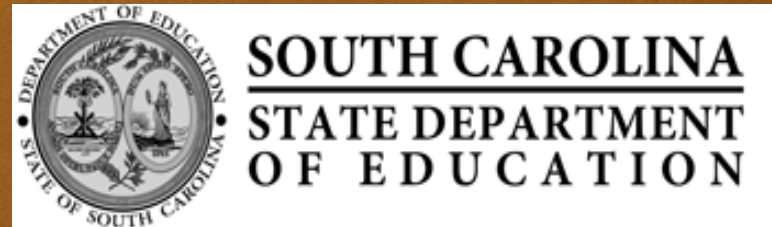
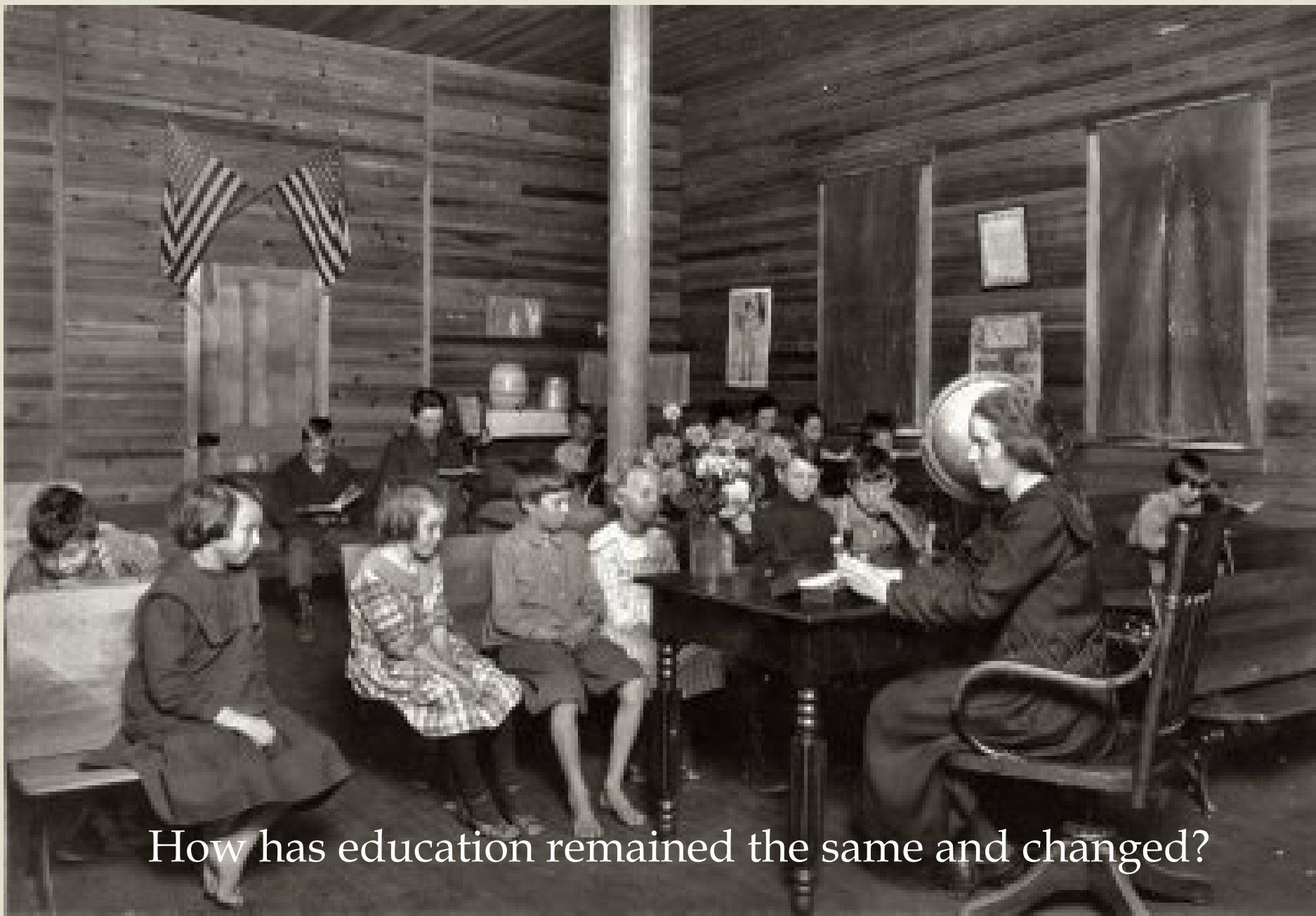


RATIO AND PROPORTIONS ~ CCSS 7.RP.2.a-d



*Laying the Foundation for
Understanding Slope ~
CCSS 8.EE.6*





How has education remained the same and changed?

