South Carolina College- and Career-Ready Standards for Mathematics High School Support Document

South Carolina Department of Education
Office of Standards and Learning
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South Carolina College- and Career-Ready Standards for Mathematics
High School Support Document
Overview

The purpose of this document is to provide guidance regarding how all the standards in Algebra 1, Foundations in Algebra, Intermediate Algebra, Algebra 2, Geometry, and Probability and Statistics may be grouped into units and how those units might look. Since this document is merely guidance, a district should implement the standards in a manner that addresses its curriculum and the specific needs of its students.

The Table of Contents below arranges the *South Carolina College- and Career-Ready Standards for Mathematics* for high school into *Course Coversheets* and *Units*.

- Each high school *Course Coversheet* organizes the high school course standards into possible instructional units and provides links to specific high school course *Units*.
- Each high school course *Unit* contains:
  - Clarifying notes related to the standards within the unit
  - New academic vocabulary in the unit
  - Prior and subsequent knowledge related to the unit
  - Description of the relationship between the standards in the unit
  - Possible instructional strategies and lessons organized by possible teaching sequence
  - Resources for the unit
  - Sample formative assessment tasks and questions organized by possible teaching sequence.
- Important notes about all *Units*:
  - ~~Strikethroughs~~ identify which piece(s) of a standard is not covered in a specific unit. ~~Strikethrough~~ portions should, however, be covered in a different *Unit* before the end of the course.
  - *Including* references content that must be mastered, while *e.g.* references possible illustrative examples. The phrase *i.e.* references the only examples or terms that should be used.
  - Asterisks (*) indicate Graduation Standards. Graduation Standards are not optional.

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South Carolina College- and Career-Ready Standards for Mathematics
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Algebra 1 Coversheet

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
Relationships Between Quantities and Expressions	Reasoning with Linear Equations and Inequalities	Modeling and Analyzing Quadratic Functions	Modeling and Analyzing Exponential Functions	Comparing and Contrasting Functions	Describing Data
Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
A1.NRNS.1*	A1.ACE.2*	A1.NRNS.1*	A1.FLQE.1*	A1.FLQE.1*	A1.FLQE.5*
A1.NRNS.2*	A1.AREI.3	A1.ASE.2*	A1.FLQE.2*	A1.FLQE.1a	A1.SPID.6*
A1.NRNS.3	A1.AREI.5	A1.ASE.3*	A1.ACE.1*	A1.FLQE.2*	A1.SPID.7*
A1.NQ.1*	A1.AREI.6*	A1.ASE.3a	A1.ACE.2*	A1.FLQE.3*	A1.SPID.8*
A1.NQ.2*	A1.AREI.6a	A1.ACE.1*	A1.FBF.3*	A1.FLQE.5*	
A1.NQ.3*	A1.AREI.6b	A1.ACE.2*	A1.FIF.1a	A1.FBF.3*	
A1.ASE.1*	A1.AREI.10*	A1.ACE.4*	A1.FIF.1b	A1.FIF.1*	
A1.AAPR.1*	A1.AREI.11*	A1.AREI.1*	A1.FIF.1c	A1.FIF.1a	
A1.ACE.1*	A1.AREI.12*	A1.AREI.4*	A1.FIF.2*	A1.FIF.1b	
A1.ACE.2*	A1.FIF.1*	A1.AREI.4a	A1.FIF.4*	A1.FIF.1c	
A1.ACE.4*	A1.FIF.1a	A1.AREI.4b	A1.FIF.5*	A1.FIF.2*	
A1.AREI.1*	A1.FIF.1b	A1.FBF.3*	A1.FIF.6*	A1.FIF.4*	
A1.AREI.3*	A1.FIF.1c	A1.FIF.1*	A1.FIF.7*	A1.FIF.5*	
A1.AREI.10*	A1.FIF.2*	A1.FIF.1a	A1.FIF.8*	A1.FIF.6*	
	A1.FIF.4*	A1.FIF.1b		A1.FIF.7*	
	A1.FIF.5*	A1.FIF.1c		A1.FIF.9*	
	A1.FIF.6*	A1.FIF.2*			
	A1.FIF.7*	A1.FIF.4*			
	A1.FIF.8*	A1.FIF.5*			
	A1.FIF.9*	A1.FIF.6*			
	A1.FLQE.2*	A1.FIF.7*			
		A1.FIF.8*			
		A1.FIF.8a			
		A1.FIF.9*			

Algebra 1 Coversheet

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Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

1. Make sense of problems and persevere in solving them. a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

Algebra 1 Unit 1 Title
Relationships Between Quantities and Expressions

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- A1.NRNS.1* Rewrite expressions involving simple radicals and rational exponents in different forms.
  - Apply properties of exponents to write equivalent expressions that include simple radicals (e.g., square roots and cube roots) and integer exponents.
  - Expand properties of exponents to write equivalent expressions that include rational exponents.
- A1.NRNS.2* Use the definition of the meaning of rational exponents to translate between rational exponent and radical forms.
  - Convert expressions with fractional exponents to equivalent radical forms and vice-versa.
- A1.NRNS.3 Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
  - Use the property of set closure to include rational and irrational numbers under addition and multiplication.
- A1.NQ.1* Use units of measurement to guide the solution of multi-step tasks. Choose and interpret appropriate labels, units, and scales when constructing graphs and other data displays.
  - Convert units of measure, as appropriate (e.g., using like units to compare or combine lengths), to solve multi-step performance tasks.
  - Apply dimensional analysis to convert units of measure.
  - Analyze the context of problems to determine the appropriate unit(s) of measure.
  - Select and interpret appropriate units of measure when solving real-world contexts involving formulas.
- A1.NQ.2* Label and define appropriate quantities in descriptive modeling contexts.
  - Identify the variables or quantities from data displayed in a given model (e.g., text, graph, picture, or algebraic formula)
  - Select the appropriate unit of measure for variables or quantities presented in a given model.
- A1.NQ.3* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in context.
  - Report solutions to problems with the appropriate level of accuracy, and with precision if necessary, for the unit of measure given in the context of the problem and/or the measuring tool used.

