SOUTH CAROLINA
STATE DEPARTMENT
OF EDUCATION

# South Carolina Department of Education Support for Implementing the Common Core State Standards for Mathematics

Rebecca L. Severin
rseverin@laurens55.k12.sc.us

June, 2012 – Regional
K-2 Content Focus
Add/Sub Fluency
Analyzing Word Problems

# Introductions

**Presenter**

**&**

**Table Teams**

**We will be looking at lessons through the lenses of a young child. To get us ready for this mind set, introduce yourself to your table team members. Let them know where you work, your position, and what you remember about math when you were about 5-7 years old!**

# *This one day institute is designed to….*

- **Provide/enhance teacher content knowledge and implementation strategies for DITs in districts that have begun transition to the CCSS (content institute)**

## *Purpose for Today's Session is to Understand. . .*

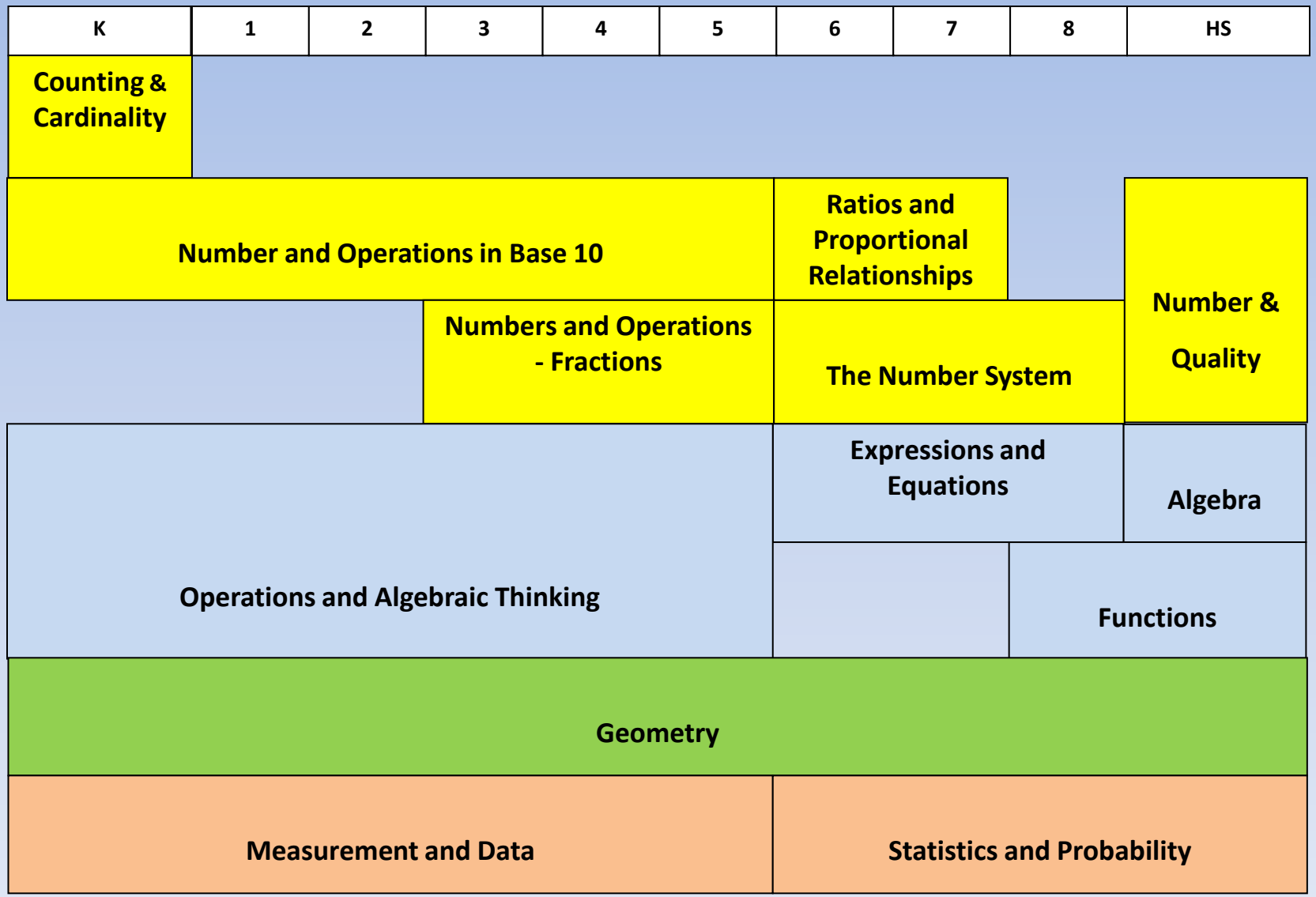
**1. The CCSS require important *prerequisite* work at levels of cognitive demand in order for students to be successful in addition and subtraction.**

**2. Lessons aligned to CCSS K-2 Operations and Algebraic Thinking that builds on developing *strategies* for operations using Mathematical Practices.**

# Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

# Domain Progression



# Early Numeracy

- **Howden (1989) described number sense as a "good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms".**
- **Unfortunately, *many textbooks move directly from beginning ideas to addition and subtraction algorithms, leaving students with a limited collection of ideas about numbers.***
- **The *result is* that children often continue to count by ones to solve simple story problems and have difficulty mastering basic facts. Early number sense requires significant attention in K-2 programs.**

# Fluency – What is it?

**At your table, discuss the idea of fluency as it relates to mathematics. Be ready to share with the whole group.**

- **Fluency with numbers is *often compared* to knowing basic facts**
- **Fluency is *MUCH* more than fact recall**
- **Fluency is knowing how a number can be composed and decomposed and using that information to be flexible and efficient with solving problems**

# Let's start with the end in mind..

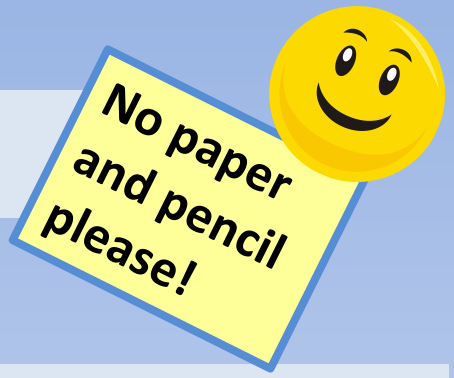
| Operations and Algebraic Thinking 2.OA |
| --- |
| **Add and subtract within 20.** |
| **2.OA.2 Fluently add and subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers.** |

**"Acquiring proficiency in single-digit arithmetic involves much *more* than memorizing."**

**What is that *"more"* ?**

Adding It Up, NCR, 2001

# Mentally solve:




$$5 + 7 + 8 =$$

- **Share how you solved this problem with a partner.**
- **Discuss strategies with your table group. Chart the different strategies.**
- **Did you solve it the same way? How was it different?**
- **Share with the whole group.**

$$5 + 7 + 8 =$$

$$5 + (5 + 2) + 8 =$$

- Knowing that 7 can be decomposed into 5 + 2 in order to think about the problem as 5 + (5 + 2) + 8 allows access to making quick tens.
- The problem can be solved as (5 + 5) + (2 + 8) through the associative property.
- In the primary grades, students work with fluency for numbers 1 through 5 and then numbers 6 through 10. Later they work to 20.

# Fluency

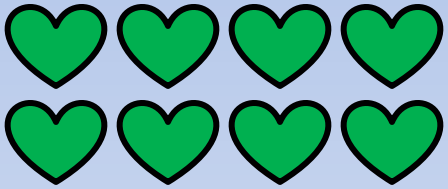
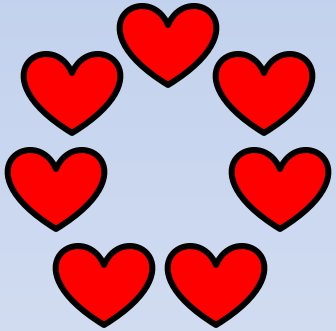
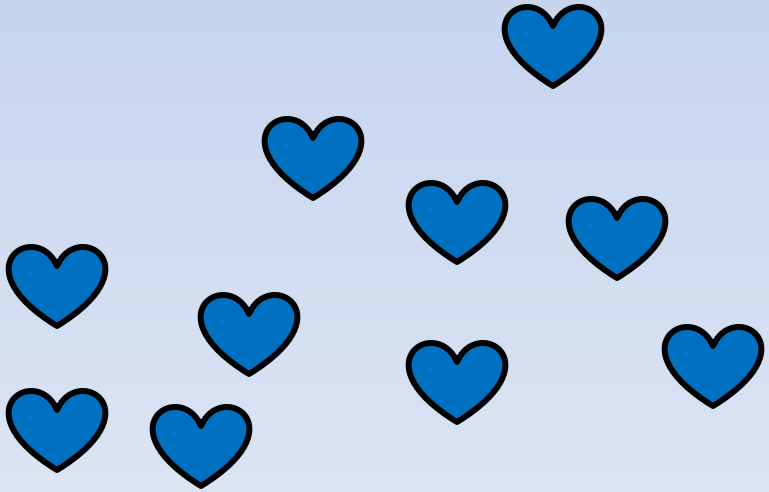
- **The word fluent is used in the Standards to mean "fast and accurate."**
- **Fluency in each grade level involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., "adding 0 yields the same number"), and *knowing some answers from the use of strategies.***

# Fluency Begins with the Counting and Cardinality Domain for Kindergarten

- **Prerequisite to Basic Operations**

- **Includes: reading and writing numerals, counting orally, rote counting, counting on and counting back, understanding quantity, counting sets of objects accurately**

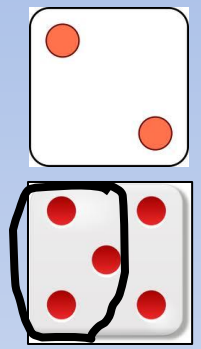
# Counting and Cardinality Progression

- Saying the counting words
- Counting objects
  - ✓ Lines 
  - ✓ Rectangular Arrays 
  - ✓ Circles 
  - ✓ Scattered Configurations 

**Which set of objects is easiest to count? Hardest? Discuss with table group.**

# CC Progression Continued

- **Counting out a given number of objects**
- **Subitizing**
  - **Perceptual**
  - **Conceptual**



- **Counting On**
- **Base–ten structure** from spoken number words → written numerals → understanding base-ten system
- **Comparison** by *matching* → comparison by *numbers* → comparisons involving *addition and subtraction*

# Subitizing

- Young children should experience using patterns of dots initially up to 5 and should associate the patterns to numbers, finger patterns, bead strings, or manipulatives

- This is a skill that helps in the development of number sense, conservation, counting on as well as composing and decomposing numbers

# Before we begin our explorations…

**As a group, decide who the table "getter" will be.**

**The person who is the table "getter" will be responsible for:**

1. **Getting materials/and or handouts**
2. **Collecting and returning materials**

# Session 1 -
# Table "getters" may….

- Get bag of materials labeled "CC" - 1 bag per 2 people sitting at your table and a number line handout (1 per 2 people).
- For web participants, materials are listed on the "Materials Needed for June Content Session p. 1-2" Upload (Session 1).