<http://iteach-and-ilearn.blogspot.com/>

Discuss With A Partner

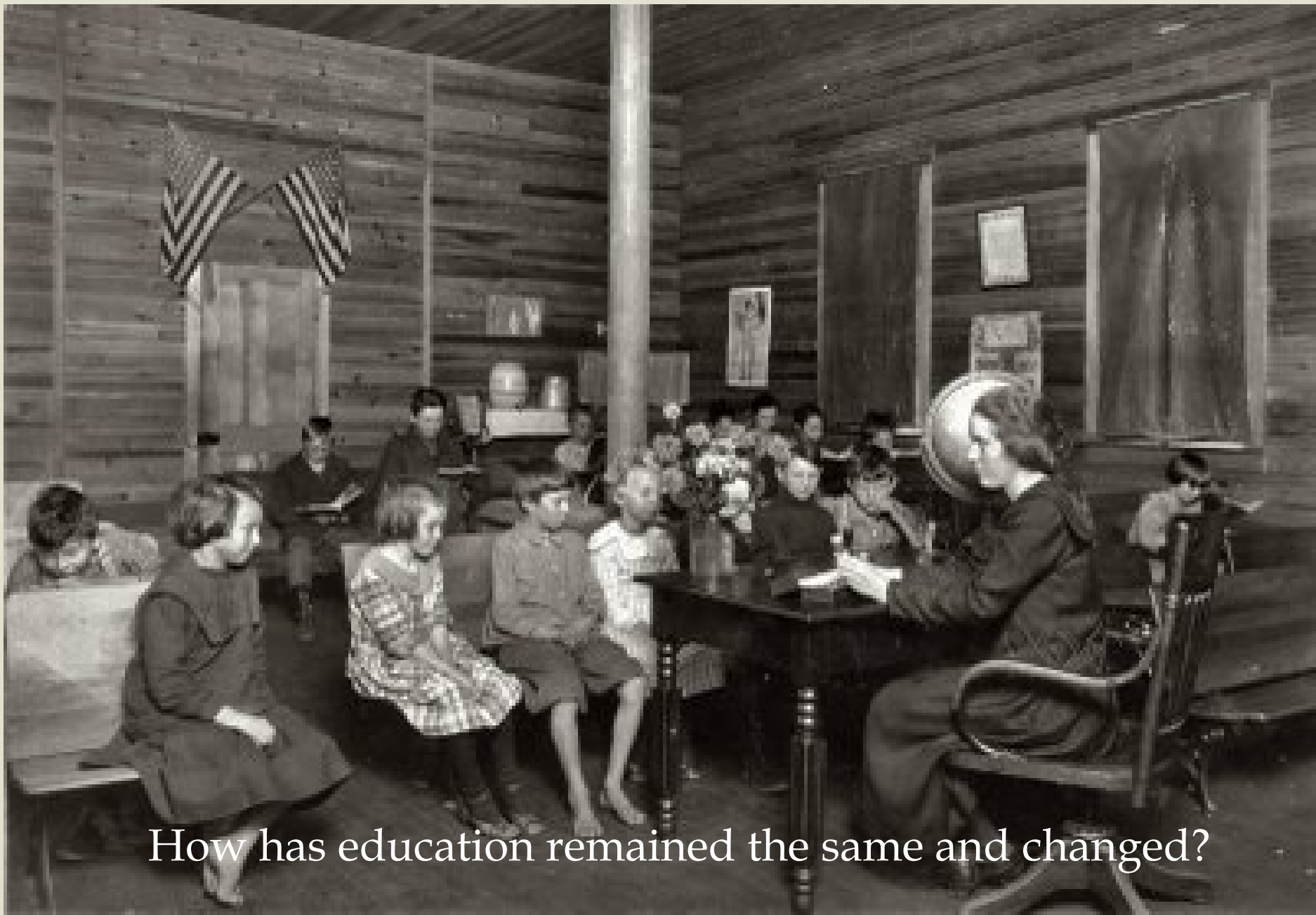
Differences

- ❧ Dress Code not an issue for students
- ❧ Teacher's dress code
- ❧ Students of different ages
- ❧ Absence of Technology
- ❧ Students sitting on benches
- ❧ Source of heat
- ❧ Other differences



Similarities

- ❧ Teacher was female
- ❧ Teacher at the front of the room giving instruction
- ❧ Differentiated Instruction
- ❧ One student off-task
- ❧ Other similarities



How has education remained the same and changed?

<http://iteach-and-ilearn.blogspot.com/>

Activity One: 6th Grade Review



Picture This In Time

Today's Objectives



- Review ratio concepts and using ratio reasoning to solve problems.
- Develop an understanding of proportional reasoning and the constant of proportionality through multiple representations.
- Recognize how understanding proportional reasoning and the constant of proportionality lay the foundation to understanding slope, and linear functions in the context of similar triangles.

Analyzing the Structure of the CCSS for Mathematics

∞ **Domain** - 6th Grade Ratios and Proportional Relationships

∞ **Cluster** - Understand ratio concepts and use ratio reasoning to solve problems.



Standards -

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Making Mathematics Relevant

Common Core State Standards

Problem Solving
Reasoning and
Proof
Communication
Connections
Representation

NCTM
Principles and
Standards for School
Mathematics (1998)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<http://www.insidemathematics.org/index.php/common-core-standards>

Discuss With Your Table



How do the 1998 NCTM Process Standards
relate to the CCSS Mathematical Practices?

