EOCEP Algebra 1

Data Review Report 2022

In fall 2022, the South Carolina Department of Education convened a panel of experts to review item data on the End-of-Course Examination Program (EOCEP) Algebra 1 test. The panel looked at items and data from spring 2022 assessments. The discussions of this year's panel yielded the recommendations that follow. The panel recognizes the hard work of SC educators, particularly during the past three years, and offers these suggestions as an addendum to those from previous years.

Algebra

- The committee felt that students chose the wrong answer because they could not identify a form equivalent to the form the student produced in their work.
- The committee felt that the phrasing on an item may have given students trouble if they focused on key words only. The sentence used the word "per" in a natural English way. The committee felt that students saw that word and it led students to think about division. In fact, "per" was meant to be "each." Items are not all written to find total cost. Items may ask for a monthly, weekly, or daily cost.
- The committee suggested that students need to practice more challenging literal equations that are multi-variable and involve several steps. One member suggested searching the internet for a "Literal Equations Name Project." This project includes 44 different literal equations students could solve. The committee considered this to be a rather comprehensive list of literal equations that Algebra 1 students should be able to solve.
- The committee also wanted to note that literal equations are not necessarily limited to roots of two or less. When rearranging variables, students may need to take a cube root or higher of the expression. In addition, students should recognize equivalent arrangements such as $\frac{4}{3}\pi = \frac{4\pi}{3}$. The committee felt that a common misconception for students is to attempt to use the Order of Operations to rearrange literal equations.
- Incorporate the process standards by analyzing other students' work. One item asked students to identify the property used to get from one step to another. Students need to be able to justify the steps in solving as well as perform the steps to solve.
- Students need more time working with graphs and finding solutions to equations of the form f(x) = g(x) graphically. Revisit this concept each time a new function is introduced to the class. Students may get two quadratics and be asked to identify the solutions. The key is for students to recognize that the intersection points are the points of solution for the systems. Many students selected the zeros as the answer for this type of item.
- Students should be able to graph inequalities with and without technology. The testing platform includes the Desmos calculator. Students should be prepared to answer questions and evaluate other student work around solving inequalities, too.
- Students need support in their conceptual understanding of the Zero Product Property. Students were asked for the solutions of a factored quadratic, with variables defined as real numbers where there would normally be numbers. Students were expected to express the zeros using the Zero Product Property. Most students understood there was a fraction involved but did not place the correct variables in the numerator and denominator.

- Completing the Square was a common problem for Algebra 1 students. Items may ask for values at various points along the path to a solution or just the value of "h" or "k" in the formula $(x-h)^2=k$. In addition, the committee had a long discussion of what value would be inserted into the "h" variable. In particular, if an item asks students "what is the value of "h" in this equation, $(x+4)^2=6$" students should report that the value is "-4." Similarly, if the assessment asks students to complete the square and fill in the following equation, "$(x-\_)^2=6$" and their worked solution was $(x+4)^2=6$, then they should select or type "-4" as their answer. Teachers should be aware that since the formula has a subtraction sign in it, a negative number needs to be substituted.
- The committee also felt that teachers need to include more fractions when completing the square. Even if the "a" term of the quadratic, $ax^2+bx+c=0$, is one, the "b" term should not always be an even number. Students need more work with fractions in Algebra 1. The committee also wanted to encourage teachers to make a strong connection between completing the square and vertex form, even though vertex form is not mentioned explicitly in the content standards.
- The committee had a discussion around the phrase, "no real solutions" and whether students understood what this means. The committee suggested rephrasing "no real solutions" to "no solutions in the real number system." Students should understand that if we limited our discussion to the integer number system, then fractions do not exist. There are real-world contexts in which fractional answers or inputs do not make sense. We cannot sell half of a physical object. Similarly, the complex number system has "imaginary" numbers. They are not actually imaginary; Rene Descartes coined the term "imaginary numbers" and meant it to be derogatory. While Algebra 1 does not include a discussion of the complex number system, it may be worthwhile to explore it only in contrast to the real number system to help give students the necessary context in which to understand their solutions.
- Students were asked to solve a quadratic equation which had irrational answers. Students could have completed the square or used the quadratic equation. Students who did well on the test and used the quadratic equation ended up dividing the denominator of "2" into the square root and incorrectly simplifying the square root. For example, students thought $\frac{\sqrt{26}}{2}$ equals $\sqrt{13}$, which is clearly incorrect.
- The committee felt that students need more practice with the quadratic formula and its parts. While the discriminant is not mentioned in the standards, there are items where knowing what the discriminant tells us would help students. The committee discouraged rote memorization of the quadratic formula and only using it to find zeros.
- When interpreting meanings of parts of equations, remember to include exponential functions. The committee thought that ASE.1 is likely covered well during linear and quadratic units but covered less well with exponentials. Ensure that this standard is continually revisited with each new function.
- Students should be able to interpret and critique the work of other people. The work of others may not appear in the same way as their own work or the work of an entire classroom of students. An item asked students to identify the part of an expression that calculated the combined area of two equal triangular faces. The given work of another student combined the "$2\cdot\frac{1}{2}$" part and just had the base times the height in their final expression. Students had trouble with this item.

- Students were given a difference of two squares. They were asked to identify the factored form and the correct justification for the form. Most students that did well on the test overall selected that it could not be factored for one reason or another. Other students were split between why it could be factored. About half of them thought that it factored into the square root of the first term minus the square root of the second term.
- The committee noted that students should know correct mathematical vocabulary. All EOCEP Algebra 1 items will use correct mathematical vocabulary. The committee also noted that answer options may have two parts. The first part is the mathematical answer, and the second part is the justification for the answer. Assessment items often pair correct math with incorrect mathematical justification. A correct answer has correct math with a correct mathematical justification.
- Students need more work reading and interpreting compound inequalities and conceptual understanding of quadratic functions. Students were given a factored quadratic with variables where there are typically numbers. They were given some information about the variables in the form of a compound inequality. When asked which graph could be the graph of the quadratic, students were not successful.
- When presented an item that assessed why a value is a solution, students had problems. Students should know why a value is a solution. Have students write why a value is a solution. These justifications focused on what happens algebraically when we substitute the x-value of a zero into the quadratic.