# Dot Card Explorations

## Whole Group Activity

- Dot Flash

Get ready for a flash….

# Perceptual

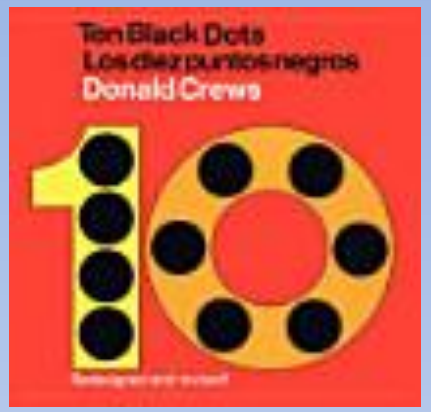
**Names the quantity without counting**

# Background Information Subitizing

- Introduces the basic ideas of cardinality
  - How many
  - More and less
  - Parts and wholes
  - Ideas of quantity

Clementes & Sarama, Learning and Teaching Early Math, 2009

# Literature Connection: Ten Black Dots




(top of p.2 handout)

- Engages students to think mathematically
- Sets the context for further explorations
- "Before, during and after" approach can be useful to use in developing math concepts

Adapted from Mastering Basic Addition and Subtraction Facts K-2

# Explorations…..

**Remember to think about this through the eyes of a 5-7 year old. One exploration could take several weeks depending how you decide to present it!**

# Dot Explorations – Part 1

**Using the dot cards provided for you, engage in the explorations**

**(listed on p. 2 of your handout):**

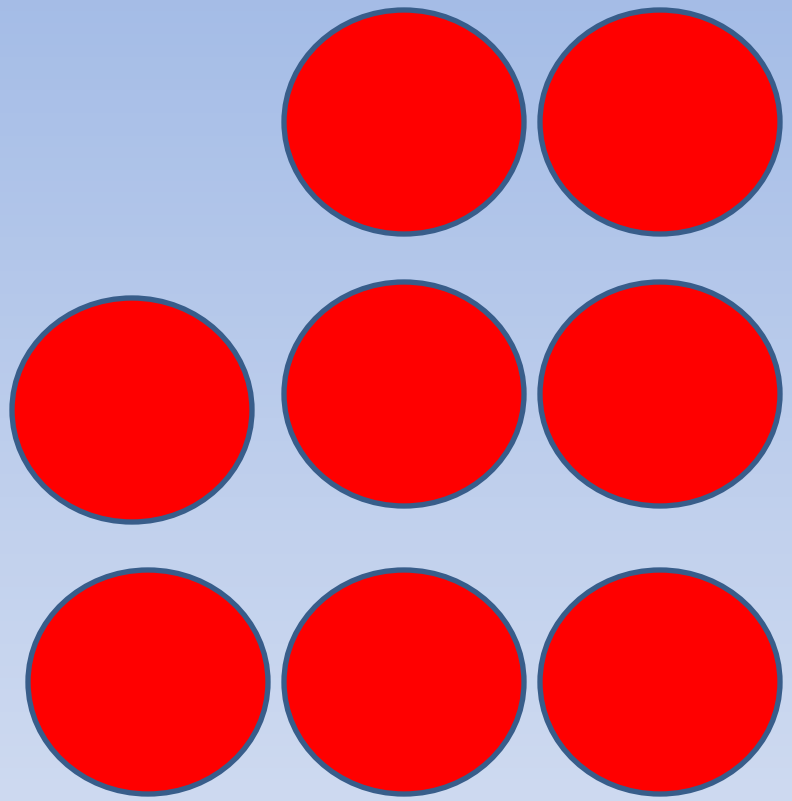
- **Dot Card Flash – Green Cards #1**
- **Dot Card Cover #2**
- **Dot Card Die Match #3**

****Think about the mathematical ideas that are present in the activities.**

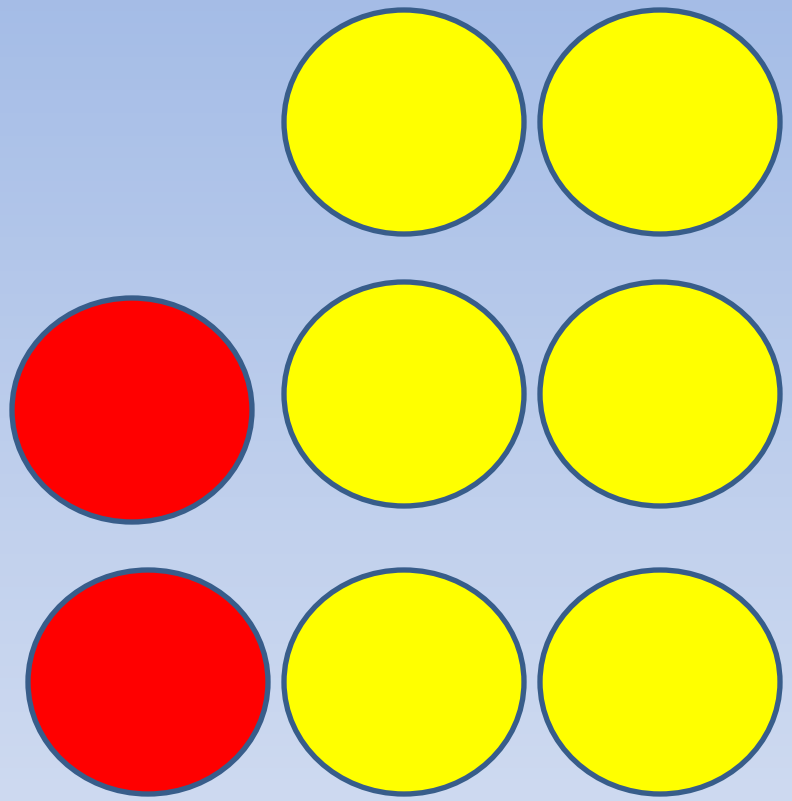
****How might they add in the development of operations for addition and subtraction?**

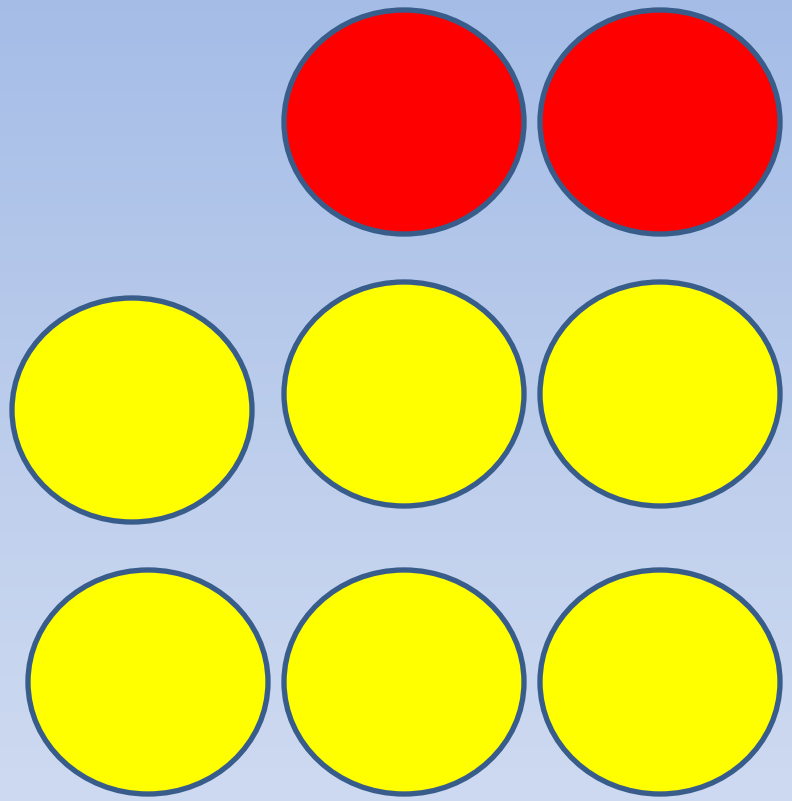
- **Most students can tell you the number of objects/dots up to 3-5 without having been taught.**
- **Students need experiences with larger dot combinations so that they will be able to *count on and begin to decompose larger numbers.***