- A1.ASE.1* Interpret the meanings of coefficients, factors, terms, and expressions based on their real-world contexts. Interpret complicated expressions as being composed of simpler expressions.
    - Limit to linear expressions for Unit 1; expand to quadratic in Unit 3 and exponential in Unit 4.
    - Rational functions are not taught in Algebra 1.
    - Recognize that an algebraic expression can be composed of multiple terms and represent unknown real number value(s).
    - Simplify or factor complicated expressions by combining like terms or extracting the Greatest Common Monomial (factor) to show equivalent expressions (e.g., $2x + 2y = 10$ is equivalent to $x + y = 5$).
- A1.AAPR.1* Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations.
    - Limit to linear in Unit 1.
    - Expand the properties and operations of real numbers to include polynomial expressions.
    - Introduce the term *polynomial*.
    - Develop foundational knowledge regarding algebraic terms and polynomial expressions and the properties of operations applied to polynomials.
- A1.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, ~~quadratic, simple rational, and exponential~~ relationships. Interpret the solutions and determine whether they are reasonable.
    - Limit to linear in Unit 1
    - Expand to quadratic in Unit 3 and exponential in Unit 4.
    - Rational functions are not taught in Algebra 1.
    - Use real-world contexts to generate and solve equations and inequalities in one variable.
    - Analyze solutions for their meaning and rationale within the given context.
    - Interpreting solutions includes solutions graphed on a number line.
- A1.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
    - Limit to linear and introduce the terms *direct variation* and *indirect variation* in Unit 1.
    - Expand to quadratic in Unit 3 and exponential in Unit 4.
    - Rational functions are not taught in Algebra 1.
    - Understand that linear equations define the relationship between two variables.
    - Generate and graph equations to represent the relationship between two variables.
    - Use appropriate labels, units, and scales to represent the relationship of two variables in a given real-world context.

- A1.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
  - Apply the properties and operations of real numbers to solve for a specified variable (e.g., solve a linear equation in standard form for y; solve $A = lw$ for w, and include other formulas from a variety of disciplines).
- A1.AREI.1* Understand and justify that the steps taken when solving simple equations in one variable create new equations that have the same solution as the original.
  - State the property or operation being applied that explains why each step of solving an equation generates an equivalent equation.
  - Verify by substitution that the variable's solution in the last step solves the equation for the given problem.
- A1.AREI.3* Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
  - Apply the properties and operations of real numbers to equations and inequalities to solve for a specified variable (e.g., solve for x in $3x - 9 = 15$; solve the slope-intercept equation $y = mx + b$ for m; solve $y \geq mx + b$ for x).
- A1.AREI.10* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.
  - Use algebraic and graphical formats to justify that the set of solutions is a one-to-one relationship, which can be graphed with ordered pairs (i.e., domain, range).

New Academic Vocabulary for This Unit
• Compound Inequalities (*notation only*) • Direct Variation • Index (Root) • Indirect Variation • Polynomial • Radical • Radicand • Rational Exponents

Prior Knowledge Required for this Unit
In earlier grades, students have developed conceptual knowledge and have had the opportunity to learn how to: • Understand and apply the constant of proportionality (7.RP.2). • Determine and apply the constant rate of change (8.F.3; 8.F.4). • Understand the relationship between independent and dependent variables (6.EEI.9; 8.F.1). • Be able to evaluate square and cubic roots, and recognize perfect and non-perfect squares as rational and irrational correspondingly (8.EEI.2). • Transform and apply the Pythagorean Theorem particularly as it relates to rational and irrational squares (8.EEI.2a, b, and d). • Understand fundamental concepts of functions, including one-to-one relationships, particularly as communicated as domain to range within ordered pairs (8.F.1). • Understand and apply the properties of exponents (8.EEI.1). • Graph the solution of one variable inequality on the number line (7.EEI.4c). • Understand that slope is a rate of change from one quantity in relation to another quantity within real world and mathematical situations (8.EEI.5b). • Understand the critical attributes of linear and nonlinear functions (8.F.3). • Represent linear functions, particularly in the form of $y = mx + b$ through table, equation, and graphical form, and identify value and meanings of slope/rate and y-intercept/initial value as found within real-world and mathematical situations (8.F.3, 8.F.4, 8.F.5, 8.EEEI.6). • Solve for a single variable in a multiple variable equation and inequalities in real-world and mathematical situations (8.EEEI.7a and d). • Generate and graph linear equations (8.F.1c and e; 8.F.4b; 8.F.5). • Understand and apply the properties of operations, equality, and inequality.

Subsequent Knowledge Related to this Unit
- Algebra 1 Unit 1 develops one- and two-variable linear equation concepts (A1.ASE.1*; A1.ASE.2*).   - Will apply and expand to include quadratic (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions) and exponential equations (Algebra 1 Unit 4: Modeling and Analyzing Exponential Functions).   - Algebra 1 Unit 1 recognizes an early form of factoring is to identify and extract the greatest common numeric or algebraic term, such as $2x + 2y = 10$ can be represented as $x + y = 5$ (A1. ACE.4*; A1.ASE.1*; A1.ASE.2*).   - Will be extended to factor and simplify polynomials, specifically quadratics (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions).   - Algebra 1 Unit 1 simplifies or factors complicated expressions by combining like terms or extracting the Greatest Common Monomial (factor) to show equivalent expressions (A1.ASE.1*).   - Will apply to primarily common monomial factoring (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions).   - Algebra 1 Unit 1 graphs one-variable inequalities on a number-line (A1.FIF.5*).   - Will extend as a possible means to express the domain and range in compound inequality notation in future units.   - Algebra 1 Unit 1 compares functions' graphical, symbolic, or tabular forms (A1.NQ.2*; A1.AREI.10*).   - Will utilize the application of the functions' graphical, symbolic, or tabular form, particularly to measure the average rate of change (A1.FIF.6* in Algebra 1 Units 2, 3, 4, and 5).   - Algebra 1 Unit 1 introduces direct and indirect variation in linear applications (A1.ACE.2*).   - Will expand study of linear relationships (Algebra 1 Unit 2: Reasoning with Linear Equations and Inequalities).   - Algebra 1 Unit 1 simplifies radicals and rationalizes denominators involving square roots and extending to cube roots as appropriate (A1.NRNS.1*).   - Will be applied in simplifying and solving literal equations and quadratics, and rationalizing will be applied in subsequent course of Geometry, such as applications with $30-60-90$ triangles (Geometry Unit 5: Right Triangles and Trigonometry).   - Algebra 1 Unit 1 defines, rewrites and explores the relationship between rational exponents and simple radicals, and the relationship of irrational numbers as subset of the entire real number systems (A1.NRNS.1*; A1.NRNS.2*; A1.NRNS.3*).   - Will develop skills found within finding square roots, completing the square, applying the quadratic formula and factoring of quadratic functions using other methods (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions).   - Algebra 1 Unit 1 rewrites and simplifies simple radicals (A1.NRNS.1*).   - Will be extended, such as simplifying square roots and rationalizing a denominator (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions).