Process Standards Comparison



Process Standards

- ☞ Problem Solving
- ☞ Reasoning and Proof
- ☞ Communication
- ☞ Connections
- ☞ Representation

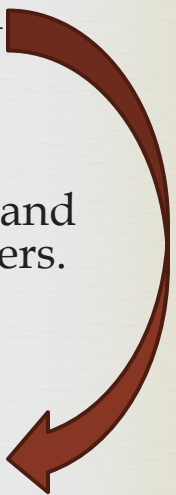
Reasoning and Explaining

Modeling and Using Tools

Seeing Structure and Generalization

CCSS Mathematical Practices

- ☞ Make sense of problems and persevere in solving them.
- ☞ Reason abstractly and quantitatively.
- ☞ Construct viable arguments and critique the reasoning of others.
- ☞ Model with mathematics.
- ☞ Use appropriate tools strategically.
- ☞ Attend to precision.
- ☞ Look for and make use of structure.
- ☞ Look for and express regularity in repeated reasoning.



Discuss With Your Partner

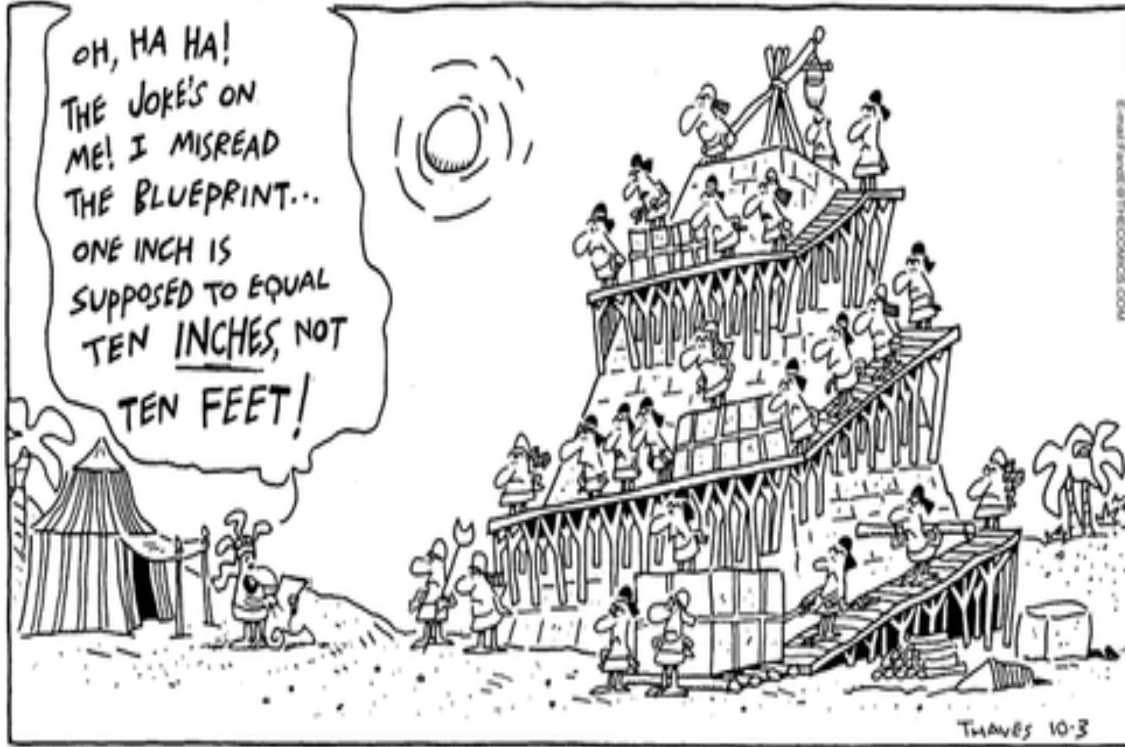


What mathematical practices were applied in
Activity 1?

Explain what it looked or sounded like.

Frank and Ernest

OH, HA HA!
THE JOKE'S ON
ME! I MISREAD
THE BLUEPRINT...
ONE INCH IS
SUPPOSED TO EQUAL
TEN INCHES, NOT
TEN FEET!



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Proportional Reasoning
Yes, it is IMPORTANT!

Importance of Vertical Progression

6th Grade Summer Professional Development Standards:

6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

Application of Conceptual Understanding

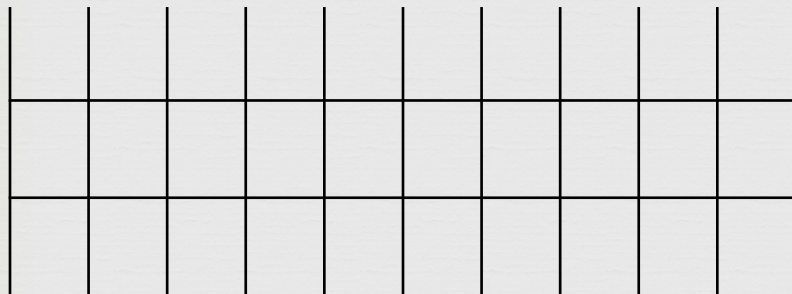


6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Diagrams



0 28 56



0 7 14

Double Number Line

Representing ratios with tape diagrams



This diagram can be interpreted as representing any mixture of apple juice and grape juice with a ratio of 3 to 2. The total amount of juice is represented as partitioned into 5 parts of equal size, represented by 5 rectangles. For example, if the diagram represents 5 cups of juice mixture, then each of these rectangles represents 1 cup. If the total amount of juice mixture is 1 gallon, then each part represents $\frac{1}{5}$ gallon and there are $\frac{3}{5}$ gallon of apple juice and $\frac{2}{5}$ gallon of grape juice.