**GET READY FOR A FLASH!!!!**



**Discuss the strategy you used to find the total number with your table group.**



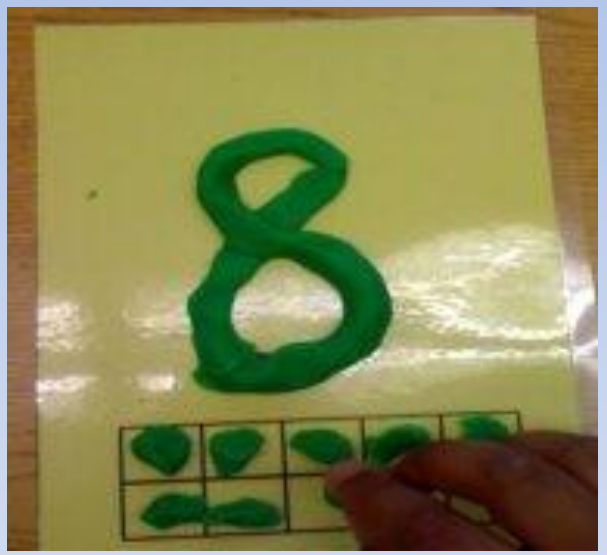


**Conceptual – Recognizes a collection of objects can be composed into 2 sets and quickly finding the cardinality of the whole collection.**

# Dot Card Explorations for Larger Numbers – Part 2

**Using the dot cards provided for you, engage in the explorations (listed in the handout p. 3).**

- **Dot Card Flash #4**
- **Dot Card Count #5**
- **Compare Dots #6**



**How do the explorations in Part 1 and Part 2 of Dot Card Explorations help children build a foundation for addition and subtraction?**

# Connect number symbols to dots with simple activities.




**Once students understand the numeral symbol relationship to the corresponding amount, they can begin representing by drawing and recording in their math notebooks.**

# Laying the Foundation in K

## Operations and Algebraic Thinking K.OA

**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** Fluently add and subtract within 5

**K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings and record each decomposition by a drawing or equation (e.g., 5=2+3 and 5=4+1).**

**K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.**

# One and Two More, One and Two Less

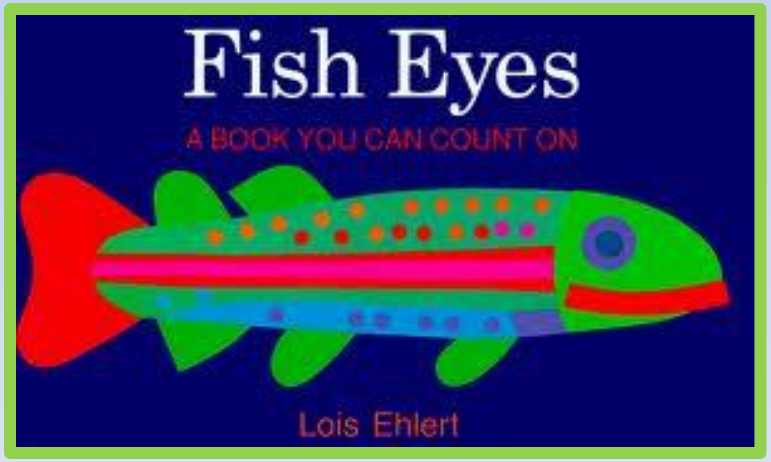
- When counting, children don't usually reflect on the way one number is related to another.
- Their goal is to match number words to objects until they reach the end of the count.
- To learn that 6 and 8 are related by the corresponding relationships of "two more than" and "two less than" requires reflection on these ideas.
- Counting on (beginning with 1 or 2) is a useful tool in constructing these ideas.

# You will need unifix cubes…

# From Counting to Counting On

**Progression of Counting on is included on p. 4 of the handout. Understanding this progression is important to build the foundation for later grades.**

**Literature:**
**While the story is reread, work with a partner and add cubes to her/his fingers as the fish are added on…..**




**Complete exploration #8 and #9 p. 5**

Adapted from Mastering the Math Facts In Addition and Subtraction K-2

**You will need a cup and 8 unifix cubes…..**

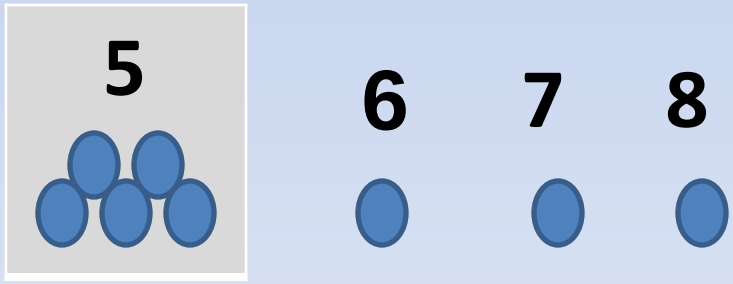
# Whole Group Exploration

**p. 5 of Handout**

**Count out 5 cubes and place in the cup. Place 3 cubes beside the cup.**

**Say, "How many are there?"**

**So let's count together…five (point to cup and count slowly), six, seven (move counters as you count)**




**•This gives students practice counting from a given number other than 1.**

Adapted from John A. Van de Walle's Elementary and Middle School Mathematics: Teaching Developmentally

# Advanced "Counting On"

- **Students begin counting the *words rather than the objects*. Number words become objects to students: 9 + ___ = 13?**

  **Niiiiine, ten, eleven, twelve, thirteen**

  **1 2 3 4**

- **Students keep track of how many things counted on with fingers, mental images, or physical actions such as head bobs.**
- **Counting on can be used to add (find a total) or subtract (find an unknown addend)**

**You will need a yellow number die and yellow dot die…**

# Counting From a Given Number –

**Roll the yellow number die and record the number on your recording sheet bottom of p. 5.**

Recording
2 +

**•Roll the yellow dot die and record the dots on your recording sheet. Solve by counting.**

2 + ● ● ● = 5

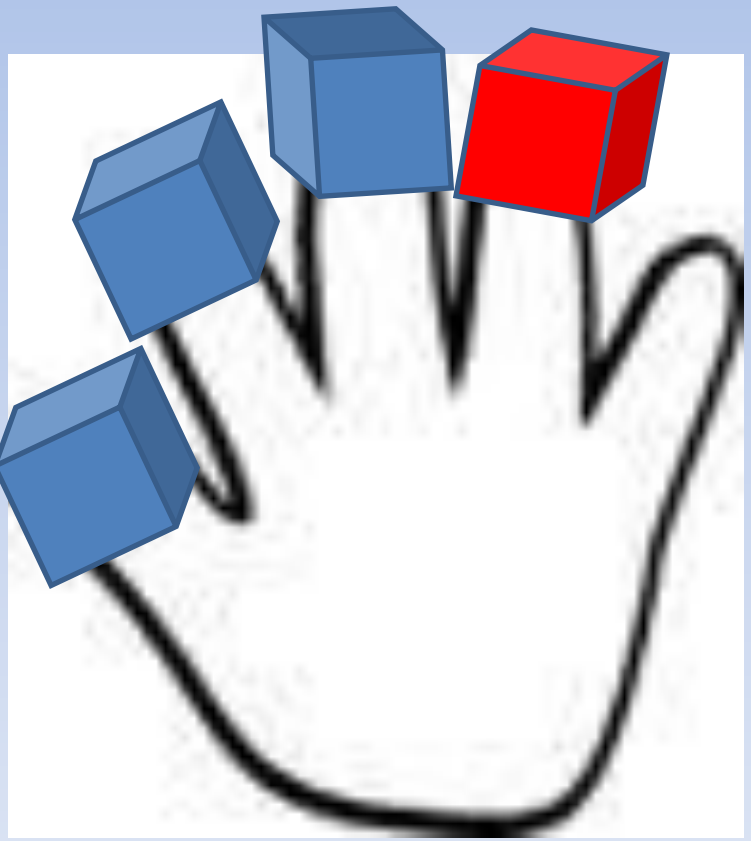
3 4 5

**Complete # 10, p. 5**

# Classroom Discussions and Visual Representations

- Helps develop oral skills which is a prerequisite to writing and representation
- Draws attention to the math concept that is the focus
- Students can learn from their classmates if strategies are shared

# Classroom Charts to Aide Discussion



3 and 1 more is 4

# Relevant Experiences- Counting On

**Provide experiences in the classroom that naturally develop counting on…**

- **Snack time – "Count one more using gold fish crackers"**
- **Calendar – "One more day"**
- **Line – "One more student may line up"**
- **Discussion/Questioning- "I wonder how many I will have if I add 1 more/2 more?"- students discuss and share answers**

# Reflection

- **Silently read and think about the questions on p. 6 of the handout about Counting/Cardinality, Operations, and the Mathematical Practices**

- **Share your thinking with your table partners.**

# Early Number Sense

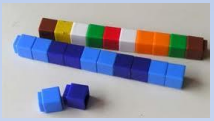
- **Provide experiences that build ideas about numbers and number relationships**
- ***Many textbooks move from early ideas to adding and subtraction***
- **Children who do not have a variety of experiences dealing with numbers often continue to count by ones and have difficulty mastering basic facts.**

# Organizing Materials –CC Bag

- Make sure all materials used are placed in the proper bag labeled CC

## List of materials:

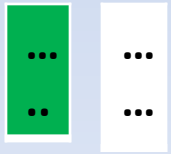
2 wooden cubes (one black dots, one orange dots)

Unifix cubes (approximately 25) 

Yellow cardstock with 1 x 5 array 

Numeral cards 3,3,4,4,5,5 


Yellow dot die, yellow number die 

Green and white dot cards 

5 circular 2-sided counters 

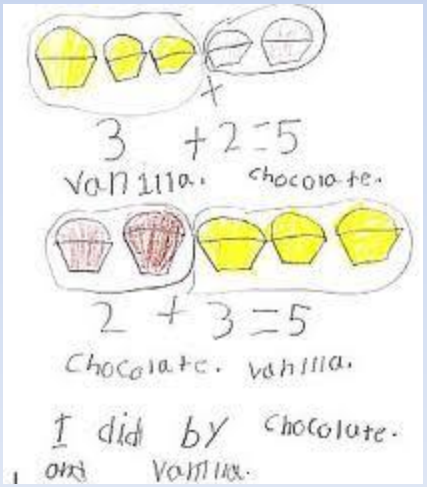
Small plastic cup

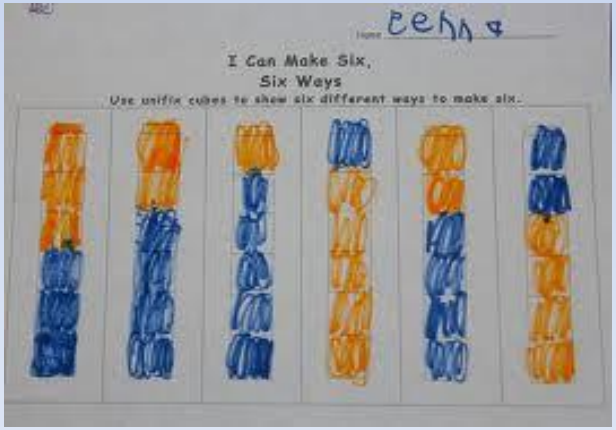
# Prepare for Session 2…Table "getter"…

- Make sure all materials are put in the CC bag correctly (for future sessions) and return.
- Collect bag labeled #2 and give to table group members. Collect 1 bag per 2 people.
- For web participants, materials are listed on the "Materials Needed for June Content Session p. 3" Upload (Session 2).