- Algebra 1 Unit 1 develops fluency in variable manipulation (A1.ACE.4*).
  - Will solve for specified variables and substitute equivalent algebraic value(s) to rewrite functions of linear equations (Unit 2; Reasoning with Linear Equations and Inequalities) and quadratic functions (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions) and to compare such functions (Algebra 1 Unit 5: Comparing and Contrasting Functions).
- Algebra 1 Unit 1 writes equations of linear functions with two variables (A1.ACE.2*).
  - Will be extended with the point-slope form and its various applications (Unit 2: Reasoning with Linear Equations and Inequalities);
  - Will expand to quadratics (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions) and exponential functions (Unit 4: Modeling and Analyzing Exponential Functions).
- Algebra 1 Unit 1 develops fluency of polynomial operations and foundational knowledge of algebraic terms and polynomial expressions (A1.AAPR.1*).
  - Will facilitate linear applications (Algebra 1 Unit 2: Reasoning with Linear Equations and Inequalities), quadratic applications and factoring (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions), and other functions in units to follow.
- Algebra 1 Unit 1 develops understanding of functions in two variables and the variables' relationship expressed in table, equation and graphical forms (A1.AREI.10*).
  - Will connect to linear functions (Algebra 1 Unit 2: Reasoning with Linear Equations and Inequalities), quadratic functions (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions), and exponential functions (Algebra 1 Unit 4: Modeling and Analyzing Exponential Functions) as students begin to use function notation.
- Algebra 1 Unit 1 (and Grade 8) limits the algebraic form of a linear function to $y = mx + b$.
  - Will apply function notation $f(x)$ in subsequent units and courses.

Relationship Among Standards in this Unit
The standards in Unit 1 provide the foundational knowledge for concepts developed in Algebra 1 in subsequent units. Students will interpret the structure of expressions, equations, and inequalities involving one or more variables and solve problems related to unit analysis. Students will construct meaning about the relationships among variables, including direct and indirect variations, through real-world contexts and through algebraic, verbal, graphic, and tabular models. Tasks will progress from simple equations and inequalities to complex equations in two or more variables. Real-world contexts, relevant to STEM-related or other career fields, will engage students in mathematical practices while applying properties and performing operations with quantities involving given units of measure. Measurement units in these contexts may necessitate a conversion, which will require students to attend to precision and accuracy. Students will write, graph, and solve linear equations or inequalities to represent the relationship between independent and dependent variables. Graphing relationships will require students to use appropriate labels, units, and scales on the axes. They will employ logic and reasoning to interpret and explain the meaning of a solution or a set of solutions. From solving linear equations or inequalities in one-variable to rewriting or solving a formula involving two or more variables, students will fluently solve for given variables. Investigation of compound inequalities (written, symbolic, and number line graphing) will empower students to express domain and range using compound inequality notation. The properties of rational and irrational numbers and operations with polynomials are included as a preparation for working with quadratic functions later in the course. Students will simplify radicals and rationalize denominators involving square roots and extending to cube roots as appropriate.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 1.*
Exponent Foundations (A1.NRNS.1*; A1.NRNS.2*; A1.NRNS.3) a. Basic Exponent Properties Review Khan Academy: Basic Exponent Properties Review b. Relationship Between Rational Exponents and Simple Radicals Khan Academy: Rational and Irrational Math Practices: Rational and Irrational MathBitsNotebook: Exponents Purple Math: Relationship Notes c. Simplifying Radicals and Rationalizing Denominators Khan Academy: Rationalizing a Denominator Video Math Open Reference: Simplifying Radicals MathBitsNotebook: Simplifying and Rationalizing Denominators NRICH: Tilted Squares Virtual Nerd: Simplifying with Square Roots
Understanding and Representing Quantity (A1.NQ.1*; A1.NQ.2*; A1.NQ.3) a. Understand the Appropriateness of Unit Size in a Real-World Context The Scale of Universe: Scale of Universe Alysion: Dimensional Analysis Notes and Summary b. Unit Conversion Analysis Virginia: Choosing Appropriate Unit of Measurement

Interpret the Meanings of Expressions (A1.ASE.1*)

a. Interpreting Expressions
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Interpreting Algebraic Expressions: Online Lesson Plan
b. Translation of Expressions
   MathBitsNotebook: Expression Translation
   MathBitsNotebook: Expression Practice
c. Evaluation of Expressions
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Classroom Challenges (Formative Assessment Lessons)
   MathBitsNotebook: Expression Evaluation
   Khan Academy: Expression Evaluation
   Virtual Nerd: Variable Substitution

Polynomial Operations (A1.AAPR.1*)

a. Polynomial Definition and Concepts
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Generating Polynomials from Patterns
   MathBitsNotebook: Defining and Classifying Polynomials
b. Polynomial Addition and Subtraction
   MathBitsNotebook: Polynomial Addition and Subtraction
   Virtual Nerd: Adding Polynomials
   Virtual Nerd: Subtracting Polynomials
   Virtual Nerd: Subtracting Polynomials (Difference in Length and Width)
c. Monomial by Polynomials
   MathBitsNotebook: Monomial by Polynomial Multiplication
   Virtual Nerd: Monomial by Polynomial Multiplication
d. Binomial Multiplication
   MathBitsNotebook: Binomial Multiplication
   Virtual Nerd: Binomial Multiplication
   Virtual Nerd: FOIL Binomial Multiplication
   Virtual Nerd: Grid Method Multiplication

e. Polynomial (more) Multiplication
MathBitsNotebook: Special Binomial Multiplication
MathBitsNotebook: Polynomial Multiplication
Virtual Nerd: Polynomial Multiplication
Virtual Nerd: Polynomial Grid Multiplication Application
Georgia Department of Education: Mathematics Framework – Polynomials Multiplication Unit

Creating and Solving Equations (A1.ACE.1*; A1.ACE.2*; A1.ACE.4*; A1.AREI.1*; A1.AREI.3*)

a. Direct and Indirect Variation Discussion
Cliffs Notes Notebook: Direct and Inverse Variation Notes and Practice Quiz
Virtual Nerd: Video Discussion on Direct Variation and How to Write an Equation in a Problem Situation
Khan Academy: Video Discussion on Direct and Inverse Variation

b. Solving One-Variable Equations
MathBitsNotebook: Solving Multi-Step Equations
MathBitsNotebook: Solving Multi-Step Equations: Practice
Virtual Nerd: Solving Multi-Step Equations: With Distributive Property
Virtual Nerd: Solving Multi-Step Equations: Clearing Fractions
Virtual Nerd: Solving Multi-Step Equations: With No Solution
Virtual Nerd: Solving Multi-Step Equations: Word Problem
Khan Academy: Solving Multi-Step Equations: Intuition Exercise
Khan Academy: Exercise Activity

c. Analyzing Multi-Step Equations
Mathematics Assessment Project – Mathematics Assessment Resource Service: Building and Solving Complex Equations

d. Linear Rates
EngageNY: Linear Rates
Mathematics Assessment Project – Mathematics Assessment Resource Service: Building and Solving Linear Equations: Online Lesson Plan

e. Solving Literal Equations
MathBitsNotebook: Solving Literal Equations
MathBitsNotebook: Solving Literal Equations: Practice Problems
Virtual Nerd: What is a Literal Equation
Virtual Nerd: Solving Literal Equations
Khan Academy: Solving Literal Equations
Khan Academy: Solving Literal Equations: Practice
Khan Academy: Celsius and Fahrenheit