[https://wiki.eee.uci.edu/index.php/Ratios_and_Proportional_Relationships_\(CCSSM\)](https://wiki.eee.uci.edu/index.php/Ratios_and_Proportional_Relationships_(CCSSM))

Tape Diagram

Expectations



Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

What does this look like?



- ❧ Collect data using a table.
- ❧ Examine and discuss data.
- ❧ Make a scatter plot to represent data.
- ❧ Analyze and interpret data by making comparisons.

Activity Two: 6th Grade Review



Games By The Dozen

Read the Problem



The ratio of the number of John's video games to the number of Bill's games is 5:2. If Bill has 12 games, how many games does John have?



Complete the Table of Equivalent Ratios

Bill	4	6	8	10	12
John	10	15	20	25	???

1. Examine the data above representing the number of games John has compared to the number of games Bill has. What patterns do you see?
2. How does the ratio 5:2 stated in the problem relate to the data in the table? Explain your conclusion by giving examples.
3. What is the unit rate comparing the number of games John has to the number of games Bill has?
4. On your graph paper, graph the ordered pairs given in the table to create a scatterplot. Label the x -axis as *The Number of Bill's Video Games* and the y -axis as *The Number of John's Video Games*.
5. On your paper, write down your observations of the data. How does the data in the chart relate to points on the scatterplot? If Bill has 12 video games, how many games does John have? Explain how you found your answer and complete the chart.
6. Discuss your work with your table.

Write On Your Chart Paper



- What patterns did you see?
- Describe the scatterplot.
- How did you decide how many games John has if Bill has 12?
- What observations and connections did you make and how do they relate to the ratio given in this problem?
- What generalization can you make when the ratio for a set of data remains the same (constant)?

6th Grade Paves the Way for 7th Grade

Expectations . . .



7.RP.2. Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7th Grade Expectations



- b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- c. Represent proportional relationships by equations.
- d. Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1,r)$ where r is the unit rate.

Today's Focus

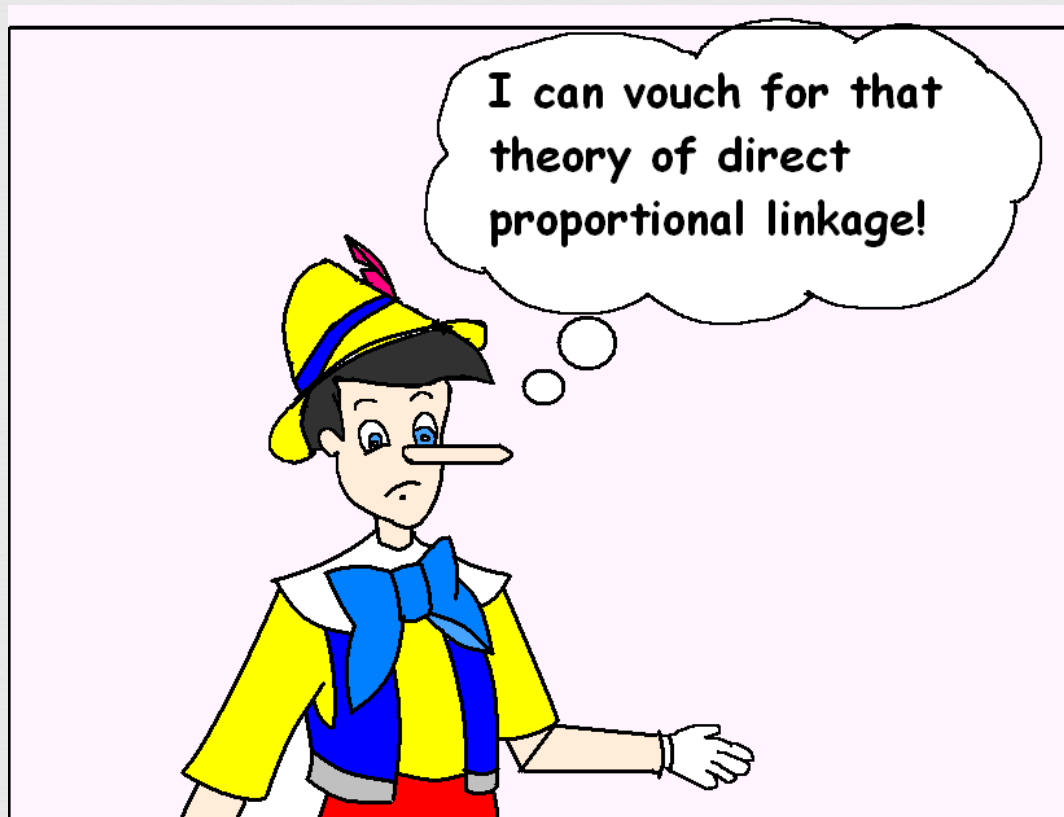
Why is the conceptual development of proportional reasoning important during the middle grades important?



According to Van de Walle, “Proportional Reasoning is usually taught in grades 6 to 9. When students are taught this concept at earlier grades, they may not be ready. This encourages students to apply rules without thinking. When this happens, the ability to reason proportionally often does not develop.”

(Van de Walle, J.A. (2007). *Elementary and middle school mathematics: Teaching Developmentally*. (6th ed.) Boston, MA: Pearson Education, p. 355)

Rules Without Reason Are Meaningless



Activity Three:

7th Grade 7.RP.2.a-d



What's The Big Idea?