# Basic Ingredients of Part-Part-Whole Explorations

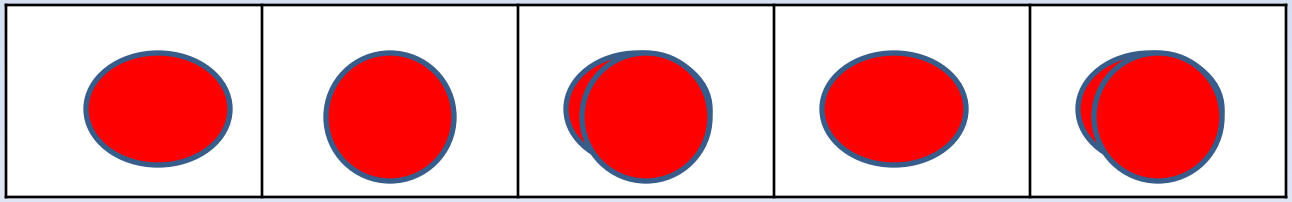
- After silently reading p. 7, "Basic Ingredients of Part-Part-Whole Explorations", discuss the important mathematical ideas embedded in Part -Part-Whole types of problems with your table members.
- Whole group sharing



# Five-Frames

- **Focuses on the relationship to 5 as an anchor for numbers**
- **Prerequisite for using Ten-Frames and Double Ten-Frames**
- **Initially use a blank five-frame and have students *place counters* randomly on the frame. Then they *describe* how they see their number in the frame (Where's the location?).**



**You and a partner will need 1 five-frame and counters….**

# Five-Frame Exploration:

(p. 7 of handout)

- **Be ready to describe the location of the counters in your five-frame to your partner.**
- **Place 4 counters randomly on your five-frame and describe the location to your partner.**

***Students may describe their five-frame as…It has a space in the middle, there are 2 counters, a space and then 2 more counters.***

- **How many more do you need to add to make 5?**

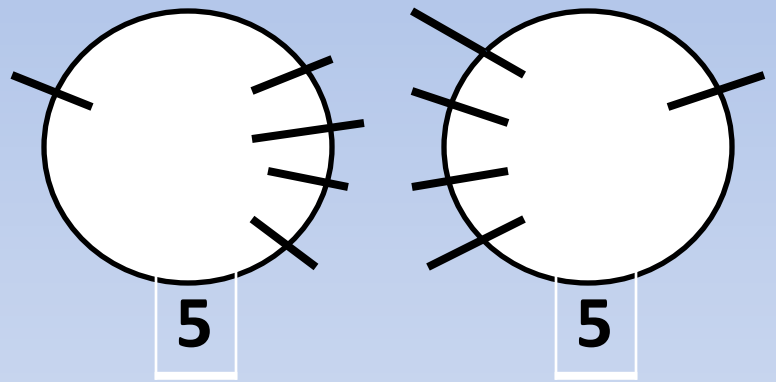
**You will need the number bracelet 5….**

# Modeling with a Number Bracelet

(p. 9 of handout)

- **Hold the number bracelet 5 in front of you.**
- **Move 1 bead to the left and 4 to the right.**
- **What do you see for 5?**
- **Now flip the number bracelet so that 4 beads are to the left and 1 bead is to the right.**
- **What do you see for 5?**


**Flip the bracelet over from left to right (1 + 4) and you should see 4 + 1.**



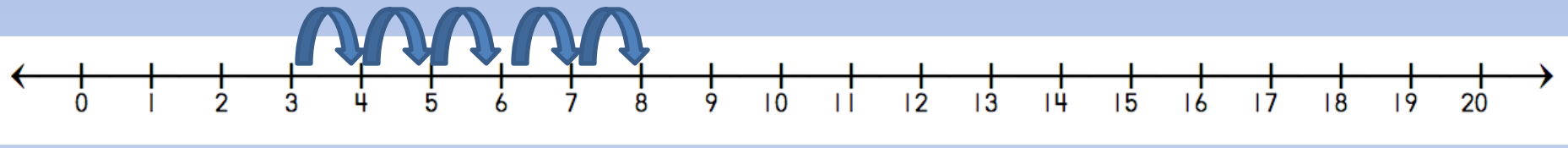

**1 + 4 = 5** **4 + 1 = 5**

- **Record all the combinations For 5 on p. 9 of the handout**
- **Compare your work with your partner. Any differences?**

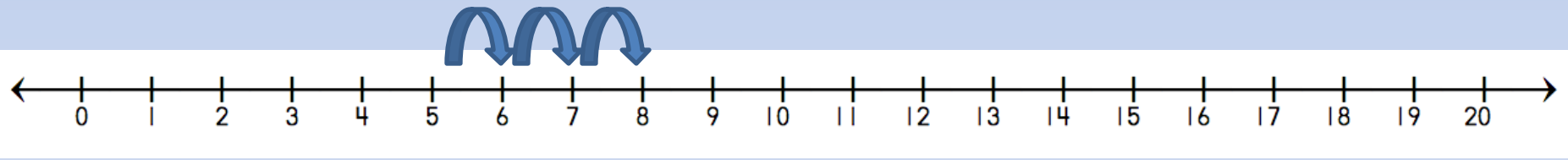
**Be ready to record on the number line….**

# Number Lines-

**Show 3 + 5 on the number line.**




**Show 5 + 3 on the number line.**




- **How are these 2 problems alike?**
- **How are these 2 problems different?**
- **How could you show that subtraction is related to the addition problems on the number line?**

# Pause and Reflect

- **With your partner, count out 10 objects and place in front of you.**
- **Any student who can count one-to-one can count out 10 objects.**
- **What is significant about the experiences is what it did not cause you to think about.**
- ***Nothing in counting a set of 10 objects will cause a child to focus on the fact that it could be made of two parts.***

# Part-Part-Whole Relationships

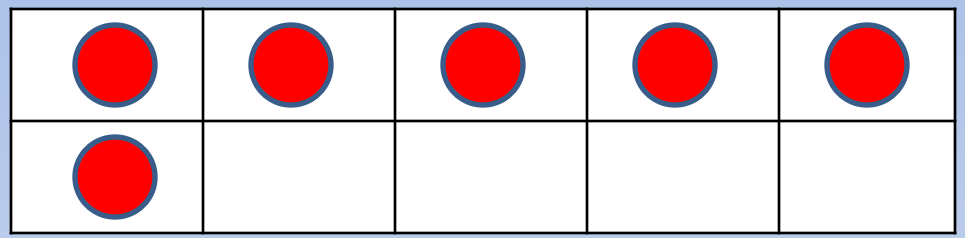
- **Separate the counters into two piles and reflect on the combination…**
  - **You could have separated your counters in a group of 5 and 5, 2 and 8, 7 and 3, 2 and 1 and 7….. Make a change in your two piles of counters and say the new combination to yourself.**
- **Resnick (1983), has stated that a major conceptual achievement of young learners *is the interpretation of numbers in terms of part and whole relationships.***

# Anchoring Numbers to 5 and 10

- We want to help children relate a given number to other numbers, specifically 5 and 10.
- These relationships are especially useful in thinking about various combinations of numbers.
- Consider how the knowledge of 8 as “5 and 3 more” and as “2 away from 10” can play a role: 5 + 3, 8 + 6, 8-2, 8-3, 8-4, 13-8.

# Five-Frame to Ten-Frame

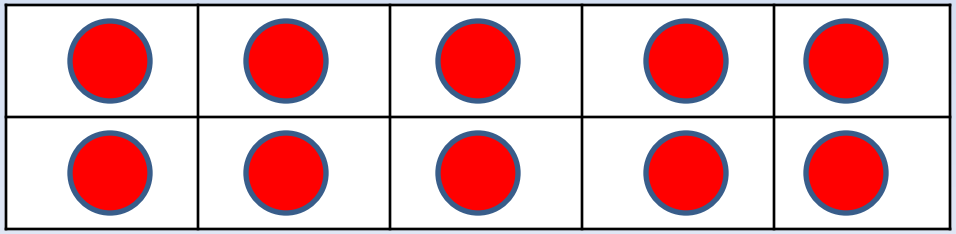
**To introduce ten-frames, play a ten-frame version of "Where's the location?"**



**Introduce the following rule for showing numbers on the ten-frame:**

**Always fill the top row first, starting on the left, the same way you read. When the top row is full, counters can be placed on the bottom row, also from the left.**

John A. Van de Walle's Elementary and Middle School Mathematics : Teaching Developmentally



**Following this rule will help in later explorations for add/sub.**

# Ten-Frames

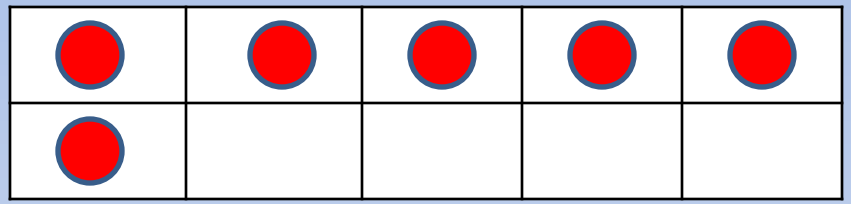
- For a while, many children will continue to count every counter on their ten-frame.
- Some will take all counters off and begin each new number from a blank frame.
- Others will soon learn to adjust numbers by adding on or taking off only what is required.
- With continued practice, all students will grow.

