SOUTH CAROLINA
STATE DEPARTMENT
OF EDUCATION

# Support for Implementing the Common Core State Standards for Mathematics

# Division of Fractions As Set Forth in Grades 5-6

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SOUTH CAROLINA
STATE DEPARTMENT
OF EDUCATION

# Office of Instructional Practices and Evaluation
# Content Knowledge Team

# "STEM" - Science, Technology, Engineering and Mathematics

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# *What will we do:* ***Teacher Focused***

1. **Take a quick look at SCDE timeline**
2. **Examine content and instructional shifts from 2007 to CCSSM (related to division of fractions)**
3. **Develop conceptual understanding of division of fractions**
4. **Relate division of fractions/problem solving to Table 2 in CCSSM**

**** Decimals**

# Implementation Timeline

**2011 – 2012 Professional Development**

**2012 – 2013 Professional Development**

**2013 – 2014 Bridge Year**

**2014 – 2015 Full Implementation**

# *Compare 2007 to CCSSM*

- **How has content related to fractions shifted?**
- **How may instruction need to shift?**

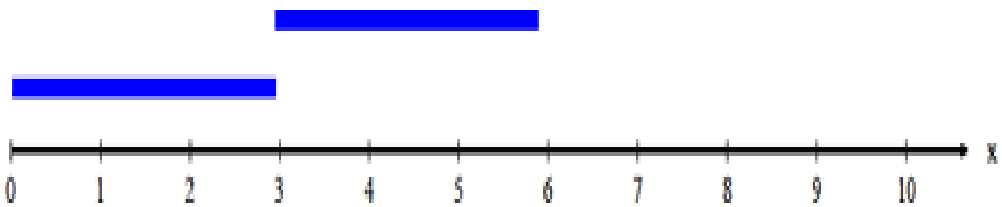
***In CCSSM Success With Division of Fractions Requires. . .***

1. **Conceptual Understanding**
   - **Translate among and relate concrete, pictorial and numerical models**
   - **Explain the relationship between whole number and fractional operations**
2. **Work in a Variety of Models**
   **Linear/measurement, area, number line, and set models**
3. **Contextual Problem Solving**
   **Partitive and Quotative Structures**

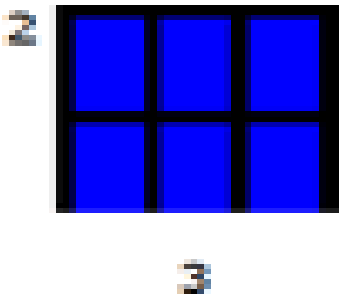
# *Work in a Variety of Models; Quotative or Partitive Structures. . .* *School-wide collaboration important* *3.OA.1-2, 3.NF.1, 3.NF.2a-b, 3.NF.3a-d, 4.NF.1, 4.NF.2, 4.NF.3* *(Habits of Mind)*

***2 x 3 or 6 ÷ 3 (6/3) or 6 ÷ 2 (6/2) ½ ÷ ¼***

***Number Line (Linear) Model***



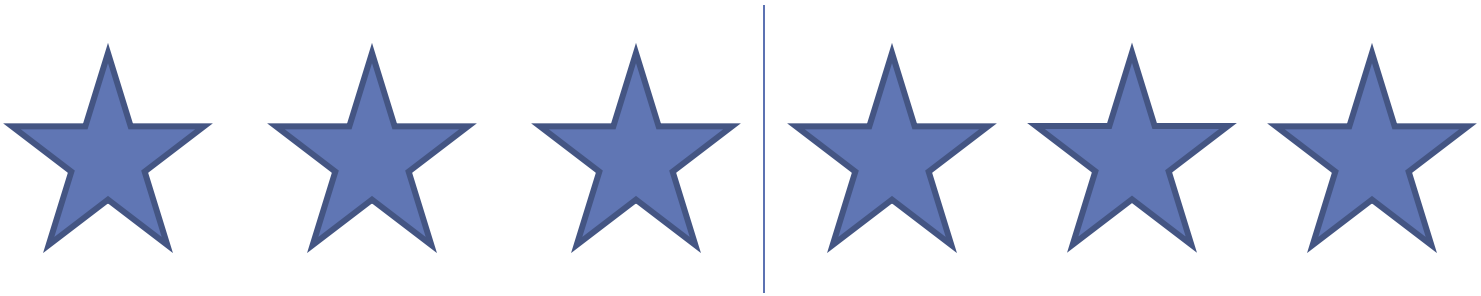

***Area Model*** ***<u>3 x 2</u>*** ***or 6 ÷ 3 or 6 ÷ 2***



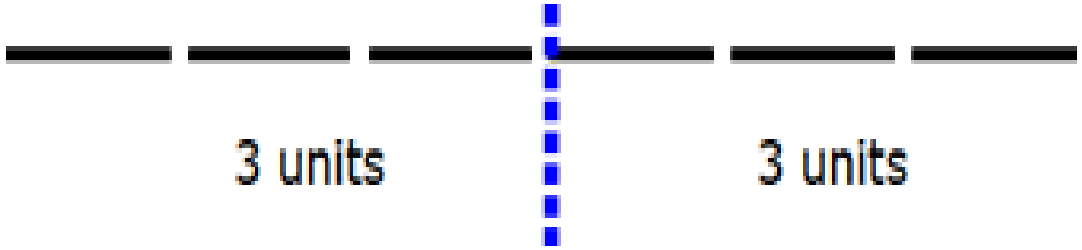
# *Work in a Variety of Models. . .*

## *2 x 3 or 6 ÷ 3 (6/3) or 6 ÷ 2 (6/2)*

***Set Model***



***Measurement (Linear) Model 2 x 3 or 6 ÷ 2 or 6 ÷ 3***



# Pre-Assessment May Be Needed. . .

**Dividing Whole Numbers. . .2007**

**4-2.5 Generate strategies to divide whole numbers by single digit divisors.**

**5-2.2 Apply an algorithm to divide whole numbers fluently.**

**5-2.3 Understand the relationship among the divisor, dividend and quotient.**

**Dividing Whole Numbers. . . CCSSM**

**3.OA.7 Fluently multiply and divide within 100. . .**

**4.OA.3 . . .including problems in which remainders must be interpreted. . .”** **(relationship of divisor, dividend and quotient.)**

