EOCEP Algebra 1
Data Review Report: Spring 2021

In fall 2021 the South Carolina Department of Education convened a panel of experts to review item data on the End-of-Course Examination Program (EOCEP) Algebra 1 test. The panel looked at items and data from spring 2021 assessments. There were no state assessments in 2020 due to the global pandemic, thus there is no Data Review Report for spring 2020. However, there are still reports from previous years that contain relevant and useful suggestions for improving instruction. The discussions of this year's panel yielded the recommendations that follow. The panel recognizes the hard work of SC educators over the past two years and offers these suggestions as an addendum to those from previous years.

The EOCEP Algebra 1 blueprint is divided into three Reporting Categories. The Reporting Categories are Algebra, Functions, and Number and Quantity; Interpreting Data. This Data Review Report is similarly divided into those three categories.

Algebra

- When studying literal equations use equations that the students may or may not have seen. Include equations with variables that may be unfamiliar to students. Chemistry and Physics were cited as disciplines in which to find formulas.
- Build standards through the lens of the three function types in Algebra 1, linear, quadratic, and exponential. Algebra 1 students should see equations literal and otherwise that are linear, quadratic, and exponential.
- Consider having students justify their steps when solving equations. Use this as an opportunity to emphasize the properties of equality.
- Emphasize that, when solving literal equations, the result may not be a number. There will likely be a single variable on one side and some type of equation on the other. The committee suggested that at first teachers remove all the letters and numbers. Have students solve an equation such as "smiley face + heart = Sun," but with actual pictures.
- Students need to be flexible using and knowing the properties of equality that generate an equivalent form. Have students find equivalent forms of common equations. For instance, give students an equation in slope-intercept form and have them find an equivalent form solved for 'x.'
- The committee also discussed emphasizing vocabulary. A variable is an unknown. It is possible to isolate any unknown in an equation. Teachers may also consider asking students to solve for a constant.
- The committee recommended using error analysis, having students identify and correct the error in an incorrectly solved equation. Another suggestion was to give students a correctly solved equation and have them provide the properties for each step. Card sorts were also suggested to have students identify equivalent forms of equations.
- The committee also suggested using whiteboards for formative assessment. Since there are often many combinations of correct steps to isolate a variable, have students solve without any guidance and then discuss each other's work.
- Give students different first steps. In this case, students are given the original equation and then each student is given a different first step. They are to examine whether the first step is valid and if it is to provide the justification. If it is not, then they correct the step and/or the justification. Then the students finish solving and providing justification for each step. This could be done in pairs or groups as well.

- The committee commented that our items have more words than their classroom items. ECOEP Algebra 1 items must carry the directions with each item. We do not give a simple direction such as, “Solve for x” and then several similar items. Rather, each item would say “Solve for x,” however, the EOCEP Algebra 1 test typically puts the formula/equation in a context. For example, if we used the equation relating distance, rate, and time we would define each variable in words before having students solve the item.
- The committee noted that students are asked to transform quadratic equations fluently between standard, factored, and vertex forms. While students are not tested directly on the names, students need to understand which form is the best for identifying the axis of symmetry, vertex, zeros, y-intercept, etc. Students also need to identify the vertex as a maximum or minimum of a quadratic function.
- The committee also stresses the interchangeability of the terms zeros, x-intercepts, and solutions. While there are appropriate mathematical usages for each term, what is important for students to recognize is all three are talking about the same features of a graph and have essentially the same meaning.
- The committee suggested giving students an item where a projectile is launched above ground level an have the students use a quadratic function where one zero is negative to find when the projectile hits the ground. This quadratic may be in factored form to start and then revisited as new forms of the quadratic are explored.
- Students must understand the Zero Product Property and how linear factors must be set equal to zero before the zeros can be determined. In addition, this relationship can be explored by giving students a graph with a quadratic graphed that has two easily identifiable zeros. Then give the students four or five linear factors and have them pick the correct ones to use to write the quadratic.
- The committee suggested using the “corners” formative assessment strategy. In this strategy students go to a corner to indicate which answer they believe is correct. They then discuss as a class the correct answer and why.
- The committee wanted to emphasize both equation notation and function notation. Students need to know how to interpret $y = 0$ and $f(x) = 0$.
- The committee also stressed that a variable may be a common factor. Students are not sure how to factor an ‘x’ out of an equation and this gives them trouble.
- Students must understand that equality goes both ways. A common misunderstanding is that equality works only in one direction, normally toward a single answer. However, that is not what equality means.
- The committee noted that the standard ASE.3 is not limited to quadratic equations. The ability to choose and produce equivalent forms must be practiced with both linear and exponential equations as well. The committee further noted that this standard is about equivalent forms of equations, not just finding the graph of a function. For instance, students may be asked to choose a form of the quadratic that reveals the vertex. The distractors may be equivalent forms, but not the one that reveals the vertex. Thus, students may not be successful if they rely solely on graphical representations.
- Students must be able to solve systems of equations with any of the three methods specified in the standards. Students may be asked to evaluate or determine the first step in one of the methods. Students may also be asked to perform error analysis of a given