John Van de Walle's Elementary and Middle School Mathematics: Teaching Developmentally

**You each will need 1 ten-frame and 6 counters….**

# Ten-Frame Explorations

- **Place 6 counters on the ten-frame starting at the top left side.**



- **Describe what you see in the ten-frame.**
- **How could you represent what you see with numbers?**

5 + 1 + ___ = 10

6 + ___ = 10

10 – ___ = 4

10 – 4 = ___

10 = 6 + 4

10 = 4 + 6

**Explore Who Has More? #13 on p. 10 of Handout. Find "Who has more?" bag.**

# Who has more?
# Variation:
# Who has less or fewer?

**This is a challenging concept to young children. They must be provided a lot of opportunities exploring objects and using the vocabulary more, less(fewer) or equal to.**

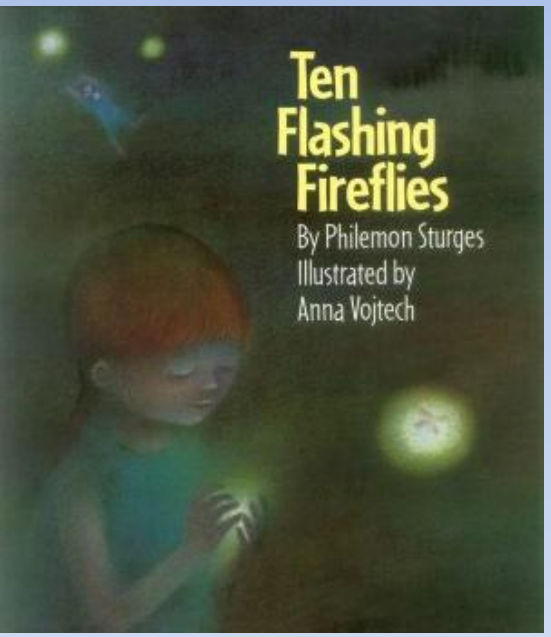
**Susan has 3 fewer coins than Jerika. Susan has 7 coins. How many coins does Jerika have?**

# Who has more cards…

**Please place the green “Who has more?” cards back in the small bag labeled “Who has more?” for future sessions. Soon you will be using the white ten-frame cards. Put aside until later. Thank you!**

# Composing and Decomposing

**(bottom of p. 10)**




**Read Aloud-What do you think this story is about?**

**With your students, you would reread the story and record the fireflies that are caught and put in the jar!**

# Read Again and Chart Actions

| Fireflies In Jar | In the Summer Night |
|---|---|
| 0 | 10 |
| 1 | 9 |
| 2 | 8 |
| 3 | 7 |
| 4 | 6 |
| 5 | 5 |
| 6 | 4 |
| 7 | 3 |
| 8 | 2 |
| 9 | 1 |
| 10 | 0 |

**Seeing combinations for numbers is a prerequisite to fact fluency!**

**10 = 0 + 10**
**10 = 1 + 9**
**10 = 2 + 8**
**10 = 3 + 7**
**10 = 4 + 6**
**10 = 5 + 5**
**10 = 6 + 4**
**10 = 7 + 3**
**10 = 8 + 2**
**10 = 9 + 1**
**10 = 10 + 0**

# Part-Part-Whole
# Connects to Add/Sub Ideas

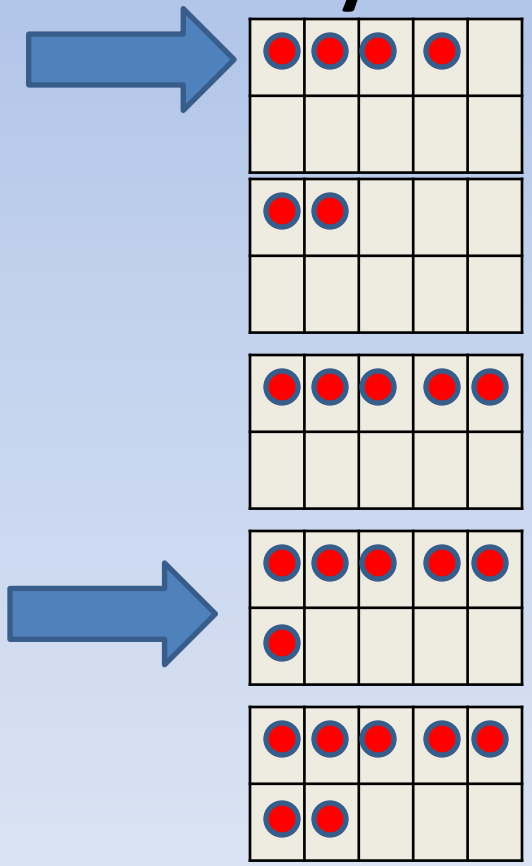
- **It is important that students build (compose) the designated quantity in two or more parts, or they can start with the full amount and separate it into 2 parts (decompose).**
- **Having students read or write the combinations of numbers serves a means of encouraging *reflective thought focused on the part-whole relationship.***
- **Writing can be in the form of drawings, numbers, or equations.**

# 10 is a Landmark Number

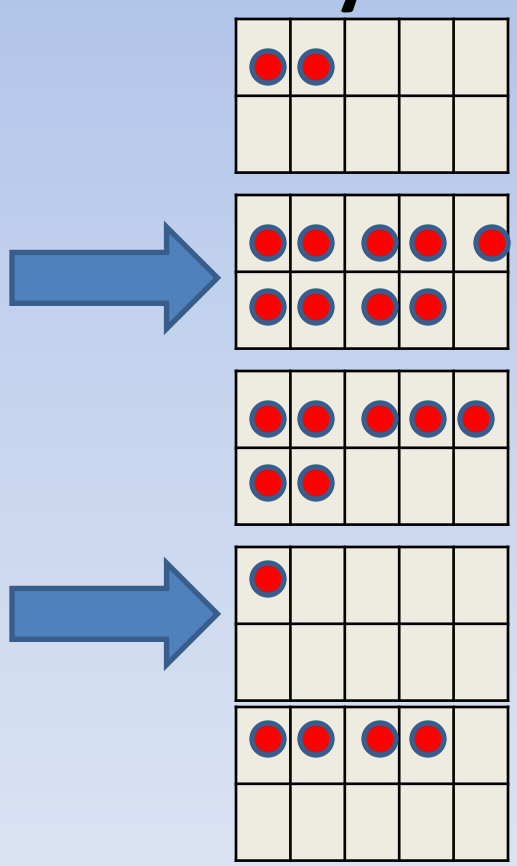
p. 11 of Handout

**Find small bag labeled "10 is a landmark #" and shuffle cards. Pass out 5 cards to yourself and 5 cards to your partner…**