# One way to get started . . . (Ruler/Yardstick) 5.NF.7(b)

**Start with easy problems students can reason out.**

**Ribbon 1 inch wide for 1/8 strips. . . # of strips?**

**1 ÷ 1/8 =**

**10 inches of pipe for 1/2 inch connectors. . . # of connectors?**

**10 ÷ 1/2 =**

**6 inch wood stick cut in 1/4 inch lengths. . .# of pieces?**

**6 ÷ 1/4 =**

- **Use the ruler as a number line and change the size of the units or "information".**
- **Use material such as pattern blocks and grid paper for area models and sets.**

# 5.NF.7b 3.OA.2, 3.NF.1

**Grandmother has three acres of land. When she divided it equally among all of her grandchildren, each got 1/2 acre. How many grandchildren does Grandmother have?**

*Write a number sentence to represent the question. Reason a solution, then draw a linear model and an area model to support your reasoning. Put your answer in a complete sentence related to the problem.*

*No free-hand drawing.* *(How many groups of size ½ are in the whole?)*

**If each grandchild gets ½ acre of land and there are three acres to be shared, then there are 6 grandchildren.**

add to list

5.NF.7b

# If this represents one pie

## and this represents the 1/3 share each friend gets,

## *how many friends will one pie feed?*

***Write a number sentence to represent the problem, reason a solution using concrete material, trace the model on paper, label the model to explain your reasoning. Write a final answer in a complete sentence related to the question.***

*(How many groups of size 1/3 are in the whole?)*

**If each friend gets 1/3 of the whole pizza, then 3 friends can be served with 1 pizza.**

Add to list

# One way to get started . . . (Ruler/Yardstick) 5.NF.7(b)

**Start with easy problems students can reason out.**

**Ribbon 1 inch wide for 1/8 strips. . . # of strips?**

**1 ÷ 1/8 =**

**10 inches of pipe for 1/2 inch connectors. . . # of connectors?**

**10 ÷ 1/2 =**

**6 inch wood stick cut in 1/4 inch lengths. . .# of pieces?**

**6 ÷ 1/4 =**

- **Use the ruler as a number line and change the size of the units or "information".**
- **Use material such as pattern blocks and grid paper for area models and sets.**

# 5.NF.7b

**Will there ever be a remainder when dividing a whole number by a unit fraction? Justify your answer.**

**Talk with your neighbor and be prepared to share.**

# 5.NF.7a

**We have 1/3 of a pizza to share equally between 2 friends. How much will each friend get? Write a number sentence to represent the question. Use a model to explain how you know.**

(How much for one? – size of group/part unknown.)

***Use multiplication to check your answer. -- We won't today.***

**2 x 1/6 =**

**2/1 x 1/6 =**

$$\frac{2 \times 1}{1 \times 6} =$$

**2/6 or 1/3**

***The two friends each get 1/2 of the 1/3 of a pizza which is the same as 1/6 of the pizza.***

***Add to list***

# Using a grid overlay. . . 5.NF.7a

**Four students sitting at a table were given 1/3 of a pan of brownies to share. How much will each student get if they share the pan equally?**

- **Write a number sentence to represent the question**
- **Use grid paper to draw a model**
- **Label the model to explain your reasoning**
- **Check your answer using multiplication - We won't today**

**If there is 1/3 of a pan to share equally among 4 students each student will get 1/12 of the pan of brownies.** Add to list

# Using a grid overlay. . . 5.NF.7a

**A strawberry grower will ship 1/5 ton of strawberries in two equal shipments. How much will each shipment weigh?**

- **Write a number sentence to represent the question**
- **Use grid paper to draw a model**
- **Label the model to explain your reasoning**
- **Check your answer using multiplication**

**Resist the temptation to get the answer using an algorithm and then work backwards to determine the model. Reason/experience the solution the way students will have to.**

**If 1/5 ton of strawberries is shipped in two equal Shipments, each of the two shipments will weigh 1/10 of a ton.**

Add to list

**5.NF.7a**

**We have 1/2 of a gallon of milk to pour equally among 3 pitchers. After pouring how much will be in each pitcher? Write a number sentence to represent the problem. Use a model to explain how you know. Explain how you can check your answer using multiplication <u>and addition</u>.**

**Each of the 3 pitchers will hold 1/6 of a gallon of milk.**

**Add to list**

# Do you notice a pattern? What's the rule?

**1 ÷ 1/8 = 8**

**10 ÷ 1/2 = 20**

**6 ÷ 1/4 = 24**

**3 ÷ 1/2 = 6**

**1 ÷ 1/3 = 3**

**1/3 ÷ 2 = 1/6**

**1/3 ÷ 4 = 1/12**

**1/5 ÷ 2 = 1/10**

**1/2 ÷ 3 = 1/6**

* Two Algorithms

For two problems you make up, test the rule using a model and link the model to the rule. Check your work using multiplication.