Solving, Interpreting, and Graphing Inequalities in One-Variable (A1.ACE.1*; A1.AREI.3*)

a. Inequality Concepts
Interpreting Solutions by Graphing One-Variable Solution
MathBitsNotebook: MathBitsNotebook: Basic Inequalities Information
MathBitsNotebook: MathBitsNotebook: Compound Inequalities

b. Solving One-Variable Inequalities
MathBitsNotebook: Solving One-Variable Inequalities
MathBitsNotebook: Solving One-Variable Inequalities Practice
Virtual Nerd: Solving One-Variable Inequalities: Two Steps
Virtual Nerd: Solving One-Variable Inequalities: Multiple Steps

c. Inequality Applications
MathBitsNotebook: Inequalities Word Problem: Practice

Graphing Linear Equations (A1.ACE.2*; A1.AREI.10*)

a. Linear Equations
Virtual Nerd: What Is A Linear Equation

b. Table Method
Virtual Nerd: Graphing Table Method
MathBitsNotebook: Graphing Linear Equations Using Table Method

c. Slope-Intercept Method Virtual Nerd: Graphing By Slope-Intercept MathBitsNotebook: Graphing Linear Equations Using Slope Intercept Method MathBitsNotebook: Graphing Linear Equations Using Slope Intercept Method - Practice Problems Math Open Reference: Lines On Coordinate Plane Math Open Reference: Slope-Intercept Math Open Reference: Slope-Intercept Applet Shodor: Slope-Intercept Slider d. Intercepts Method Virtual Nerd: Graphing By Intercepts Method MathBitsNotebook: Graphing Linear Equations Using Intercept Methods (including calculator guide) e. Modeling and Applications MathBitsNotebook: Graphing Linear Equations - Mixed Practice Problems NCTM Illuminations: Bathtub Water Levels (Slope-Intercept: Negative Slope) f. Rate of Change Virtual Nerd: Rate Of Change
a. Inside Mathematics: Performance Assessment Tasks

Resources
Application Resources (Downloadable Lessons, Video, Applets, and Online Algebra Notes) a. Algebra 1 Skills: Algebra 1 Skills b. Cliff Notes: Algebra 1 c. Emergent Math: Emergent Math d. EngageNY: Algebra 1 Module 1 e. Georgia Department of Education: 9 - 12 Standards Framework f. Khan Academy: Introduction to Algebra g. Math Open Reference Math Open Reference h. MathBitsNotebook: Algebra 1 i. Mathematics Assessment Project – Mathematics Assessment Resource Service: Assessing 21st Century Mathematics j. Virtual Nerd: Algebra 1
Dictionaries, Calculators, and Templates (Graphs and Graphic Organizers) a. A Maths Dictionary for Kids: Math Charts b. A Maths Dictionary for Kids: Math Dictionary c. Math Open Reference: Calculator d. Math Open Reference: Full-Size Calculator e. North Central Regional Educational Laboratory: Graphic Organizers f. University of Georgia Mathematics Education Program: Interactive Mathematics Dictionary g. Web 2.0: Calculator
Teaching Strategies a. Creative Educator: Authentic Tasks - Write a Great Authentic Task b. Creative Educator: Project-Based Learning c. Illustrative Mathematics: Illustrative Mathematics Homepage d. Math Video Instructional Development Source: Authentic Contexts e. Power Up What Works: Math Strategies that Work Research

Sample Formative Assessment Tasks/Questions
Arithmetic with Polynomials and Expressions (A1.AAPR.1*) a. Illustrative Mathematics Task: Powers of 11 b. NRICH: Quadratic Patterns c. NRICH: Square Number Surprises
Building Functions, Solving Equations and Inequalities, and Describing Relationships (A.NQ.2*; A1.ACE.1*; A1.ACE.2*; A1.AREI.10*) a. Illustrative Mathematics: Cash Box b. Illustrative Mathematics: Equations and Formulas c. Illustrative Mathematics: Reasoning with Linear Inequalities d. Illustrative Mathematics: Rewriting Equations e. Illustrative Mathematics: Same Solutions f. Illustrative Mathematics: Traffic Jam g. MathBitsNotebook: Solving One-Variable Equations - Summary Practice
Graphing Equations (A1.ACE.2*; A1.AREI.10*) a. MathBitsNotebook: MathBitsNotebook - Practice Graphing Linear Equations
Interpret the Meanings of Expressions (A.ASE.1*) a. Illustrative Mathematics: Animal Populations b. Illustrative Mathematics: Delivery Trucks c. Illustrative Mathematics: Delivery Trucks (this is a different approach) d. Illustrative Mathematics: Equivalent Expressions e. Illustrative Mathematics: Mixing Candies f. Illustrative Mathematics: Seeing Dots g. MathBitsNotebook: Basic Algebraic Expression Assessment

Real Number System (A1.NRNS.1*; A1.NRNS.2*; A1.NRNS.3)

a. Illustrative Mathematics: Calculating the Square Root of 2
b. Illustrative Mathematics: Checking a Calculations of a Decimal Point
c. Illustrative Mathematics: Evaluating a Special Exponential Expression
d. Illustrative Mathematics: Evaluating Exponential Expressions
e. Illustrative Mathematics: Operations with Rational and Irrational Numbers
f. Illustrative Mathematics: Rational or Irrational?
g. Illustrative Mathematics: Sums of Rational and Irrational Numbers
h. New Zealand Maths: It Sounds Like Mah Jong

Algebra 1 Unit 2 Title
Reasoning with Linear Equations and Inequalities

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- A1.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
  - Limit to linear in Unit 1. Expands to quadratic in Unit 3 and exponential in Unit 4. Rational functions are not taught in Algebra 1.
  - Understand that linear equations define the relationship between two variables, and graph equations to represent that.
  - Write an equation of a line given a point and slope, both algebraic in model and application
  - Write an equation of a line given at least two points, both algebraic in model and application
- A1.AREI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
  - Apply the properties and operations of real numbers to equations and inequalities to solve for a specified variable (e.g., to solve a slope-intercept equation or point-slope equation for x or another specified variable; or $y \geq mx + b$ for x).
  - Apply the properties and operations of real numbers to the standard form of a linear equation to find the x-intercept, the y-intercept, or the slope of the function.
- A1.AREI.5 Justify that the solution to a system of linear equations is not changed when one of the equations is replaced by a linear combination of the other equation.
  - Define *system of equations* and *solution of a system*.
  - Multiply by the same number on both sides of the equal sign to produce equivalent equations.
  - Replace one equation with the sum of that equation and a multiple of the other to create a system with the same solutions as the original equation.
  - Substitute the common solution (if there is one) into a system to validate every equation.