## Player A

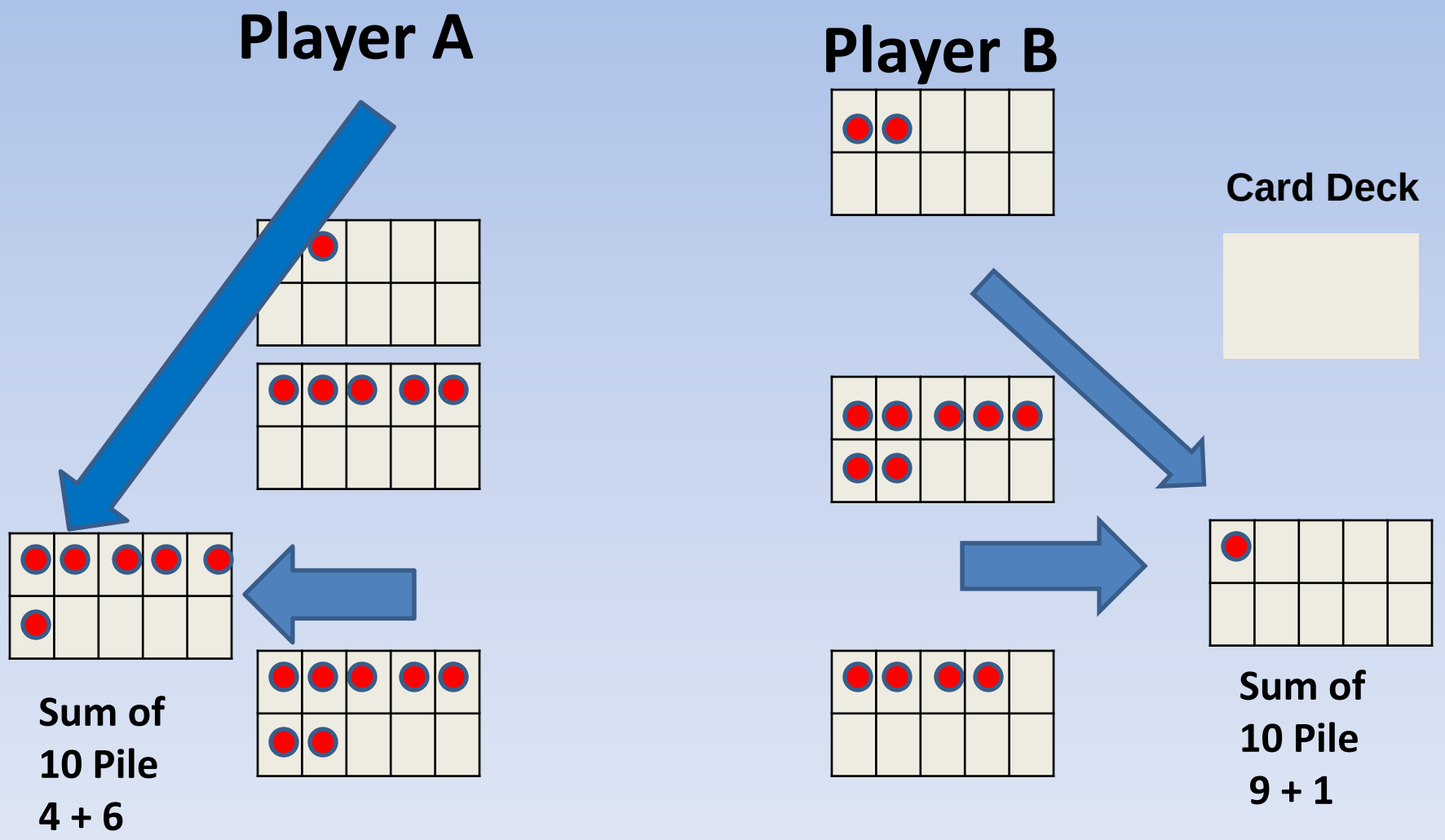


## Player B



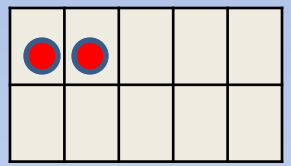
**Extra Card Deck Face down…**

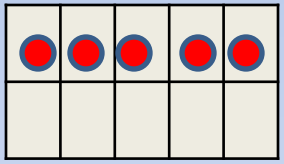
# 10 is a Landmark Number

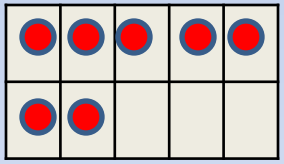


# 10 is a Landmark Number

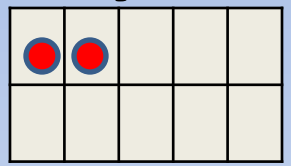
## Player A

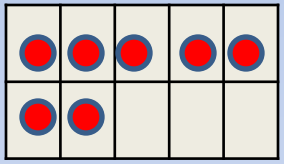


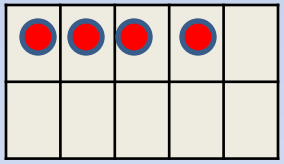




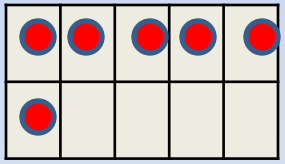
## Player B







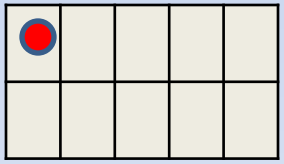
Card Deck



Sum of
10 Pile
4 + 6

**Now you are ready to begin!**

- Read remaining game directions and play on p. 11 under "Now you are ready to begin!"
- Play for 5-10 minutes and discuss with your partner how this helps in developing a sense of ten. Record on p. 12.



Sum of
10 Pile
9 + 1

# 10 is a Landmark # Cards

**Please place “10 is a Landmark #” cards back in the small bag for future sessions. Thank you.**

# Writing the Equation

p. 13 of Handout

- **Complete exploration #14 with a partner using the 1 set of ten-frames (white cards).**
- **Record the equation(s) on the recording sheet on p. 13.**
- **Are your equations different or alike from your partner?**
- **How could Ten-Frames be used to show the reverse of addition and subtraction?**

# White Ten-Frame Cards

**Please place the white ten-frame cards in the small bag labeled “Who has more?” for future sessions.**

# Missing Parts

**Silently read p. 14 of Handout and exploration #15-#16**

**How do these explorations aide in subtraction and/or addition?**

- **Provides reflection on the combinations of a number.**
- **They also serve as the forerunner to subtraction concepts.**
- **With a whole amount of 8 but with only 3 showing, the student can later learn to write:**

**8 – 3 = 5 or 5 + 3 = 8**

# Continuing the Foundation in Grade 1

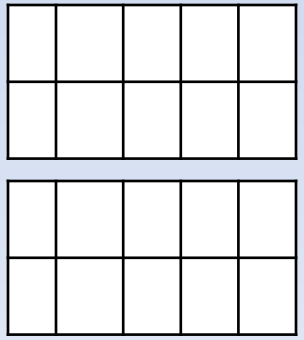
| Operations and Algebraic Thinking 1.OA |
| --- |
| **Understand and apply properties of operations and the relationship between addition and subtraction.** |
| **1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).** |

**Fluently add and subtract within 10**

# Strategies for Single Digit Addition

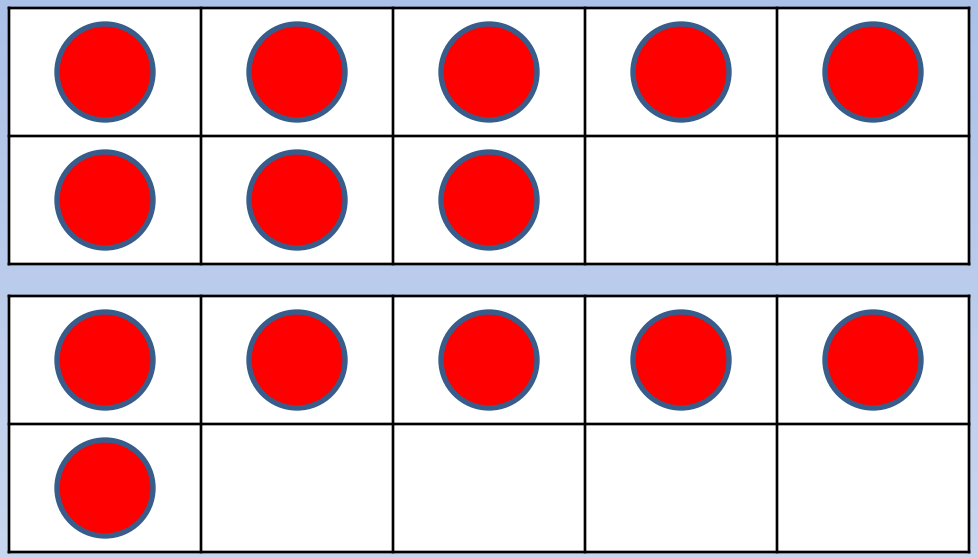
- Use fives
- Make a ten
- Use double
- Use an easier equivalent problem
- Use reverse operations

Each participant will need a
double ten-frame and 14 counters.



# Solving 8 + 6

**Put 8 counters on your first frame beginning at the top left corner and 6 counters on your second frame.**

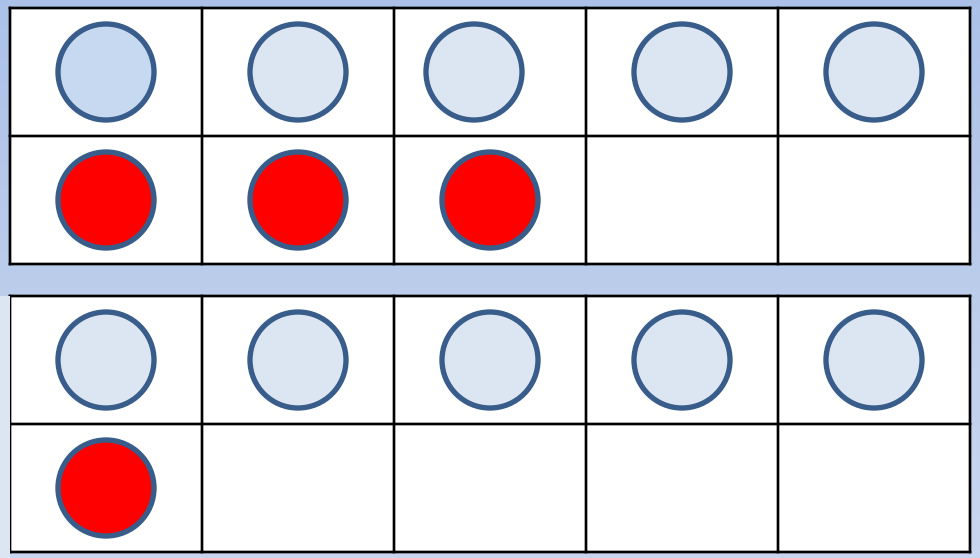


**What earlier explorations helped in the development of using a double ten-frame?**

## Strategies for solving:

- **Use fives**
- **Use a double**
- **Make a ten**
- **Use some other equivalent problem**