Functions

- There were a few items where students were asked about parts of functions. The committee felt that students had trouble reading, identifying, and writing inequalities and interval notation to express where a key feature of a function is located. Include the infinity symbol when writing interval notation. Provide students with opportunities to practice all the verbal and symbolic representations of where functions are demonstrating key features. Use this to reinforce using x-values in all forms to describe parts of functions.
- Students need to pay attention to the variables used as inputs for functions and the variables used in the equation. Students were asked to find a function in terms of a given variable. The students chose an answer option where the input did not match the variable used in the equation. For example, students chose $f(x) = 3p - 4$ rather than $f(x) = 3x - 4$.
- Students need more work using function notation. There was an item where students were asked how a value would be substituted into a function. Students were not sure where a given value is substituted. If the item asked to substitute into $f(x) = y$ students placed the number in every place possible, $3f(x) = y$, $3(x) = y$, $f(3) = y$, and $f(x) = 3$.
- When examining key features of graphs compare and contrast those key features with other graphs. Do two graphs increase on the same interval? Are they both positive over the same interval?
- The committee wanted to encourage teachers to use tables with skipped domain values. When students are given a table of values they should consider plotting the points. Students can use graph paper or technology to plot points. Once they are plotted many key features of the graph become clear.
- Students need practice limiting graphs to real-world contexts. Teachers should focus on whether the domain is discrete or continuous and which quadrants make sense for the

real-world context. The committee felt that sometimes teachers limit the graph for the students automatically. Rather, the students should graph the full function in all quadrants and then limit the domain and range to fit the real-world situation themselves.

- The committee suggested giving information through multiple representations in one item. Give one function as a graph and the other as a verbal description or equation. Then ask students to compare key features or values.
- Students need more help understanding average rate of change. The committee encouraged teachers to revisit average rate of change for both quadratics and exponentials. While the average rate of change is the same as the constant rate of change, which is the same as the slope for linear equations, this is the least interesting case. The concept of average rate of change becomes interesting with non-linear equations.
- In addition to calculating the average rate of change for non-linear functions, students must also be able to interpret or explain the average rate of change in terms of a real-world context. Students may be given the value and asked for the interpretation or asked to calculate and interpret.
- The committee suggested that teachers should use verbal and tabular representations when teaching. Use these representations with the familiar algebraic and graphical representations to build understanding.
- Students need to know which form of an equation is best to extract certain information. For instance, vertex form is the best form for quickly determining the vertex. While students are not tested on the names, they do need to know the forms and their uses.
- Students need more work on going from a description of a function to the equation of the function. For example, in quadratics tell students the zeros and the leading coefficient and have them write the quadratic. Then ask students to find a key feature, value, etc. The committee thought there is too much time spent on going from the equation to zeros, vertices, etc.
- The committee wanted teachers to know that Algebra 1 students need to know the quadrant names. Students also should know how to interpret a vertically shifted exponential function. Teachers and students should use the correct terminology as well. Rather than always saying, “the a value” students should know to say, “the leading coefficient” for quadratics.
- Items asking students to identify situations modeled by either linear or exponential functions will include percent language or symbols in more than one answer option. One of the options will be a distractor. Include items where students need to analyze the situation rather than relying on key words.
- The committee recommended that teachers work on word problems every day.
- Students need more time working with the linear, quadratic, and exponential functions and discussing what happens as they approach infinity. How do the end behaviors compare? The committee recommended retyping functions into Desmos if students want to see the graph in color.
- Give students multiple representations in the same item. Give students the algebraic expression and the table and then have them interpret both to identify key features. Students can always look at the graph, especially when students are given the algebraic representation.

Number and Quantity; Interpreting Data

- The committee highlighted the idea that when items are in a real-world context, and the output is labeled profit or loss, then negative values are possible. Students must be able to connect an equation to the real-world context it is modeling. Have students ask, what is the input, what is the output, what are the labels being used in this context?
- The committee highly recommended that students write something on paper for every item regardless of presentation format. The committee further suggested using this to simulate the computer-based testing process with paper classroom test sets. Instruct students to write on a separate sheet of paper, rather than on the hard copy of the test, to practice computer-based testing.
- Students were asked for an "appropriate level of accuracy" and they read right past that phrase. Students calculated a percent and found the answer regardless of the context in the item.
- The committee wanted to make sure that teachers justify answers to exponent items by working exponent items "backwards" too. Do not always just simplify. Students need to work from simplified form back to radical form. This is an important foundational skill that will be applied in future classes.
- The committee suggested that typically teachers give one item for rewriting radical expressions and one item for simplifying radical expressions. Give students items where they need to rewrite and simplify radical expressions. For example, write $m^{\frac{10}{3}}$ as a simplified radical expression as $m^3\sqrt[3]{m}$. This is also true of negative exponents. Students may be asked to rewrite and simplify negative exponents.
- When simplifying radical expressions students should be prepared for multiple variables and more than one step in simplification. However, the assessment will not include an egregious number of steps or variables.
- The committee wanted to make sure teachers are taking larger roots of numbers. The fourth root could be used when simplifying radical expressions. In addition, indices may be mixed in items. Items may include a fourth root and an unlabeled radicand. Students should know that the unlabeled radicand is a square root.