- A1.AREI.6* Solve systems of linear equations algebraically and graphically focusing on pairs of linear equations in two variables.
    - Determine the approximate solution to a system of linear equations by graphing both equations and estimating the point of intersection.
    - Solve a system of linear equations algebraically (by substitution or elimination/linear combinations) to find an exact solution.
    - Explain why some linear systems have no solutions and identify linear systems that have no solutions.
    - Explain why some linear systems have infinitely many solutions and identify linear systems that have infinitely many solutions.
    - Understand that linear systems can be solved multiple ways and that one method might be more efficient than others. (e.g., $y_1 = mx + b$ and $y_2 = mx + b$ suggests the graphing or substitution method, $y_1 = mx + b$ and $Ax + By = C$ suggests the substitution method).
    - Graph the linear equations of a system to determine if the system has one, none, or infinitely many common solutions (points of intersection).
    - Manipulate the equations within a linear system algebraically (through substitution or elimination) to determine the common solution, if any exists.
    - Verify by substitution that the variables' solutions (x, y) solve the original equations.
- A1.AREI.6a Solve systems of linear equations using the substitution method.
    - Solve and verify the exact solution of a system of equations using substitution.
- A1.AREI.6b Solve systems of linear equations using linear combination.
    - Eliminate a variable algebraically to find an exact solution for a system of linear equations.
    - Verify by substitution that the variable's solutions (x, y) solve the original equations.
- A1.AREI.10* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.
    - Verify that any point on a graph will result in a true equation when their coordinates are substituted into the equation.
- A1.AREI.11* Solve an equation of the form $f(x) = g(x)$ graphically by identifying the x-coordinate(s) of the point(s) of intersection.
    - Understand that point of intersection on the graph of a system of equations, $y = f(x)$ and $y = g(x)$, represents a solution to both equations.
    - Infer that since $y = f(x)$ and $y = g(x)$, $f(x) = g(x)$ by the substitution property.
    - Verify that the x-coordinate of the points of intersection for $y = f(x)$ and $y = g(x)$ are also the solutions for $f(x) = g(x)$.
    - Use a graphing calculator to determine the approximate solutions to a system of equations $f(x)$ and $g(x)$.
- A1.AREI.12* Graph the solutions to a linear inequality in two variables.
    - Graph a linear inequality on a coordinate plane, resulting in a boundary line (solid or dashed) and a shaded half-plane.

- A1.FIF.1a Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
  - Define a function as a relation in which each input (domain) has exactly one output (range).
  - Determine if a graph, table, or set of ordered pairs represents a function.
  - Determine if stated rules (both numeric and nonnumeric) produce ordered pairs that represent a function.
- A1.FIF.1b Represent a function using function notation and explain that $f(x)$ denotes the output of function f that corresponds to the input x.
  - Introduce the function notation $f(x)$ to represent the output or range values of a function.
  - Understand that $f(x)$represents the corresponding output of the function when x is an element of the input of a function.
- A1.FIF.1c Understand that the graph of a function labeled as f is the set of all ordered pairs (x, y) that satisfy the equation $y = f(x)$.
  - Explain the relationship between the graph of f and the graph of the equation $y = f(x)$.
- A1.FIF.2* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.
  - Decode function notation and explain how the output of a function is matched to its input. (e.g., the function $f(x) = 3x^2 + 5$ squares the input, triples the square, and adds five to produce the output).
  - Use order of operations to evaluate a function for a given domain (input) value.
  - Analyze the input and output values of a function based on a problem situation.
  - Identify the real numbers that are not in the domain of a function.
  - Recognize that the domain may change depending upon the context of problem.
- A1.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative~~; relative maximums and minimums; symmetries; end behavior and periodicity~~.
  - Limit to linear in Unit 2. Expand to quadratic in Unit 3 and exponential in Unit 4.
  - Convert a table, graph, set of ordered pairs, or description into function notation by identifying the rule used to turn inputs into outputs and writing the rule.

- A1.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - Limit to linear in Unit 2. Expand to quadratic in Unit 3 and exponential in Unit 4.
  - Analyze the input and output values of a function based on a problem situation.
  - Identify the numbers that are not in the domain of a function recognizing that the domain may change depending upon context of problem.
  - Write the domain and range in various formats (e.g., compound inequalities and $\mathbb{R}$).
- A1.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.
  - Limit to linear in Unit 2. Expand to quadratic in Unit 3 and exponential in Unit 4.
  - Explain the relationship between the average rate of change and $m = (y_2 - y_1)/(x_2 - x_1) = \Delta y/\Delta x$
  - Calculate the average rate of change of a function.
  - Compare the rates of change of two or more functions.
- A1.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; ~~relative maximums and minimums; symmetries; end behavior and periodicity~~. Graph simple cases by hand and use technology for complicated cases.
  - Limit to linear in Unit 2. Expand to quadratic in Unit 3 and exponential in Unit 4.
  - Identify that the parent function for lines as the line $f(x) = x$.
  - Identify the point-slope form of a linear function as $y - y_1 = m(x - x_1)$.
  - Graph a line in point-slope form and use the graph to show where the starting point (x_1, y_1) and the slope (m) are represented on the graph.
  - Identify the slope-intercept form of a linear function as $f(x) = mx + b$.
  - Graph a line in slope-intercept form and use the graph to show where the y-intercept (b) and the slope (m) are represented on the graph.
  - Explain the effects of change of slope (m) and y-intercept (b) on linear functions ($f(x) = mx + b$).

- A1.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function.
  - Limit to linear in Unit 2. Expand to quadratic in Unit 3 and exponential in Unit 4.
  - Identify the point-slope form of a linear function as $y - y_1 = m(x - x_1)$.
  - Identify the slope-intercept form of a linear function as $f(x) = mx + b$.
  - Identify the standard form of a linear function as $Ax + By = C$.
  - Use definitions of x-intercept and y-intercept to find the intercepts of a standard form line.
  - Relate the constants A, B, and C to the values of the x-intercept, y-intercept, and slope.
- A1.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal.
  - Limit to linear in Unit 2. Expand to quadratic in Unit 3 and exponential in Unit 4.
  - Use equations, verbal descriptions, graphs, and tables to analyze the relationship between quantities or the properties of two functions.
- A1.FLQE.2* Create symbolic representations of linear ~~and exponential~~ functions, including arithmetic ~~and geometric sequences~~, given graphs, verbal descriptions, and tables.
  - Limit to linear in Unit 2. Expand to exponential in Unit 4.
  - Determine if a function is linear given a graph, table of values, or a description of the relationship.
  - Write a linear function algebraically from a graph, table of values, or a description.
  - Describe the algebraic process used to construct a linear function from two given points.