**Trace pattern blocks, use a ruler or grid paper, etc. - no "free-hand" drawing allowed.**

*Division does not always come out "even". . .* *<u>4.OA.3</u>, 5.NF.3, 5.NF.7*

**We have 11 gallons of milk to fill pitchers that hold 5 gallons each. How many pitchers can we fill completely and how much of the partial pitcher can we fill?** **11 ÷ 5 or 11/5**

$$\begin{array}{r} 2 \\ 5\overline{)11} \\ \underline{10} \\ 1 \end{array}$$

**What do we do with the "1"?** **5.NF.3** **Divisor, Dividend?**

**We have 1 out of the 5 parts needed to fill the next pitcher. So, 11 gals of milk can completely fill 2 pitchers that hold 5 gals each and 1/5 of the next pitcher.** **2 groups of 5 and 1/5 of another group.**

**5.NF.3 Interpret a fraction as division of the numerator by the denominator. . .**
**5.NF.7 Apply and extend <u>previous understanding of division</u> to divide unit fractions. . . .**

***Apply whole number reasoning to fractional parts. . .*** ***4.OA.3, 5.NF.3, 7***

**We have 3 pizzas to be shared equally among 4 friends. How much will each friend get?** **3 ÷ 4 or 3/4**

$$\begin{array}{r} 0 \\ 4\overline{)3} \\ \underline{0} \\ 3 \end{array}$$

**What do we do with the "3"?** **5.NF.3 Divisor, Dividend?**

**Since there are 4 friends, each friend gets a ¼ share of each pizza. Since there are 3 pizzas, that means each friend gets a total of ¾ of a pizza. Prove using a model.**

**<u>5.NF.3</u> Interpret a fraction as <u>division of the numerator by denominator</u>. . .Between what two whole numbers does the answer lie?**

**5.NF.7 Apply and extend <u>previous understanding of division</u> to divide unit fractions. . . .**

# 5.NF.7a

We have 1/3 of a pizza to share equally between 2 friends. How much will each friend get? Use a model to explain how you know.

Anna said she can make two batches of cookies with 1/3 cup of sprinkles. How much sprinkles are needed to make one batch of cookies?

***How do these two problems differ?***

***How might that impact teaching?***

***Teaching Considerations***

# 5.NF.7a

Anna said she can make two batches of cookies with 1/3 cup of sprinkles. How much sprinkles are needed to make one batch of cookies?

How many batches of cookies can be made from 1/3 cup of sprinkles if each batch takes 2 cups?