Critical Ideas



- ∞ constant of proportionality (unit rate) - 7.RP.2b, d
- ∞ proportional relationships - 7.RP.2b, c, d

Critical Ideas



- b. Identify the **constant of proportionality (unit rate)** in tables, graphs, equations, diagrams, and verbal descriptions of **proportional relationships**.
- c. Represent **proportional relationships** by equations.
- d. Explain what a point (x,y) on the graph of a **proportional relationship** means in terms of the situation, with special attention to the points $(0,0)$ and $(1,r)$ where r is the **unit rate**.

Discuss - *Constant of Proportionality*



- ❧ What prior learning does *constant of proportionality* refer to in this standard?
- ❧ How is *constant of proportionality* to be represented?
- ❧ What specific algebraic form of *constant of proportionality* is named in this standard?

Discuss – *Proportional Relationships*



- ❧ How is the expectation of **understanding** proportional relationships different from the expectation of **identifying and solving** a proportion?
- ❧ What are three proportional relationships that students need to **understand**?

Mathematics Teachers help shape the future...



*In this changing world, those who understand and can do
mathematics will have significantly enhanced opportunities and
options for shaping their futures.*

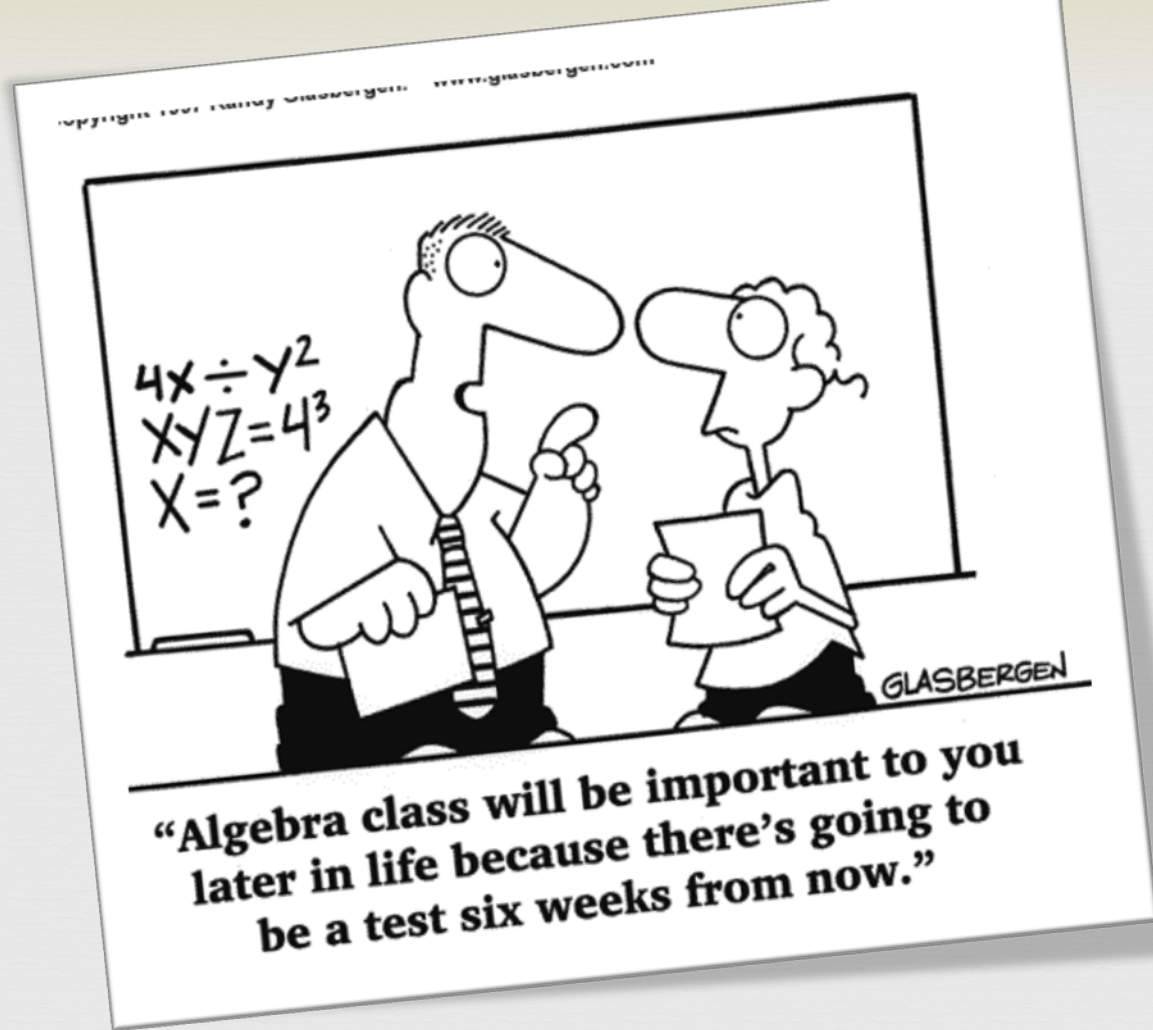
(NCTM Executive Summary Mathematics Principles and Standards for School <http://www.nctm.org>)

Why is it important for teachers in grades 6-7 to intentionally develop students' proportional reasoning skills in context?