New Academic Vocabulary for This Unit
• Average Rate Of Change • Boundary • Combinations Method • Function Notation ($f(x)$ *notation for* y) • Half-Plane • Interval • Linear Inequality • Point-Slope Form • Relation • Standard Equation Form (*for linear functions*) • Substitution Property

Prior Knowledge Required for this Unit
In earlier grades/units, students have developed conceptual knowledge and have had the opportunity to learn how to: • Extend previous understandings of Order of Operations (7.EEI.3). • Expand foundational knowledge of inequality solution sets, including compound equalities (6). • Understand function concepts, including one-to-one domain to range, particularly as communicated within ordered pairs (8.F.1). • Apply linear functions (particularly in the form of $y = mx + b$)through table, equation, and graphical form, and identify value and meanings of slope/rate and y-intercept/initial value as found within real-world and mathematical situations (8.F.3; 8.F.4; 8.F.5; 8.EEEI.6). • Extend Grade 8's conceptual knowledge of linear functions and expand the definition of linear function, which was limited to $y = mx + b$, to include function notation $f(x)$. • Solve for a variable in a multiple variable equation or inequality in real-world and mathematical situations (Algebra 1 Unit 1 - A1.ACE.1*; A1.ACE.2*; A1.ACE.4*). • Investigate the concept of linear systems through graphs or algebraic (substitution and elimination) methods. • Understand when one, none, and infinitely many solutions arise through application or inspection of linear systems (8.EEI.7b and c; 8.EEI.8). Unit 2 in Algebra 1 will scaffold this specific concept to improve mastery and extend understanding of linear systems and their solution sets. For example, in Algebra 1, the difficulty of the solution of a linear system (e.g. non-integer answers) may increase, such as expected in standard A1.AREI.11 (which compare $f(x)$ and $g(x)$ functions for linear equations). The additional standards in Unit 2 of Algebra 1 makes linear systems more rigorous in Algebra 1 than when introduced in Grade 8. • Understand and write the concept of inequalities from within real-world and mathematical situations (7.NS.4b).

Subsequent Knowledge Related to this Unit

• Algebra 1 Unit 2 includes creating one and two variable linear equations extended from Algebra 1 Unit 1, which studied the relationships among variables (Algebra 1 Unit 1: Relationships Between Quantities and Expressions) (A1.ASE.1; A1.ASE.2).
    ○ Will be applied and expanded to include quadratic (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions) and exponential (Algebra 1 Unit 4: Modeling and Analyzing Exponential Functions).
• Algebra 1 Unit 2 deepens knowledge of linear functions through graphical, symbolic, or tabular forms and how to measure the average rate of change (A1.FIF.6).
    ○ Will utilize the application of the functions' graphical, symbolic, or tabular form, particularly to measure the average rate of change (A1.FIF.6* in Algebra 1 Units 3, 4, and 5).
• Algebra 1 Unit 1 (A1.ACE.4) requires fluent variable manipulation and empowers students in Algebra 1 Unit 2 to rewrite functions of equations (A1.ACE.2*; A1.AREI.3).
    ○ Will be extended to rewrite quadratic functions (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions) and to compare such functions (Algebra 1 Unit 5: Comparing and Contrasting Functions) by solving for and substituting equivalent algebraic value(s).
• Algebra 1 Unit 1 (A1.AAPR.1) involves the development of foundational knowledge regarding algebraic terms and polynomial expressions and the properties of operations applied to polynomials. In Algebra 1 Unit 2, algebraic fluency of these concepts deepens and broadens while studying linear applications.
    ○ Will be extended to quadratic applications and factoring (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions) and other functions in units to follow.
• Algebra 1 Unit 2 explores linear function relationships in two variables expressed in table, equation and graphical forms (A1.AREI.10).
    ○ Will continue to be developed as students explore quadratic (Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions), exponential (Algebra 1 Unit 4: Modeling and Analyzing Exponential Functions), and other function relationships expressed in table, equation and graphical forms.
• Algebra 1 Unit 2 applies the function notation $f(x)$ (A1.FIF.1*).
    ○ Will extend the function notation $f(x)$ in all subsequent units and courses.
• Algebra 1 Unit 2 integrates linear systems and linear inequality graphing, lending towards extension of linear inequality systems (A1.AREI.12*).
    ○ Will develop linear inequality systems formally in Algebra 2.

- Algebra 1 Unit 2 develops skills with writing equations of lines (A1.FIF.7*; A1.FIF.8*).
  - Will be extended in Geometry to build on this foundation, such as when writing of lines that are parallel and perpendicular (G.GGPE.5*).

Relationship Among Standards in this Unit
The standards of Unit 2 expand students' prior knowledge of functions, specifically linear functions. Students learn function notation, analyze concepts of domain and range, and explore linear relationships through graphic, tabular, and algebraic representations. Students will identify the critical attributes of linear functions (e.g., rate of change, intercepts, domain, and range) and understand that arithmetic sequences are linear functions. As students further extend their prior knowledge of systems of linear functions and whether or not they have one, none, or infinitely many common solutions (8.EEI.7), they transfer conceptual understandings to apply other algebraic methods (i.e., substitution, combination, and elimination). Students study the relationships between variables and linear functions and linear inequalities in tabular, graphing, and algebraic formats and in real world and mathematical situations. Students also graph two-variable linear inequalities in preparation for subsequent courses. They communicate their understanding of the solution sets of inequalities in algebraic, verbal, and graphic representations. Unit 2 limits standards A1.ACE.2*, A1.FIF.4* through A1.FIF.9*, and A1.FLQE.2* to their linear function applications. This conceptual knowledge, however, is foundational for studies in subsequent units applied to quadratic functions (Unit 3) and exponential functions (Unit 4); therefore, these standards are located in multiple units within this Algebra 2 support document.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 2.*
Function Notation and Evaluative Meanings (A1.FIF.1*; A1.FIF.2*; A1.FIF.5*) a. Instructional Materials Building Functions Solving Equations and Inequalities Describing Variables Mathematics Assessment Project – Mathematics Assessment Resource Service: Best Buy Tickets b. Exploring Functions Georgia Department of Education: Fiona Task
Linear Modeling, Creating Equations with Two Variables, and Graphing Linear Functions (A1.ACE.2*; A1.FIF.5*; A1.FIF.6*) a. Average Rate of Change Virtual Nerd: Average Rate of Change Review, Slope, and Table-Method b. Creating Equations and Graphing Linear Functions Khan Academy: Linear Function Graphing Videos on Relationships Discussion, Slope Discussion, Equations Example 1, and Equations Example 2
Writing Equations of Lines (A1.FIF.7*; A1.FIF.8*) a. Point-Slope Form Khan Academy: Introduction To Point-Slope Video Virtual Nerd: Rewriting From Slope-Intercept Into Standard Or Point-Slope Formats Virtual Nerd: Writing Equations Of Lines In Point-Slope Format, Finding “B” When Given Point And Slope, and Given A Parallel Line And Point b. Two-Points Khan Academy: Writing Equation Of A Line Given In Point-Slope Given Two Points c. Applications Open Middle: Write A Linear Function