***How do these two problems differ?***

***How might that impact teaching?***

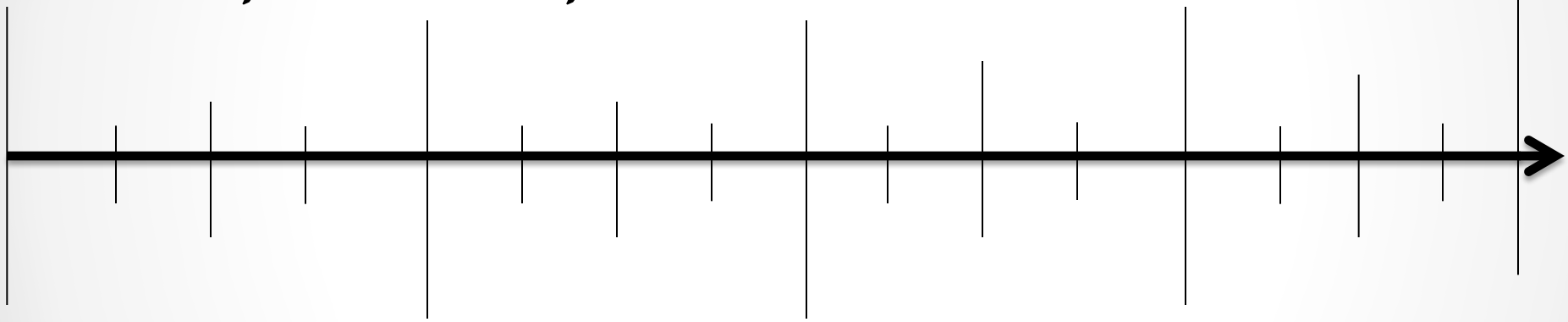
***CCSSM Table 1-2***

***Teaching Considerations***

# Yard Stick Number Line

**5.NF.7, <u>5.MD.2</u> and 4.NF.3c**

**1/4 c. , 3/4 c., 2/4 c. in 3 beakers**



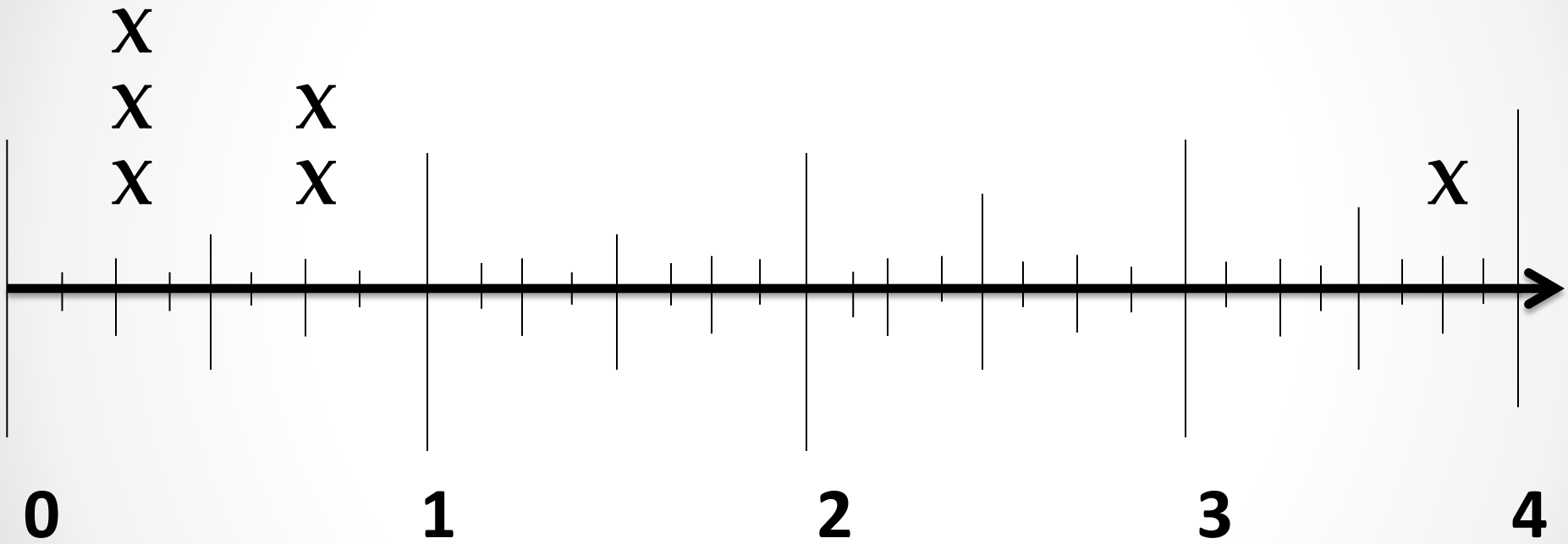
**0 1 2 3 4**

**How much in each beaker when redistributed equally?**

Line Plot

5.NF.7, 5.MD.2 and 5.NF.1

# Liquid in Beakers




**Amount of Liquid in Liters**

**How much in each beaker when redistributed equally?**

Struggling in mathematics is not the enemy any more than sweating is in basketball, it's a clear sign you are in the game.

Kim Sutton

Based on Carol Dweck's research on "Mindset and the Impact on Student Achievement."

# 6.NS.1 Fraction by a fraction

**Pre-assessment may be necessary -**
**How many quarter hours are in three hours?**
**How many thirds of a cup are in two cups?**
**How many half meters are in five meters?**

- **Start with easy problems that students can reason and model -- 2/3 ÷ 1/3, 1-2/4 ÷ 2/4**
- **Always ask problems in context at the beginning then move to "naked" numbers.**

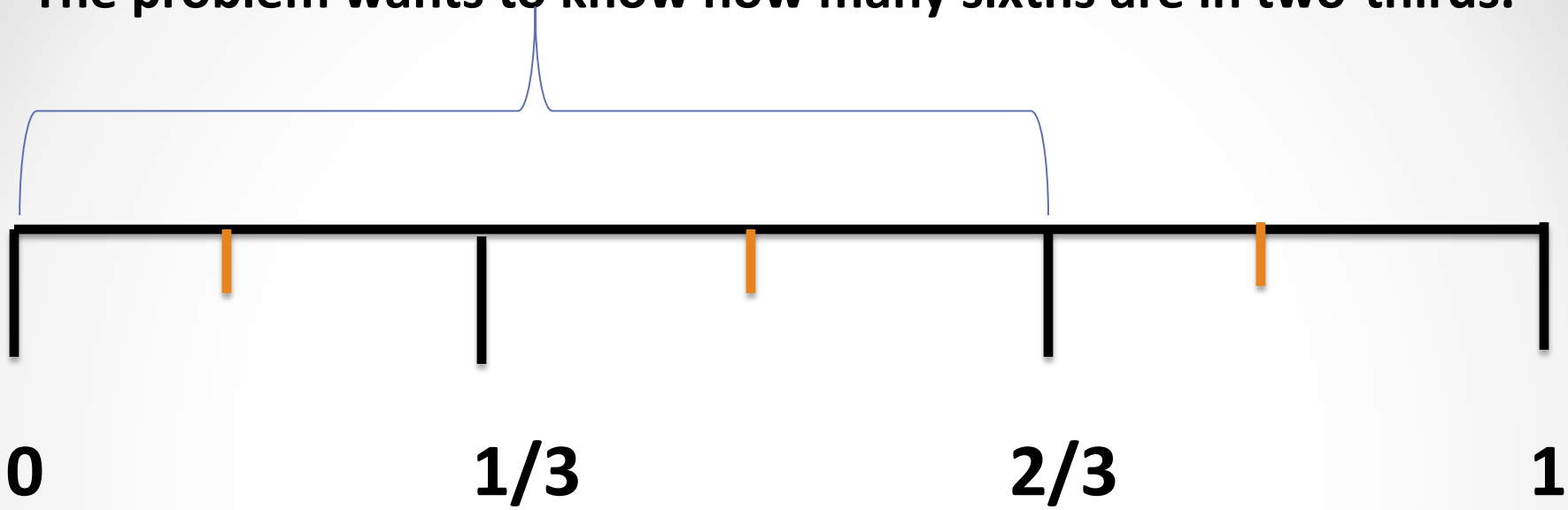
# 6.NS.1 Fraction by a fraction

**Taylor has 2/3 of an hour left to make cards. It takes her about 1/6 of an hour to make each card. About how many cards can she make in the time left?**

**Model the problem.**

**Write a number sentence and show how to solve the problem. Relate the number sentence to the model you drew.**

The problem wants to know how many sixths are in two-thirds.




If Taylor has 2/3 of an hour to make cards and it takes her about 1/6 of an hour per card, then she can make about four cards in the time left.

**Anna has 2-5/8 meters of yarn that she needs to cut into ½ meter lengths. How many lengths of yarn can Anna cut?**

**Write a number sentence to represent the problem. Draw and label a visual model to represent a solution. Write your solution in a complete sentence that relates to the question.**

**Anna can make 5-1/4 lengths of yarn that measure ½ meter each from 2-5/8 meters of yarn.**

**Anna has 2-5/8 acres of land that she needs to cut into ½ acre plots. How many ½ acre plots will she have?**

**Write a number sentence to represent the problem. Draw and label a visual model to represent a solution. Write your solution in a complete sentence that relates to the question.**

**Anna can make 5-1/4 lengths of yarn that measure ½ meter each from 2-5/8 meters of yarn.**

**David has 2 1/2 pizzas. He wants each friend to get a 1/8 pizza serving. How many friends can David serve?**

**Write a number sentence to represent the problem. Draw and label a visual model to represent a solution. Write your solution in a complete sentence that relates to the question.**

**If David has 2-1/2 pizzas and each friend gets 1/8 of a pizza serving, then David can serve 20 friends.**

**David has 2 ½ pizzas. He wants each friend to get a 1/3 pizza serving. After cutting into those serving size pieces, will there be pizza left over? If so how much? If we had a fraction of person, how many people could David serve?**