# Fives: 8 + 6



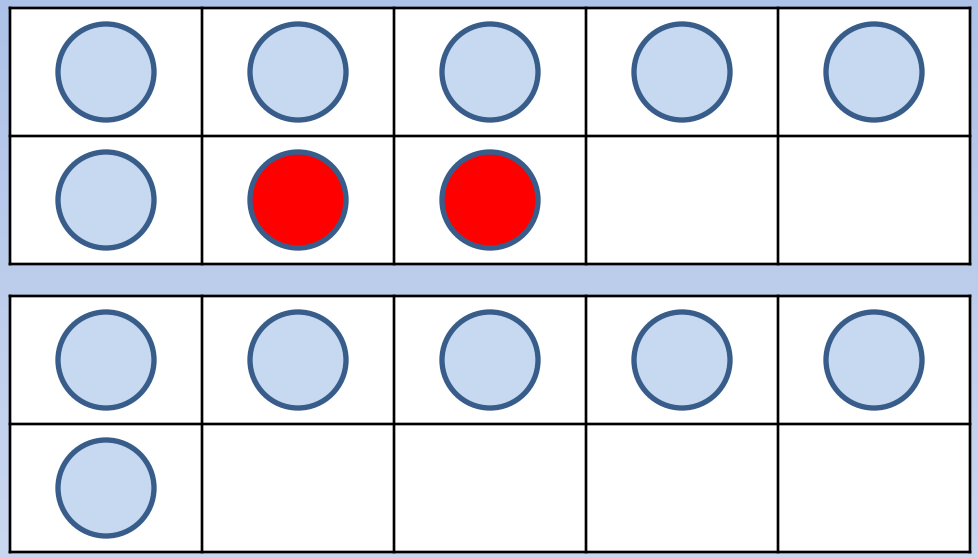
**Can you see some fives?**

**Write an equation(s):**

**Did your equation look like…**
**8 + 6 = 5 + 5 + 3 + 1 = 10 + 4?**

# Use a double: 8 + 6

**What doubles do you see?**



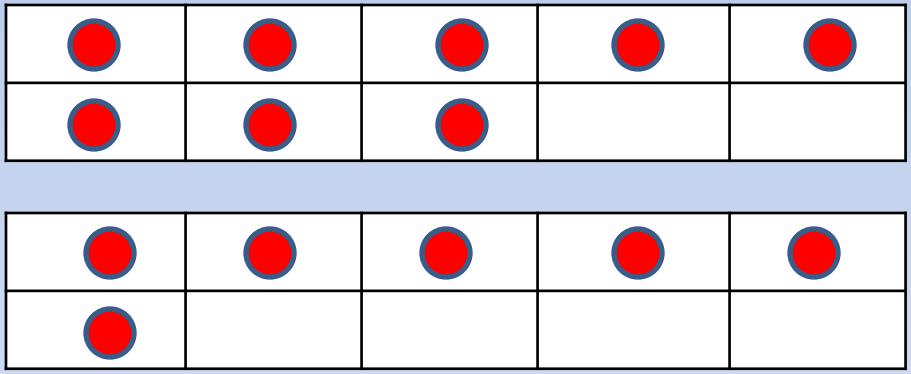
8 + 6

2 + **6 + 6**

# Make a ten: 8 + 6

## 8 + 6 Becomes 10 + 4

**The most common model for exploring this relationship is the double ten-frame.**



**Write an equation: 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14**

**Later similar relationships can be used in the development of mental computation skills on larger numbers such as 68 + 7.**

5 + 9  6 + 7

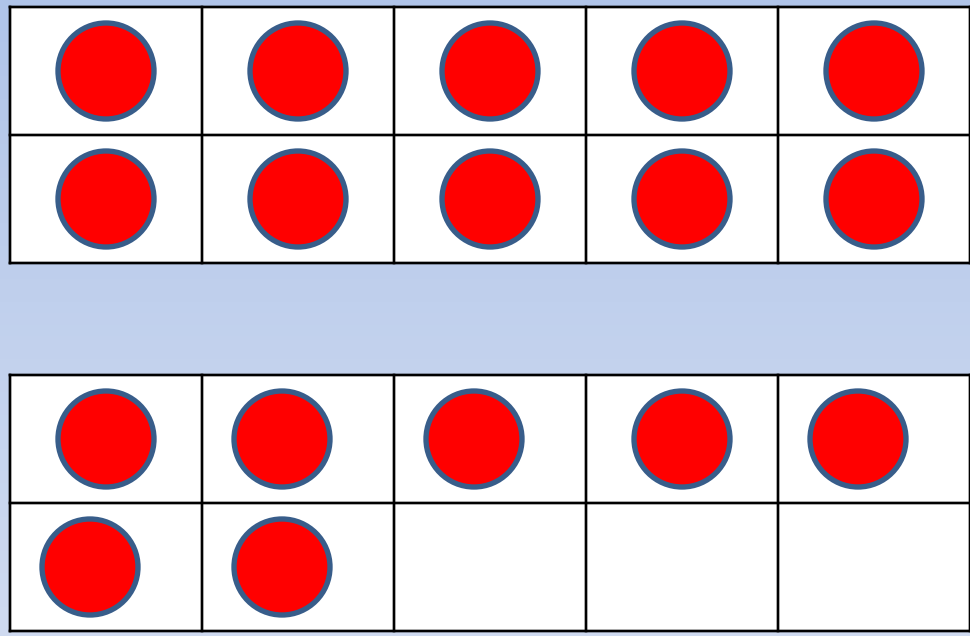
- Select a problem.
- Select a strategy for the group-

  Make a ten, make doubles, use fives, or make an easier equivalent problem
- Use the double ten-frames and counters to reason through the strategy and write an equation.
- Share, compare, and discuss as a group.
- Can you repeat using a different strategy?
- Be ready to discuss the strategy that worked best for each problem.

**With a partner, count out 17 counters and work together using 1 double ten-frame….**

# Decompose to ten: 17 - 8

- Place 17 counters on the double ten-frame by completely filling the top frame and placing 7 on the bottom frame.



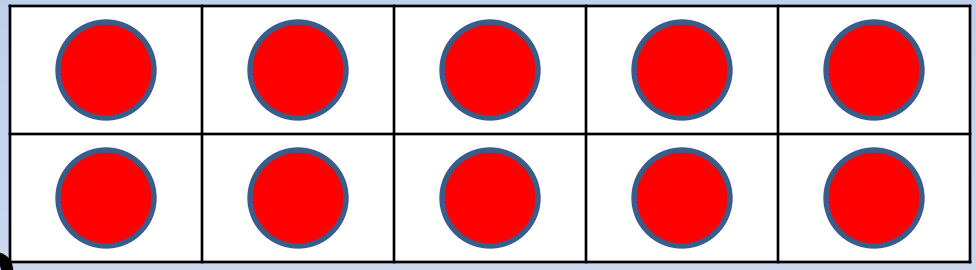
# Decompose to ten: 17 - 8

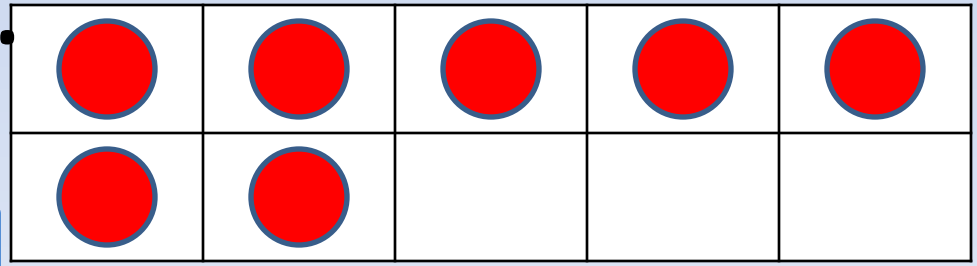
7 1

- How can you remove 8 counters in parts by decomposing it in a way that gets you a ten?
- Remove 7 counters to get to ten (17 – 7 = 10).
- Remove 1 more (10 – 1).
- Write an equation(s).

17 – 7 = 10 -1 = 9

*Or* 17 – 7 – 1 = 9





# Now you try it!: 13 – 6 15 - 6

- With a partner, use the double ten-frame and counters to reason through the "Decompose to Ten" strategy.
- Write an equation(s) to show your reasoning.
- Share and discuss your reasoning and equation(s) at your table. Were they alike? Were they different?

# Fluency Connections K-2

- The deep extended experiences students have with addition and subtraction in Kindergarten and Grade 1 culminate in Grade 2 with students becoming fluent in single-digit additions and the related subtractions using mental strategies as needed.
- So fluency in adding and subtracting single-digit numbers has progressed from numbers within 5 in Kindergarten to within 10 in Grade 1 to within 20 in Grade 2.

- **How have the strategies explored thus far help in developing Operations and Algebraic Thinking by the end of Second Grade?**

- **Partner Share**

- **Whole Group Share**

| **Operations and Algebraic Thinking 2.OA** |
| --- |
| **Add and subtract within 20.** |
| **2.OA.2 Fluently add and subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers.** |

# Organize Materials..
# Please place objects and frames in bag #2 for future sessions.

**Don't forget:**

2 double ten-frames

2 ten-frames

1 five-frame

Approximately 25 counters

1 bag with "10 is a Landmark #" cards

1 bag with "Who has more?" cards

# For the last time!
# Table "getters"….

- **Collect materials in bag #2 and return. Get Story Problem bag #3 and pass out to table members. *Please pass out 1 bag per 2 people.***
- **Partners *initially only* need scissors from bag and 1 sheet of story problems (1-10).**

TABLE 1. Common addition and subtraction situations.[6]

| | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| **Add to** | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?<br>2 + 3 = ?<br>K | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?<br>2 + ? = 5<br>1st | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?<br>? + 3 = 5<br>1st-2nd |
| **Take from** | Five apples were on the table. I ate two apples. How many apples are on the table now?<br>5 – 2 = ?<br>K | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?<br>5 – ? = 3<br>1st | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?<br>? – 2 = 3<br>1st-2nd |
| | | | |
| **Put Together/ Take Apart[2]** | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown[1]** |
| | Three red apples and two green apples are on the table. How many apples are on the table?<br>3 + 2 = ?<br>K | Five apples are on the table. Three are red and the rest are green. How many apples are green?<br>3 + ? = 5, 5 – 3 = ?<br>1st | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?<br>5 = 0 + 5, 5 = 5 + 0<br>5 = 1 + 4, 5 = 4 + 1<br>5 = 2 + 3, 5 = 3 + 2<br>K |
| | | | |
| **Compare[3]** | **Difference Unknown** | **Bigger Unknown** | **Smaller Unknown** |
| | ("How many more?" version):<br>Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?<br>1st<br>("How many fewer?" version):<br>Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?<br>2 + ? = 5, 5 – 2 = ? | (Version with "more"):<br>Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?<br>1st-2nd<br>(Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?<br>2 + 3 = ?, 3 + 2 = ? | (Version with "more"):<br>Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?<br>1st-2nd<br>(Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?<br>5 – 3 = ?, ? + 3 = 5 |