Linear Systems (A1.FIF.9*; A1.AREI.5; A1.AREI.6*; A1.AREI.6a*; A1.AREI.6b*; A1.AREI.10*; A1.AREI.11*; A1.FIF.4*) a. Linear Systems - General Understanding Cliffs Notes: Linear Systems General Information EngageNY: Creating Systems Of Equations Georgia Department of Education: Methods For Solving Systems Of Equations – Georgia Standards Of Excellence Unit 2 – Graphic Organizer Georgia Department of Education: Solve Systems Of Linear Equations By Elimination – Graphic Organizer MathEdPage: Systems With Teacher Notes And Applications Mathematics Assessment Project – Mathematics Assessment Resource Service: Maximizing Profit - Online Lesson Plan Mathematics Assessment Project – Mathematics Assessment Resource Service: Solving Linear Equations In Two Variables - Online Lesson Plan
Graphing Inequalities with Two-Variables (A1.AREI.12*) a. Graphing Two-Variable Inequalities EngageNY: Graphing Inequalities with Two-Variables Mathematics Assessment Project – Mathematics Assessment Resource Service: Representing Inequalities Graphically - Online Lesson Plan Virtual Nerd: Graphing Two-Variable Inequalities Videos - Determining Boundary Line and Graphing Two-Variable Inequalities

Resources
• A Maths Dictionary for Kids: Math Charts • Algebra 1 Skills: Algebra 1 Skills • Cliff Notes: Algebra 1 • Desmos: Explore Math with Desmos • Emergent Math: Emergent Math • EngageNY: Algebra 1 • Georgia Department of Education: 9 - 12 Standards Framework • Georgia Department of Education: Georgia Instructional Framework Teacher's Edition (TE) • Graphing Stories: Graphing Stories Homepage • Illuminations: Resources for Teaching Math By NCTM • Khan Academy: Introduction to Algebra • Math Education Page: Math Education Homepage • Math Open Reference: Math Open Reference Homepage • MathBitsNotebook: Algebra 1 Online Study Resources • Virtual Nerd: Algebra 1 • Virtual Nerd: Algebra Skills Videos

Sample Formative Assessment Tasks/Questions
Solving Linear Equations and Systems of Equations (A1.ACE.2*; A1.AREI.1*; A1.AREI.3*; A.AREI.6*; A.AREI.11*) a. Illustrative Mathematics: Accurately Weighing Pennies I b. Illustrative Mathematics: Collinear Points (A1.AREI.10*) c. Illustrative Mathematics: Estimating a Solution Via Graphs d. Illustrative Mathematics: Find a System e. Illustrative Mathematics: Fishing Adventures 3 (systems of linear inequalities) f. Illustrative Mathematics: Pairs of Whole Numbers g. Illustrative Mathematics: Solution Sets (systems of linear inequalities) h. Illustrative Mathematics: Solving Two Equations in Two Unknowns i. Illustrative Mathematics: Taxi (A1.AREI.10*) j. New Zealand Maths: Renting a Car k. NYC Department of Education: The Cycle Shop l. Task: A Typical Envelope

Interpreting Functions (A1.FIF.1a; A1.FIF.1b; A1.FIF.1c; A1.FIF.2*; A1.FIF.4*; A1.FIF.5*; A1.FIF.6*; A1.FIF.7*; A1.FIF.8*; A1.FIF.9*)

a. Illustrative Mathematics: 10000 is Half of 2000 (A1.FIF.6*)
b. Illustrative Mathematics: Cell Phones (A1.FIF.2*)
c. Illustrative Mathematics: Domains
d. Illustrative Mathematics: Function Notation I (A1.FIF.1b)
e. Illustrative Mathematics: Hoisting the Flag I
f. Illustrative Mathematics: Hoisting the Flag II
g. Illustrative Mathematics: How is the Weather? (A1.FIF.5)
h. Illustrative Mathematics: Interpreting the Graph
i. Illustrative Mathematics: Laptop Battery Charge (A1.FIF.6*)
j. Illustrative Mathematics: Mathemafish Population (A1.FIF.6*)
k. Illustrative Mathematics: Pizza Place Promotion
l. Illustrative Mathematics: Playing Catch
m. Illustrative Mathematics: Points on a Graph
n. Illustrative Mathematics: Random Walk I (A1.FIF.2*)
o. Illustrative Mathematics: Random Walk II (A1.FIF.2*)
p. Illustrative Mathematics: Temperature Change (A1.FIF.6*)
q. Illustrative Mathematics: The Customers (A1.FIF.1a)
r. Illustrative Mathematics: The High School Gym (A1.FIF.6*)
s. Illustrative Mathematics: The Parking Lot (A1.FIF.1a)
t. Illustrative Mathematics: Warming and Cooling
u. Illustrative Mathematics: Words-Tables-Graphs (A1.FIF.9*)
v. Illustrative Mathematics: Yam in the Oven (A1.FIF.2*)
w. Illustrative Mathematics: Your Father (A1.FIF.1a)
x. Practice Task - Understanding and Using Function Notation (A1.FIF.2)