**Are the numerical responses to the above questions the same? Why or why not?**

**This represents 1**

**Using that model for one, what is 1-3/4 ÷ 1/2 ?**

**Reason out a solution, make a solution model, and write an equation to represent the problem.**

**There are 3 and ½ pieces of size ½ in 1-3/4.**
**1-3/4 ÷ 1/2 = 3-1/2**

**David works for the humane society and had to buy food to feed the dogs. He bought 5-1/2 pounds of dog food. He feeds each dog about 3/4 of a pound. How many dogs can he feed? Will there be leftovers; if so, how much?**

**Write a number sentence to represent the problem. <u>Reason it out</u> and then draw a model to confirm your reasoning.**

**With 5-1/2 pounds of dog food, David can feed 7 dogs and have 1/3 of the needed ¾ pound serving left over.**

***At the beginning choose your problems wisely – not all problems are easily reasoned nor modeled.***

How many 3/4 cup servings are in 2/3 of a cup of yogurt?

How many 3/4 yard of ribbon can we get from 7/12 of a yard?

If David can run 5 miles in 3/4 of an hour, how long will it take him to run one mile?

How many 2/3 gallon servings are in 1-1/4 gallon of lemonade?

**Will the answer be less than or greater than one? How do you know?**

***Not all problems are easily reasoned nor modeled.***

**How many 3/4 cup servings are in 2/3 of a cup of yogurt?**

**How many 3/4 yard of ribbon can we get from 7/12 of a yard?**

**If David can run 5 miles in 3/4 (1/4) of an hour, how long will it take him to run one mile?**

**How many 2/3 gallon servings are in 1-1/4 gallon of lemonade?**

**For each question write a number sentence and show how to solve using a model.**

# Two algorithms for dividing fractions

- Common Denominators
- Invert and Multiply (Multiply by the Reciprocal)

**Both are applicable to division of and by unit fractions as well – but the common denominator method is too cumbersome when working with division by unit fractions. However, it may be helpful to some students.**

# *Common Denominator Method Linked to Division of Whole Numbers*

## How many 2/3 gallon servings are in 1-1/4 gallons of lemonade?

**Is the question asking how many groups or the size of each group (how much for one)?**

**Step 1 -**

## Get common denominators for 5/4 and 2/3

## 5/4 = 15/12; 2/3 = 8/12

## What is 15/12 ÷ 8/12?

# *Common Denominator Method Linked to Division. . .*

**5.NF.3, 5.NF.7 and 6.NS.1**

## 15/12 ÷ 8/12 =

**Step 2 - Divide the numerators and divide the denominators.**

$$\frac{15 \div 8}{12 \div 12}$$

$$\begin{array}{r} 1 \\ 8\overline{)15} \\ \underline{-\,8} \\ 7 \end{array}$$

**So 15 ÷ 8 is 1 with 7 out of the needed 8 left;**
**1-7/8 servings of size 2/3 gal are in 1-1/4 gal.**

***Discuss size of fractions - If we have 1-1/4 gal & need 2/3 gal servings, would you expect the answer to be more or less than 1? Between what two numbers would you expect the answer to lie and why?***