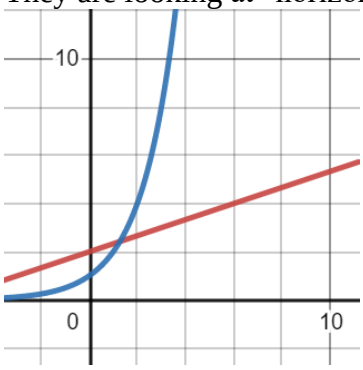
incorrectly solved item. Students must understand the correct steps in each method to critique another student's work.

- The committee stated that after one day on solving systems they focus on items with a context. Their focus is typically on interpreting the answer over the method of solution. For AREI.6 and the sub-standards, both the method of solution and interpretation of the answer are important.
- The committee noted that when using linear combinations, it is typical to multiply an equation by a constant so that the next step is addition and the addition will eliminate a variable. While this is the most efficient method, subtraction is also a method of linear combination. Students should understand subtraction as a valid method in linear combinations.

Functions

- The committee noted that function notation is introduced in Algebra 1 and teachers should not assume that students know what f(2) = 8 means. Students need practice working with function notation. Students need to know how to interpret f(x) = 3 vs f(2) = 8 in terms of x and y and in the context of a real-world item.
    - The committee suggested using the graph of a quadratic function and asking students two questions related to the quadratic to help them.
        - What value(s) satisfy f(2) = ?
        - What value(s) satisfy f(x) = 4?
    - After students begin to understand the concrete representation above ask students similar questions related to tables of values, equations, and graphs of linear, quadratic, and exponential functions.
    - Consider the context of tossing a ball into the air, the variables may be *h* for height and *t* for time with the function being defined as $h(t)$. Have students discuss the input, output, domain, and range. When discussing domain and range discuss the domain of the function in a purely mathematical sense (devoid of the context) and then again with respect to the real-world situation it describes.
    - Relate the real-world context back to interpreting function notation. Ask students what $h(2) = ?$ and what $h(t) = 4$ mean in the context of the item. Give students an ordered pair from the graph and ask for a correct interpretation in the context of the situation.
- Students must be able to distinguish between constant differences and constant factors. It was the committee's experience that students have spent a lot of time studying arithmetic sequences and this becomes their default answer whenever they see a pattern in a sequence of numbers. Students need to have time to compare arithmetic and geometric sequences to see how they grow differently.
- The committee also brought up the EOCEP Algebra 1 Sample Release Item number 15. They pointed out that students should see items where there is a percent in an item that is describing a linear situation. They suggested giving students a variety of scenarios and have them answer if the scenario is best described by a linear or exponential function.
- When practicing the difference between linear and exponential functions, discuss what type of function could be defined when there are just two points. Then, challenge students to answer what type of function it could be when a third point is added.

- When discussing linear and exponential functions, the committee recommended asking students if they would rather have a penny doubled every day for 30 days or $1,000 a day for 30 days. Students need an item to use as a hook to understand the power of exponential functions. Use this same item to explain how exponential functions will eventually exceed, in value, both linear and quadratic functions. Find the best example for your students and refer to it often.
- The committee identified a possible misconception for students. Students are misunderstanding what the "value of the function" means on a graph. The value of the function is the y-value (output) not the x-value (input). Students were presented with two graphs, a linear and exponential both graphed on the same axes and asked which function will eventually exceed and remain larger than the other. Students tend to choose the linear function.
    - Consider two functions, $f(x) = \frac{1}{3}x + 2$ and $g(x) = 2^x$. We know that the exponential obtains larger values faster than the linear function. However, for students who are unsure of how the value of a function is expressed on a graph, they seem to select the function that appears to cover the x-axis values faster. They are looking at "horizontal growth" which is a serious misconception.



