behavior need to be discussed more than once.
- Teach function concepts and how they each relate to one another in linear, quadratic, and exponential. Discuss end behavior and compare graphs. Be sure to compare exponential and quadratics using tables, graphs, equations, and verbal descriptions. Additionally, compare across those various forms (i.e. one tabular and one graphical). Include functions written in function notation.
- Ensure students understand the phrase the "xth" or "nth" term when discussing sequences. Students should understand that these terms refers to the position of the term and not the value of that term.

Quantities (NQ)

- Students need help recognizing rate of change from story items and graphs. Students must be able to name the independent and dependent variables. The panel suggested labeling the slope-intercept form of a linear equation with the units in order to help make sense of units.
- Students should be asked to define variables themselves. Additionally, students need to work with items that include simple unit conversions. For instance, an axis may be labeled with integers, while the verbal axis label includes "in thousands," "in hundreds," or "in thousandths." Students should be able to handle this type of item under this key concept and also under the SPID key concept.

Real Number System (NRNS)

- Students are missing items involving the properties of exponents. Largely, students are multiplying the exponents regardless of the operation involved. A portion of those students also multiply the bases. This skill needs to be reviewed in Algebra 1. Build off the concepts introduced in seventh and eighth grades. Include rational exponents when reviewing these concepts.
- Students need a conceptual understanding of exponents as well. For instance, "What does it mean for an expression to be raised to the power of 1/3?" In this mathematical discussion, emphasize that the properties of exponents do not have a direction. We use them to rewrite expressions.
- When discussing re-expressing between radical and rational exponent forms, use roots other than square roots. Students in Algebra 1 need a solid conceptual understanding of the exponent rules and fluency in their use.

Interpreting Data (SPID)

- Students need work on real-world items involving lines of best fit and extrapolating or interpolating. Practice giving a line of best fit in slope-intercept form or standard form and the scatterplot. Then have students interpolate or extrapolate for a given input value.
- Teachers should allow students to define the variables and interpret scales in items with scatterplots. Students should be comfortable working with integer scales labeled "in thousands" when defining variable or substituting values.
- Students require more work understanding and interpreting the correlation coefficient. Students should know that the correlation coefficient is not slope. They need to understand that it is a number that measures the strength of a linear association. Give students many different visuals for determining the correlation coefficient. There are plenty of websites students can visit and play games where they guess the correlation. Then have students help define what a strong, moderate, and weak correlation is generally. Seeing many visuals should help with this concept.
- Require students to write a complete sentence describing each parameter and r in the context of the item. Use these standards to tie back to earlier concepts. Have students discuss the reasonableness of the y-intercept in terms of the item. Encourage mathematical discussions in this unit.