***Practice using Common Denominator Method. . .***

**How many 3/4 cup servings are in 2/3 cup of yogurt?**

**Use the common denominator method to solve, link to the previous model and check your answer using multiplication.**

**Is the question asking the size of each group (how much for one) or the number of groups?**

# *Practice using Common Denominator Method. . .*

**How many 3/4 cup servings are in 2/3 cup of yogurt?**

**3/4 = 9/12 2/3 = 8/12**

**8/12 ÷ 9/12**

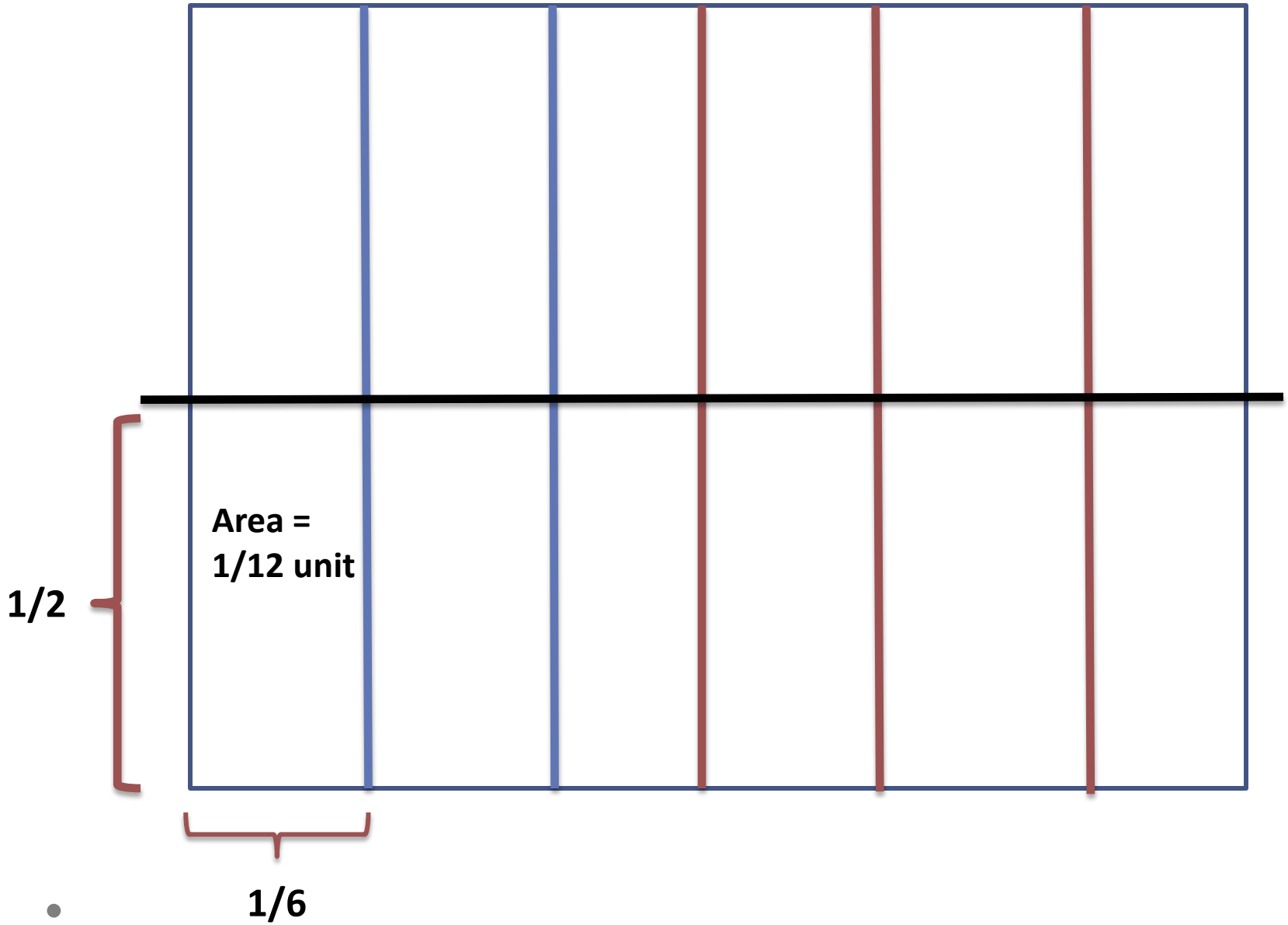
$$\frac{8 \div 9}{12 \div 12}$$

$$\begin{array}{r} 0 \\ 9\overline{)8} \\ \underline{-0} \\ 8 \end{array}$$

**8/9**

**We can get 8/9 of a ¾ cup serving is in 2/3 of a cup of yogurt.** **Discuss size of fractions - If we have 2/3 of a cup is that enough for a serving size of ¾ cup? 5.NF.3, 5.NF.7 and 6.NS.1**

# 5.NF.4 and 4b  Fractional side lengths. . .



**Invert and Multiply (Multiply by the Reciprocal) Algorithm -- How is that possible?**

**While it is the most commonly used method for division of fractions, it is the least understood.**

***To know rote is not knowledge it is retention of that which is entrusted to the memory.***

**Montague**

# *Constructing Understanding. . .* *Teacher knowledge*

# There are two explanations:

**2/5 ÷ 1/2**

$$\frac{2/5}{1/2}$$

**Don't think "complex fraction"**

**Make it easier by getting rid of the denominator. How?**

$$\frac{2/5}{1/2} \times \left[\frac{2/1}{2/1}\right]$$

$$\frac{2/5 \quad x \quad 2/1}{1/2 \quad x \quad 2/1}$$

**2/5 x 2/1 =**

**4/5**

# Constructing Conceptual Understanding Why Do We Multiply by the Reciprocal (Invert and Multiply)? Teacher Knowledge

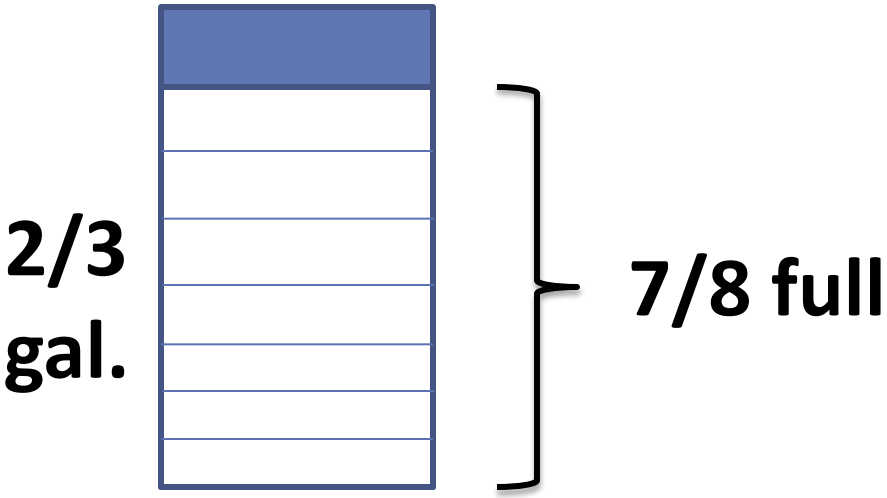
**A small pail can be filled 7/8 full using 2/3 of a gallon of water. How much will the pail hold if filled completely?**