“It is estimated that more than half the adult population cannot reason proportionally. This means that for most people, maturation and experience, even when they are supplemented by current instruction, are not sufficient to develop sophisticated mathematical reasoning.”

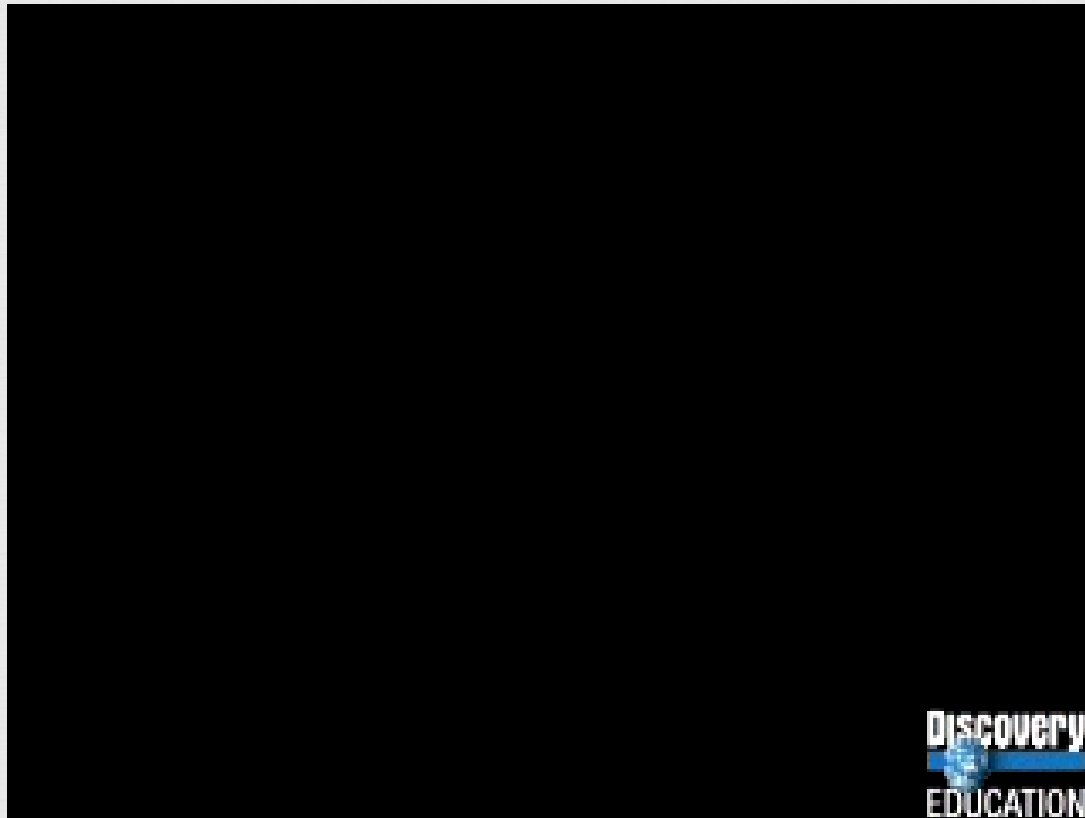
(Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers, Lamon, 1999, page 5)



That is to say, "Proportional thinking is developed through activities involving comparing and determining the equivalence of ratios and solving proportions in a wide variety of situations and contexts" (Van de Walle, p. 353).

It is not developed simply by growing older.

Proportional Thinkers



DISCOVERY
EDUCATION

Proportional Thinkers

According to Lamon (1999),
proportional thinkers:



According to Van de Walle (2007),
proportional thinkers understand:

- understand relationships in which two quantities vary together and are able to see how the variation in one relates the variation in another.
- recognize proportional relationships as distinct from non-proportional relationships in **real-world contexts**.
- develop a wide variety of strategies for solving proportions or comparing ratios, most of which are based on **informal strategies** rather than prescribed algorithms.
- understand ratios as distinct entities representing a relationship different from the quantities they compare.

- A ratio is a multiplicative comparison of two quantities or measures. A key developmental milestone is the ability of a student to begin to think of a ratio as a distinct entity, different from the two measures that made it up.
- Ratios and proportions involve **multiplicative rather than additive** comparisons. Equal ratios result from **multiplication or division**, not from addition or subtraction.

6th Grade Paves the Way for 7th Grade

Expectations . . .



7.RP.2. Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Activity Four: 7th Grade
7.RP.2.a



Miles and Miles To Go

Write a description and an Equation



1. In your own words, describe the relationship between the number of miles driven and the number of gallons of gas consumed.
2. What is the unit rate comparing the number of miles to one gallon of gas?
3. Write an equation that uses the constant of proportionality to model the relationship between the number of miles driven and the number of gallons of gas consumed?

7th Grade Expectations



- b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- c. Represent proportional relationships by equations.
- d. Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1,r)$ where r is the unit rate.