    - Encourage students to use a table of values when given a comparison of two functions. Using a table places the focus back on the value of the function (y-values). Then, tie the table back to the graph and have students explain how they are seeing the growth on the graph and how it relates to the growth in the corresponding table.
    - With calculators and spreadsheets, it is possible to challenge students to find a linear or quadratic that will exceed a given exponential for large values of x.
    - Challenge students to find between which two integers the exponential overtakes other functions.
- Another area where students are confusing the "value of the function" is when students are asked to identify the value of the maximum or minimum. Students give the x-value as the value of the vertex.
- When discussing increasing and decreasing students tend to mistakenly describe "increasing to the left." Students should be exposed to the phrase "as x increases" and know how that relates to graphs. Generally, students need help reading graphs. Students need help knowing that all the points need to be graphed to graph a function.

Number and Quantity; Interpreting Data

- The committee noted that the support document for Algebra 1 mentions dimensional analysis aligned to NQ.1. They suggested that teachers look at the support document for this standard. Show students how units cancel in dimensional analysis. Further, expose students to items with multiple conversions in a single item without creating an answer at each step. In this type of long conversion have the units in the first step cancel with units in the third step. Students need some flexibility in their thinking within unit conversion.
- Students need practice determining what goes on the x-axis and what goes on the y-axis. Have students determine the independent and dependent variables. The committee suggested using contextual items to practice this without solving the item.
- The committee also suggested having students give answers in complete sentences or phrases when working on a contextual item. For instance, instead of x = 5 and y = 8 have students say "…bought 5 daisies and 8 roses."
- Have students practice graphing mathematical stories and writing plausible stories from graphs. The most common of these use a distance vs time graph with a line already on the graph and have students make up the story that describes the graph and vice versa.
- Expose students to graphs that have different scales on the x- and y-axes.
- Students need help with decimal expansions of real numbers. Generally, students need help understanding the number system and the interaction between rational and irrational numbers. Students need to understand that there are an infinite number of numbers after the decimal in an irrational number. Students must also know that this does not change if we multiply an irrational number by an integer. For instance, 3π still has a nonterminating nonrepeating decimal expansion.
- Students need to recognize when a number under a radical results in a rational number and when a number under a radical results in an irrational number. Students need more time identifying that $\sqrt{2}$ is irrational whereas $\sqrt{16}$ is rational. The committee suggested using card sorts to help students.
- The committee suggested touching on the property of closure when discussing adding, subtracting, multiplying, dividing, and taking roots of both rational and irrational numbers. They also suggested having students write and justify their thinking. Have students create foldables to organize their thinking.
- Students must have a good working knowledge of the number system. Students must know what happens when irrational and rational numbers are part of an operation. The committee suggested giving students an answer with one of the initial terms and then asking the students what type of number the missing number must have been. Another question may be given irrational outputs and an operation what type of inputs could there have been.
- When working with scatterplots students need to be able to describe an association using the words strong, moderate, weak, no, positive, and negative. This concept relates to slope. However, the correlation coefficient is not the slope of the line of best fit.
- Teach students how use technology when working with scatterplots, fitting models to scatterplots, and when finding the correlation coefficient of a linear fit. Students should be able to use a familiar calculator or the online Desmos calculator. The Desmos calculator is the calculator used in the online test engine.

- The committee suggested using real-world data to ask questions. There are many online repositories of data that can be analyzed for the statistics standards. The committee also recommended searching online for correlation matching games so students can get a feel for what a graph with a particular correlation coefficient looks like.
- The committee wanted to emphasize that students must understand correlation is not a percentage and that a negative correlation coefficient is not necessarily weak. They suggested looking at data sets with different possible lines of fit. Then have students pick the line of best fit. This can be done by visual inspection. (Students do not need an understanding of least squares.)