# Examining Simple Word Problems

**Word problems provide a strong foundation for students' conceptual understanding of addition and subtraction. When students encounter addition or subtraction problems based on familiar situations, they are able *to develop their own strategies* for finding the answers. Although we often think of word problems as being more difficult to solve than the basic arithmetic problems, the reverse is actually true.**

Carpenter et al., 1999; Kilpatrick, Swafford & Findell, 2001; Van de Walle, 2009

# Research

**Research tells us that when children are learning to add and subtract, they benefit from simple and straightforward contexts that help them to conceptualize and model the situation.**

Carpenter et al., 1999; Kilpatrick, Swafford & Findell, 2001; Van de Walle, 2009

# With a partner, cut the 10 cards apart-

## Read the story problems and sort the cards into 2 piles: addition and subtraction

| | | |
|---|---|---|
| 1<br>Joe had some candy. His mother gave him 2 more pieces of candy. How many pieces of candy does Joe have now? | 2<br>Susan has 3 gerbils and 2 birds. How many animals does Susan have? | 3<br>Bobby had 5 video games. His sister gave him some more video games. Now Bobby has 7 video games. How many video games did his cousin give him? |
| 4<br>Linda had 6 book markers. She gave 3 of her book markers to her friend Lea. How many book markers does she have now? | 5<br>Sammy has 7 cars: 3 are gray and the rest are purple. How many purple cars does Sammy have? | 6<br>Johna found some crayons. She found 3 more crayons. Now she has 8 crayons. How many crayons did she have to start with? |
| 7<br>Katie has 7 marbles and Johnny has 3 marbles. How many more marbles does Katie have than Johnny? | 8<br>Rachel has 7 gummy bears. Some of them got eaten. Now she has 4 gummy bears. How many gummy bears did she eat? | 9<br>Lori has 8 balls. Sean has 2 more balls than Lori. How many balls does Sean have? |
| 10<br>Lisa ate some pie. She ate 2 pieces of pie. Now there are 8 pieces of pie left. How many pieces of pie was there to start with? | **Record your sorting on the cards**<br>**Addition (+) Subtraction (-)**<br>**Rank the problems by difficulty and sort.**<br>**E-easy, M- moderate, C-Challenging** | |

Adapted from Teaching by Design in Elementary Mathematics

# Compare and discuss at your table….

- What factors did you consider when deciding your rankings?
- What features do the “easier” problems have in common?
- What features do the “challenging” problems have in common?
- Have you used word problems like these with your teachers or students?
- How might the contexts of the problems help students to find the solutions?

**Story Situation:**

**Susie had 6 cars. Her sister gave her some more cars. Now Susie has 8 cars. How many cars did her sister give her?**

**Student: Teacher, I'm stuck. How do you do this problem?**

**Teacher: Do you think you should add or subtract to find the answer?**

**Student: I don't know. Is it a plus problem because the cousin gave him some cars?**

**Teacher: No. no. It's not adding. This is a subtraction problem. You see, you need to take 6 away from 8 because we don't know how many cars Susie's sister gave her.**

**Student: But the story says her cousin gave her some more. That means I should add.**

**Teacher: No, in this problem you need to subtract.**

Adapted from Teaching by Design in Elementary Mathematics

**The natural approach in young children for solving word problems is typically not to view the problems in terms of the operation that might be used to calculate an answer, but to respond to the *action or relationship* and then consider the outcome of that action or relationship.**

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (1999).

**Student: Teacher, I'm stuck. How do you do this problem?**

**Teacher: Do you think you should add or subtract to find the answer?**

**Student: I don't know. Is it a plus problem because the cousin gave him some cars?**

**Teacher: No. no. It's not adding. This is a subtraction problem. You see, you need to take 6 away from 8 because we don't know how many cars Susie's sister gave her.**

**Precludes him from thinking of this problem in terms of the operation of subtraction.**

**Student: But the story says her cousin gave her some more. That means I should add.**

# Action Vs. Relationship

**We will resort these cards according to what researchers have identified as a young child's way of thinking.**

# Action problem –

Can you find a problem card that shows an action…does everyone agree?

Describes a sequence over time that includes a beginning, middle and end.

There is a starting quantity, a change to that quantity, and a resulting amount. Add to/Take From

# Relationship problem –

Can you find a problem card that shows a relationship…does everyone agree?

There is NO time sequence for the information, no change over time, no beginning, middle or end. These problems require the student to make sense of how quantities are related. Consider how parts relate to the whole set or making comparisons.

Put Together/Take Apart/Compare

# Resort: Action or Relationship

**Resort the 10 cards into 2 groups (Table 1 can help):**

1. **Problems that describe an <u>action</u>:**

   **Take to/Take From - Result Unknown, Change Unknown, Start Unknown**

2. **Problems that describe a <u>relationship</u>**

   **<u>Put Together/Take Apart</u> -Total Unknown, Addend Unknown, Both Addends Unknown**

   **<u>Compare</u> - Difference Unknown, Bigger Unknown, Smaller Unknown**

**(Hint: 6 are action and 4 are relationship)**

# Classifying Problems by Difficulty

**With a partner, place the problems in each section in order according to difficulty (from easy to most challenging).**

**Which section(s) contain most of the problems identified as challenging?**

**Start Unknown and Compare problems are very challenging for 1st & 2nd Grade students.**

TABLE 1. Common addition and subtraction situations.[6]

| | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| **Add to** | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?<br>2 + 3 = ?<br>#1 K | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?<br>2 + ? = 5<br>#3 1st | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?<br>? + 3 = 5<br>#6 1st-2nd |
| **Take from** | Five apples were on the table. I ate two apples. How many apples are on the table now?<br>5 – 2 = ?<br>#4 K | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?<br>5 – ? = 3<br>#8 1st | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?<br>? – 2 = 3<br>#10 1st-2nd |

| | Total Unknown | Addend Unknown | Both Addends Unknown[1] |
|---|---|---|---|
| **Put Together/ Take Apart[2]** | Three red apples and two green apples are on the table. How many apples are on the table?<br>3 + 2 = ?<br>#2 K | Five apples are on the table. Three are red and the rest are green. How many apples are green?<br>3 + ? = 5, 5 – 3 = ?<br>#5 1st | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?<br>5 = 0 + 5, 5 = 5 + 0<br>5 = 1 + 4, 5 = 4 + 1<br>5 = 2 + 3, 5 = 3 + 2<br>K |

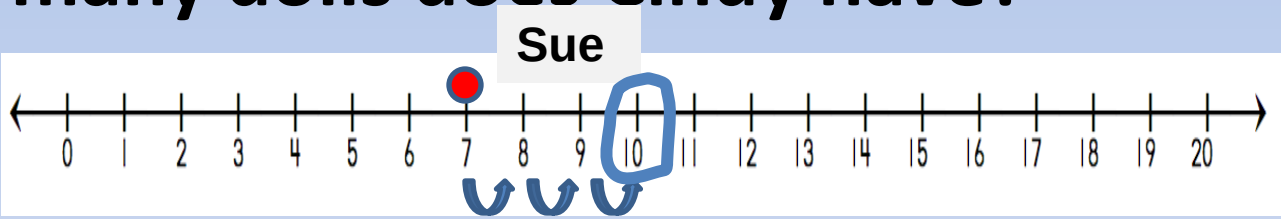
| | Difference Unknown | Bigger Unknown | Smaller Unknown |
|---|---|---|---|
| **Compare[3]** | ("How many more?" version):<br>Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?<br>#7 1st<br>("How many fewer?" version):<br>Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?<br>2 + ? = 5, 5 – 2 = ? | (Version with "more"):<br>Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?<br>#9 1st-2nd<br>(Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?<br>2 + 3 = ?, 3 + 2 = ? | (Version with "more"):<br>Julie has three more apples than Lucy. Julie has five apples. How many apples does Luc<br>1st-2nd<br>(Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?<br>5 – 3 = ?, ? + 3 = 5 |

# Model Concretely, in a Drawing, and with Numbers

**Sue has 3 fewer dolls than Cindy. Sue has 7 dolls. How many dolls does Cindy have?**

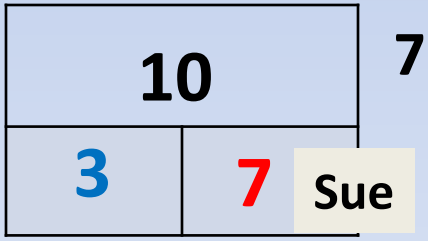
**What is known?**

**Sue has 7 dolls.**




**What is the question?**

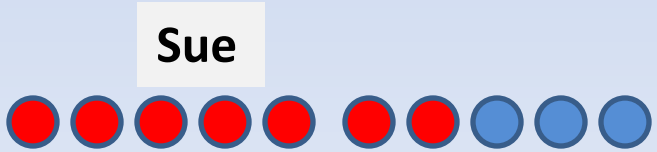
**How many dolls does Cindy have?**




**7 + 3 = 10**

**What other information is there?**

**Sue has 3 fewer dolls than Cindy.**



# Common Core Problem Situations

- **With a partner, choose 2 problems to solve. Decide how you will work together to create strategies to solve the problem. <u>Problem situations, as well as materials to help solve them, are included in your bag.</u>**
- **Represent the problem in at least 2 ways: model, drawing, equation, etc.**
- **Be ready to share your findings with table members.**

# Fluency Again

- **The word fluent is used in the Standards to mean "fast and accurate."**
- **Fluency in each grade level involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., "adding 0 yields the same number"), and *knowing some answers from the use of strategies.***

# Questions?

**Please complete the exit slip for "DIT's Role in Implementing the Common Core State Standards for Mathematics K-2 Content Session"!**

**Thank you!**