Today's Focus

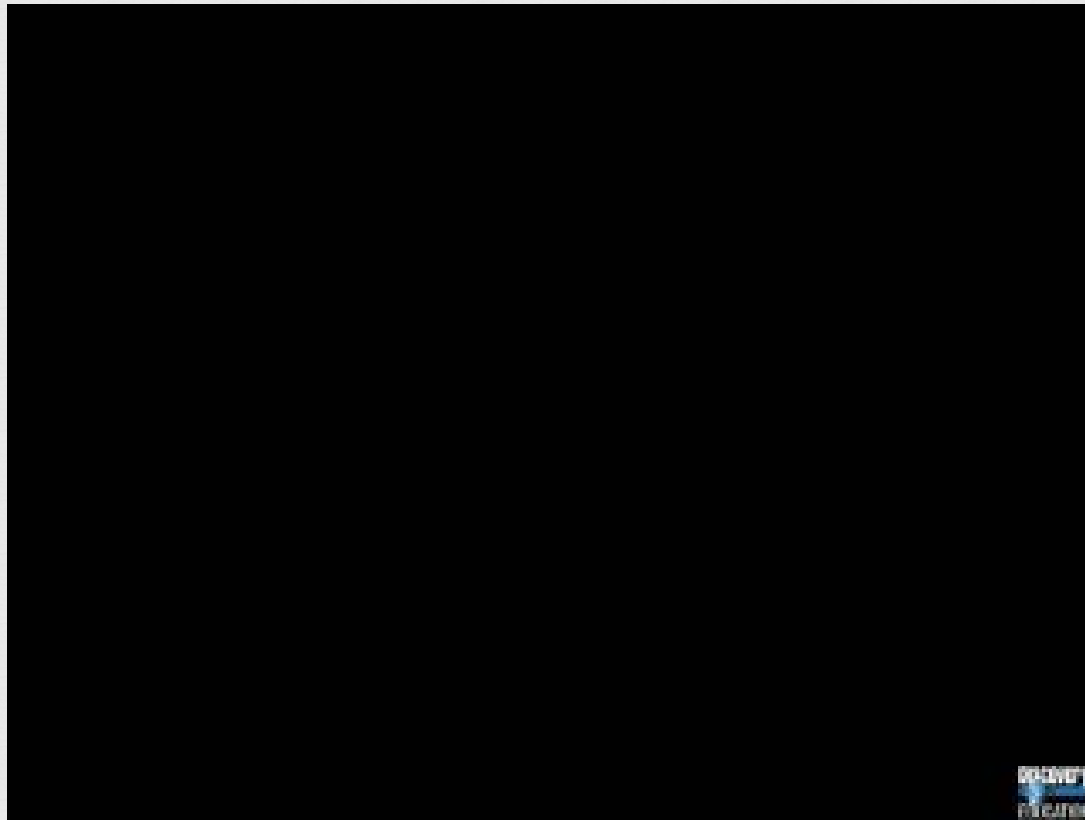
Activity Five: 7th Grade

7.RP.2b-d



The Price at the Pump

Constant of Proportionality



What does this look like?



- ❧ **Compare ratios in a table or on a graph to determine if they are in proportional relationship.**
 - ❧ Equal ratios result from multiplication or division.
 - ❧ Graphs are linear and pass through the origin.
- ❧ **Identify the unit rate (constant of proportionality) in different representations.**
 - ❧ Tables
 - ❧ Graphs
 - ❧ Equations
 - ❧ Diagrams
 - ❧ verbal descriptions
- ❧ **Write equations for proportional relationships.**
- ❧ **Describe the proportional relationship of points on a graph in terms of the context, specifically the coordinates of the points that represents the unit rate (constant of proportionality) and the initial value (origin).**

7th Grade Conceptual Development of Ratios and Proportional Reasoning is Critical to 8th Grade Conceptual Understanding of proportional similarity, slope, and linear functions.



Proportional relationship

- ❧ Two quantities change, or vary, directly with each other.
- ❧ The comparison between the ratios and proportions is multiplicative. In other words, if one item is doubled, the other quantity is also doubled.
- ❧ This relationship is a direct variation.
 - ❧ The equation is $y = mx$ in which
 - ❧ x represents one quantity, *the independent variable*.
 - ❧ y represents the other quantity, *the dependent variable*.
 - ❧ The graph is a line that passes through the origin.
 - ❧ m is the **slope (unit rate or rate of change)** of the line. It is also called the *constant of proportionality* of the function.

Laying the Foundation



8th Grade Expressions and Equations
Line and Linear Equations
FUNCTIONS 8.F.4

http://secc.sedl.org/common_core_videos/grade.php?action=view&id=778

8th Grade

Domain - EXPRESSIONS AND EQUATIONS



Cluster -

Understand the connections between *proportional relationships*, lines, and linear equations.

Standard -

8.EE.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

Similar Triangles



8th Grade

Domain - EXPRESSIONS AND EQUATIONS



Cluster -

Understand the connections between *proportional relationships*, lines, and linear equations.