**Reason out the solution with a friend.**

**Share solutions.**

**What do we know. . .**

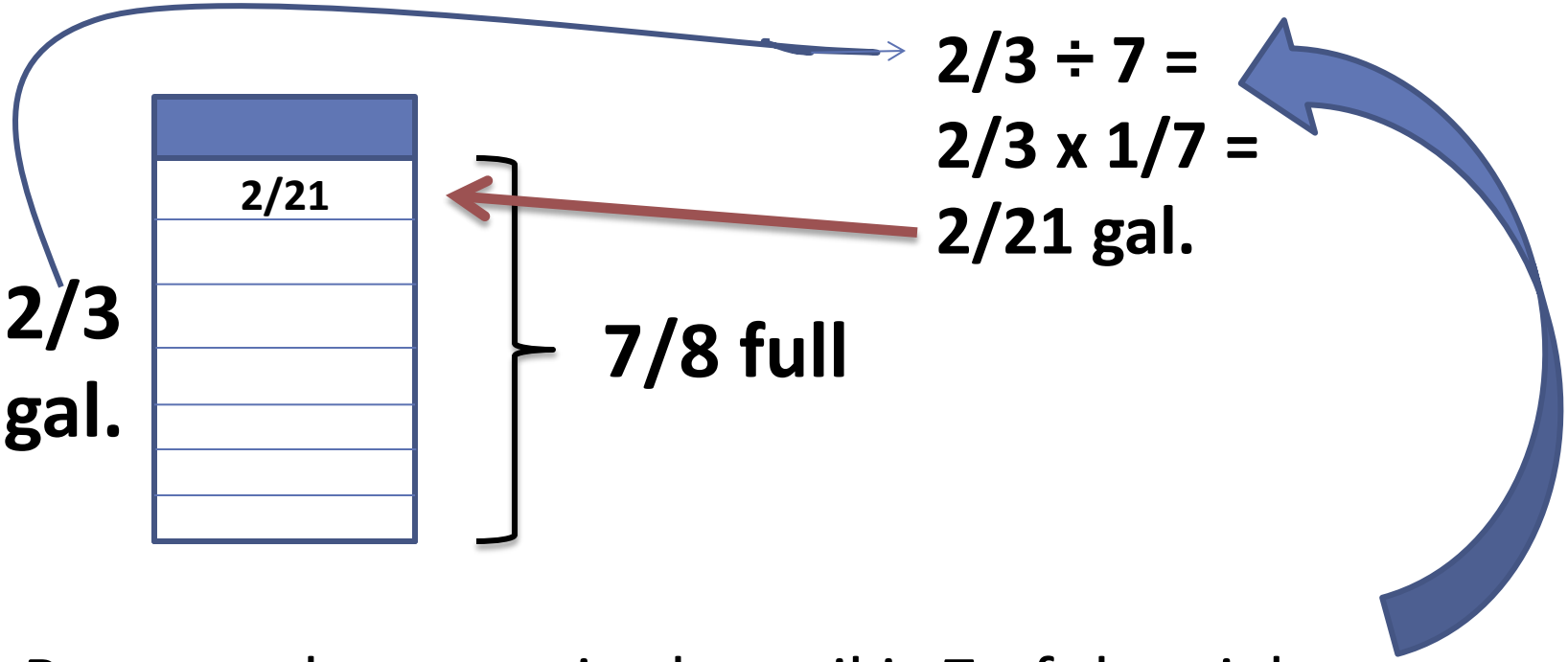
**A small pail can be filled 7/8 full using 2/3 of a gallon of water. How much will the pail hold if filled completely?**




**We know the partial (2/3 gal) so the task is to find out the whole.**

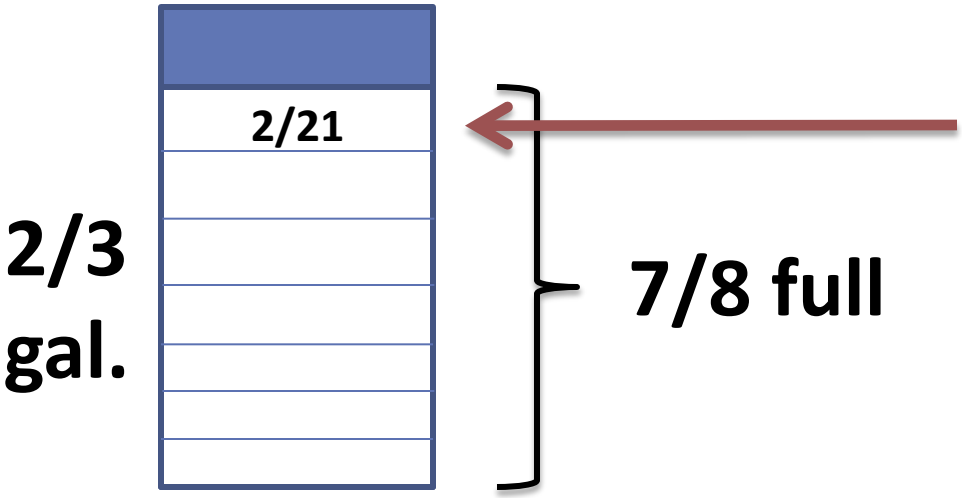
Because the water in the pail is 7 of the eight parts needed to fill the pail, dividing the water by 7 tells us the size of each part.

**A small pail can be filled 7/8 full using 2/3 of a gallon of water. How much will the pail hold if filled completely?**




Because the water in the pail is 7 of the eight parts needed to fill the pail, dividing the water by 7 tells us the size of each part.

**A small pail can be filled 7/8 full using 2/3 of a gallon of water. How much will the pail hold if filled completely?**

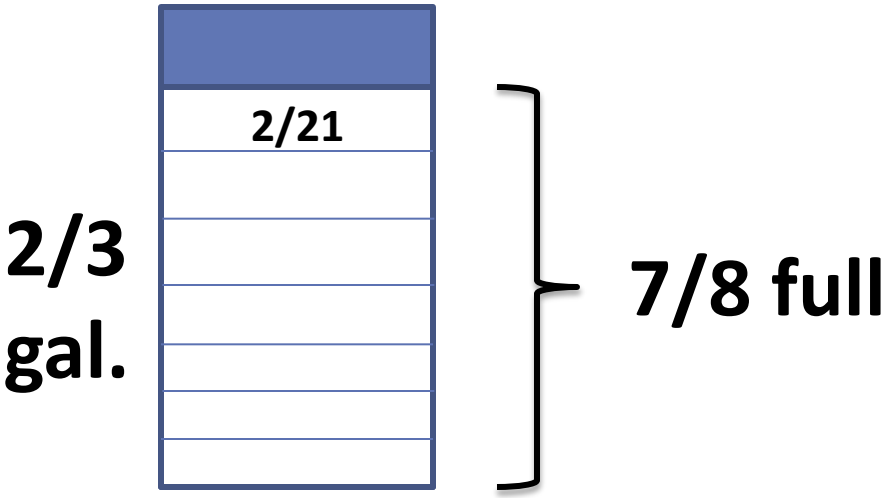



2/3 ÷ 7 =
2/3 x 1/7 =
***2/21 gal.***

*Check it to see that 7 pieces of size 2/21 is 14/21 or 2/3.*

**As a result of dividing by 7, the numerator of 7/8 now becomes the divisor.**

**A small pail can be filled 7/8 full using 2/3 of a gallon of water. How much will the pail hold if filled completely?**