Standard -

8.EE.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

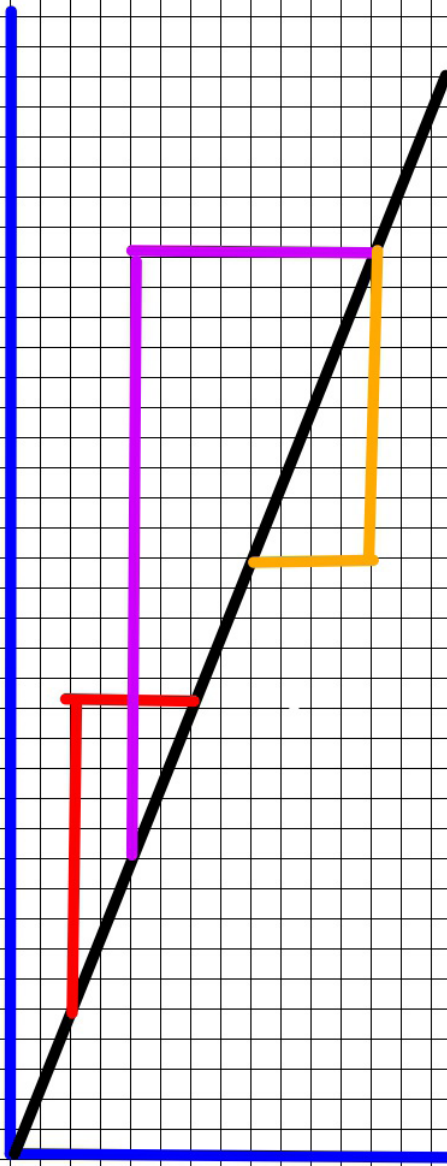
Activity Five: 8th Grade

8.EE.6



Hitting the Slopes

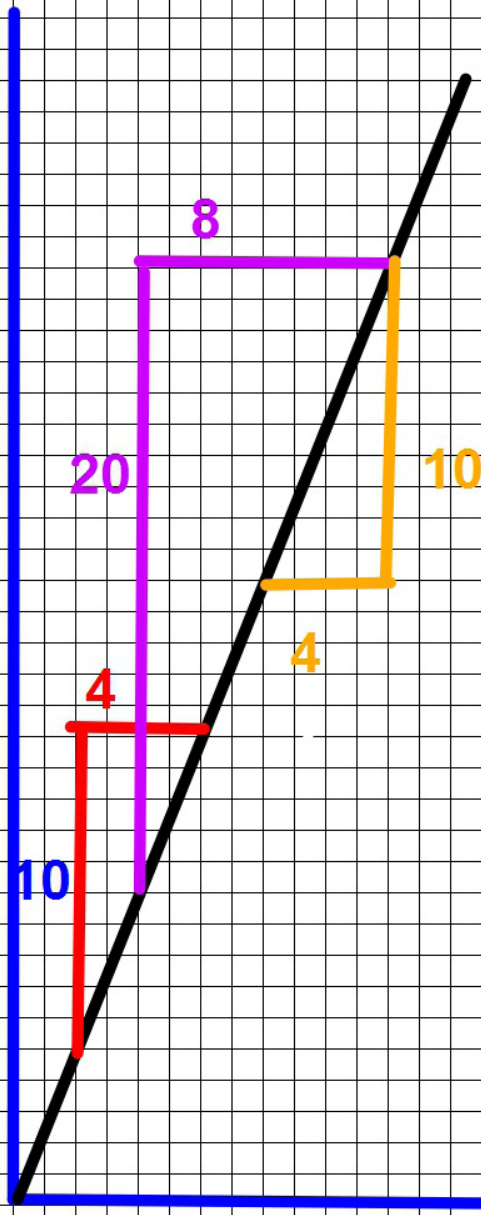
<http://illuminations.nctm.org/LessonDetail.aspx?id=L728>



Notice the graph from *Games by the Dozen*.

Choose any 2 points and draw a triangle. Draw a second triangle.

How could you determine if the triangles are similar?



Do the ratios demonstrate constant proportionality?

$$10/4 \quad 20/8 \quad -10/-4$$

What is the unit rate?

Is that the slope of the line?

...derive the equation $y = mx$ through the origin...



- Refer to the graph of Games by the Dozens.
- Choose an ordered pair.
- Substitute the ordered pair into the equation and simplify.
- What is the relationship between the slope and x ?
- Are the slope of the line and the constant proportionality the same value?
- How would you write an equation in the form of $y=mx$, if $m=2$.

$$Y = \left(\frac{5}{2}\right) X$$

- Is the equation true.

...derive the equation $y = mx + b$ for a line intercepting the vertical axis at b (*y-intercept*).



- ❧ If a line intersects the origin, what is the initial value?
- ❧ What if the initial value is not $(0,0)$?
- ❧ Consider the following problem...Mike earns a base salary of \$500.00 per week plus a 20% commission on all sales. Define x , the total sales, such that,
 $0 < x < 2500$.
- ❧ How does this graph look different from previous graphs?
- ❧ Where does this graph intersect the y -axis?
- ❧ How does this affect the equation of the function?

Other Resources on Ratio and Proportions



8th Grade Unit -
8.EE.5-6 and 8.F.2,4

<http://effectiveness.ed.sc.gov/content-knowledge/scripts/stemstandards.cfm>

Activity Six: Connecting Ideas



Concept Map

http://scde.mrooms.org/file.php/1/Teacher_Effectiveness/TE-_Mathematics/Secondary_Resources_List.pdf

Resources



- ❧ Lamon, Susan J. *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers. 1999. Print.
- ❧ Van De Walle, John A., and LouAnn H. Lovin. *Teaching Student-Centered Mathematics Grades 5-8*. Vol. 3. Boston, MA: Pearson Education, 2006. Print.
- ❧ Van de Walle, John A. *Elementary and middle school mathematics: Teaching Developmentally*. (6th ed.) Boston, MA: Pearson Education, 2007. Print.
- ❧ SEDL <http://secc.sedl.org>
- ❧ Materials to support implementation for grades 6-8
http://www.livebinders.com/play/play_or_edit?id=380192

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