***Since 8/8 is needed to fill the pail, multiply***

**8 x 2/21 =**

**16/21 *gal. in pail when full***

***Check to see if the answer is reasonable – Is 16/21 of a gallon more than 2/3 of a gallon?***

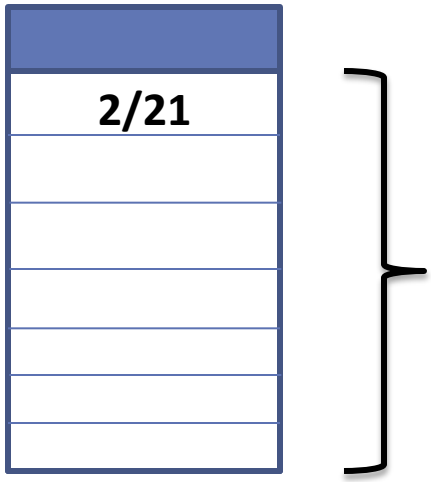
**The denominator of 7/8 now becomes the multiplier.**

**So, 2/3 ÷ 7/8 =**

- **2/3 x 8/7 = 16/21.**

**A small pail can be filled 7/8 full using 2/3 of a gallon of water. How much will the pail hold if filled completely?**

**2/3 gal.**




**7/8 full**

*2/3 ÷ 7 = 2/21*

*8 x 2/21 16/21 OR*

*2/3 ÷ 7/8 =*
*2/3 x 8/7 =*

$$\frac{2 \times 8}{3 \times 7}$$

**6.NS.1 . . . In general** $(a/b) \div (c/d) = ad/bc$

$$\frac{a \times d}{b \times c}$$

- *c/d has been inverted -- multiplying the reciprocal of c/d*

**6.NS.1** **Teacher content knowledge**

**How many 3/4 cup servings are in 2/3 of a cup of yogurt?**

**2/3 ÷ 3/4**

2/3 ÷ 3 =
2/3 x 1/3 =
2/9

**The numerator becomes the divisor**

2/9 x 4 =
<u>8/9</u>

**The denominator becomes the multiplier.**

**Traditional algorithm**

2/3 ÷ 3/4 =
2/3 x 4/3 =
<u>8/9</u>

**(a/b) ÷ (c/d) = ad/bc**

***Division can be thought of in terms of multiplying by the inverse. The multiplicative inverse of every number, except 0, is the reciprocal of that number; and the product of a number and its reciprocal is 1.***

# Pulling it all together. . .

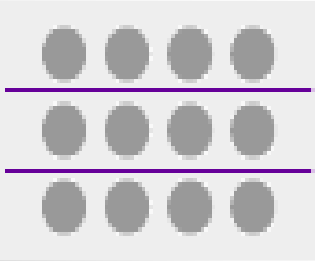
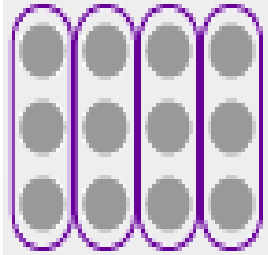
John is building a patio. Each section requires 2/3 of a cubic yard of concrete. The concrete truck holds 2-1/4 cubic yards of concrete. If there is not enough for a full section at the end, John can put in a divider and make a partial section. How many sections can John make with the concrete in the truck?

- Reason it out
- Use repeated subtraction
- Use common denominators
- Use invert and multiply (multiply by the reciprocal) algorithm
- Use a model

- Share answers for each method
- Size of each group or the number of groups?

**Two common structures to division problems. One is about partitioning or doling out equal shares (the size of the group – how much for one) (partitive) and the other is about measuring out partitions of a fixed size (how many groups) (quotative).**

# Handout. . .

| **Partitive:** | **Quotative:** |
|---|---|
| $12 \div 3 = 4$ | $12 \div 3 = 4$ |
| Partition into 3 groups. | Repeatedly subtract 3. |
| There are 4 in each group. | There are 4 groups of 3 in 12. |
| Twelve apples, 3 bags -- how many in each? | Twelve apples, 3 in a bag -- how many bags? |
|  |  |

| | Unknown Product | Group Size Unknown (“How many in each group?” Division) | Number of Groups Unknown (“How many groups?” Division) |
|---|---|---|---|
| | $3 \times 6 = ?$ | $3 \times ? = 18$, and $18 \div 3 = ?$ | $? \times 6 = 18$, and $18 \div 6 = ?$ |
| Equal Groups | There are 3 bags with 6 plums in each bag. How many plums are there in all?<br>*Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?<br>*Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed?<br>*Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| Arrays,[4] Area[5] | There are 3 rows of apples with 6 apples in each row. How many apples are there?<br>*Area example*. What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row?<br>*Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?<br>*Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |
| Compare | A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?<br>*Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?<br>*Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?<br>*Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |
| General | $a \times b = ?$ | $a \times ? = p$, and $p \div a = ?$ | $? \times b = p$, and $p \div b = ?$ |

# Pulling it all together. . .

**We have 4-1/2 pizzas to share equally between 2 friends. How much will each friend get?**

**How many 2/3 gallon servings are in 1-1/4 gallons of lemonade?**

***How do these questions differ?***

***Are the questions asking the size of each group (how much for one) or how many groups?***

***Will that impact how students approach a solution or write an answer?***

# Problem Solving Tips That Lead to Conceptual Understanding. . . (Handout)

- ✓ ***Start with easy problems for which students can reason a solution***
- ✓ ***Provide <u>well-chosen</u> problems in context***
- ✓ ***Require students to use multiple solution approaches***
- ✓ ***Use Guided <u>Productive</u> Talk***

# My Contact Information

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