Symbolic Representation of Linear Function (A1.FLQE.2*)

a. Illustrative Mathematics Task: Do Two Points Always Determine a Linear Function?
b. Open Middle problem - Write a Linear Function

a. Inside Mathematics: Performance Assessment Tasks

Algebra 1 Unit 3 Title
Modeling and Analyzing Quadratic Functions

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- A1.ASE.2* Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions.
  - Apply models for factoring and multiplying polynomials.
  - Justify why equivalent expressions are equivalent applying the laws of algebra.
  - Identify patterns in the structure of expressions (e.g., like terms, common factors, difference of squares, perfect squares).
  - Write expressions in multiple ways to reveal something about the problem it represents.
- A1.NRNS.1* Rewrite expressions involving simple radicals and rational exponents in different forms.
  - Use the operations of addition, subtraction, and multiplication to simplify expressions involving radicals (e.g., square roots and cube roots).
  - Factor out perfect squares to simplify expressions involving radicals.
  - Rationalize a denominator.
  - Expand properties of exponents to write equivalent expressions that include radicals (e.g., square roots and cube roots) and rational exponents.
- A1.ASE.3* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
  - Illustrate properties of a function represented through expressions written in multiple ways.
  - Predict whether a quadratic will have a minimum or a maximum based on the value of a.
  - Identify the maximum and minimum of a quadratic written in the form $a(x-h)^2 + k$
  - Complete the square to rewrite a quadratic expression ($ax^2 + bx + c$) with the form $a(x-h)^2 + k$.
- A1.ASE.3a Find the zeros of a quadratic function by rewriting it in equivalent factored form and explain the connection between the zeros of the function, its linear factors, the x-intercepts of its graph, and the solutions to the corresponding quadratic equation.
  - Connect the linear factors of a quadratic expression ($ax^2 + bx + c$) to the graphic and algebraic representations of the zeros of the function.
- A1.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable. (Limit to linear; quadratic; ~~exponential with integer exponents~~.)
  - Identify the variable and quantities in real-world contexts.
  - Use context to choose the applicable algebraic model (linear equation, linear inequality, or quadratic equation).
  - Write, solve, and interpret the solution of an equation or a linear inequality.

- A1.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; exponential with integer exponents; direct and indirect variation.)
  - Identify the variables and quantities in real-world contexts.
  - Use context to choose the applicable algebraic model (e.g., linear, quadratic).
  - Write and graph equations in two or more variables, and interpret relationships among variables.
- A1.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
  - Isolate variables to rewrite equations and formulas in equivalent forms.
- A1.AREI.1* Understand and justify that the steps taken when solving simple equations in one variable create new equations that have the same solution as the original.
  - Apply order of operations and inverse operations to solve equations.
  - Construct an argument to justify a solution process.
  - Demonstrate that the solution of the equation solves the equations created for each step of the process.
- A1.AREI.4* Solve mathematical and real-world problems involving quadratic equations in one variable.
  - Identify and interpret the meaning of the y-intercept.
  - Identify and interpret the meaning of the x-intercepts.
  - Explain why some functions have more than one x-intercept.
  - Define "increasing interval" as a set of function inputs for which the output increases as the input increases.
  - Define "decreasing interval" as a set of function inputs for which the output decreases as the input increases.
  - Identify and explain the contextual meaning of increasing and decreasing intervals and the relative maximum or minimum of a table or graph.
  - Identify the reflective symmetry in a table or graph.
  - Use the problem situation to explain why the function has symmetry.
- A1.AREI.4a* Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-h)^2 = k$ that has the same solutions. Derive the quadratic formula from this form.
  - Identify a quadratic expression, $ax^2 + bx + c$.
  - Identify a perfect-square trinomial by noticing if *a* and *c* are perfect squares and if $b = 2ac$.
  - Factor a perfect-square trinomial.
  - Complete the square of $ax^2 + bx + c$ to write the quadratic in the form $(x-p)^2 = q$.
  - Derive the quadratic formula by completing the square of $(ax^2 + bx + c)$.

- A1.AREI.4b* Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b. (Limit to non-complex roots.)
  - Determine the best method to solve a quadratic equation in one variable (e.g. by inspection, finding square roots, completing the square, applying the quadratic formula, or factoring).
  - Explain that complex solutions result when the radicand is negative in the quadratic formula ($b^2 - 4ac < 0$)
- A1.FBF.3* Describe the effect of the transformations $f(x)$, $f(x) + k$, $f(x + k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to linear; quadratic; ~~exponential with integer exponents;~~ vertical shift and vertical stretch.) (Limit to linear and quadratic in unit 3; expands to exponential with integer exponents in units 4 and 5).
  - Explain why $f(x) + k$ translates the original graph of $f(x)$ up k units and why $f(x) - k$ translates the original graph of $f(x)$ down k units.
  - Explain why $f(x + k)$ translates the original graph of $f(x)$ left k units and why $f(x - k)$ translates the original graph of $f(x)$ right k units.
  - Describe the transformation that changed a graph of $f(x)$ into a different graph when given pictures of the pre-image and the image.
  - Determine the value of k given a graph of a transformed function.
  - Graph transformations of the graph of $f(x)$ when given the value(s) of k [$f(x)\pm k$, $f(x\pm k)$, and $f(x\pm k)\pm k$].
  - Generate and compare examples of functions with different k values.
- A1.FIF.1* Extend previous knowledge of a function to apply to general behavior and features of a function.
- A1.FIF.1a Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
  - Define a function as a relation in which each input (domain) has exactly one output (range).
  - Determine if a graph, table, or set of ordered pairs represents a function.
  - Determine if stated rules (both numeric and nonnumeric) produce ordered pairs that represent a function.
- A1.FIF.1b Represent a function using function notation and explain that $f(x)$ denotes the output of function f that corresponds to the input x.
  - Recognize that when x is an element of the input of a function, $f(x)$ represents the corresponding output of the function.
- A1.FIF.1c Understand that the graph of a function labeled as f is the set of all ordered pairs (x, y) that satisfy the equation $y = f(x)$.
  - Explain that the graph of f is the graph of the equation $y = f(x)$.

- A1.FIF.2* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.
  - Decode function notation and explain how the output of a function is matched to its input. (e.g., "The function $f(x) = 3x^2 + 5$ squares the input, triples the square, and adds five to produce the output").
  - Use order of operations to evaluate a function for a given domain (input) value.
  - Analyze the input and output values of a function based on the context of a problem.
  - Identify the real numbers that are not in the domain of a function.
  - Recognize that the domain might change depending upon the context of problem.
- A1.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; ~~end behavior and periodicity~~. (Limit to linear; quadratic; ~~exponential~~. Limit to linear and quadratic in Unit 3; expands to exponential with integer exponents in Units 4 and 5.)
  - Connect this content to the content of A1.AREI.4*.
  - Create a graph that matches the description and shows all of the key features of the function, if applicable, the y-intercept, x-intercept(s), increasing interval, decreasing interval, relative minimum, relative maximum, and symmetry.
  - Explain the meaning of all the key features included in a graph or verbal description.
- A1.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. (Limit to linear; quadratic; ~~exponential.~~ Limit to linear and quadratic in Unit 3; expands to exponential with integer exponents in Units 4 and 5).
  - Analyze the input and output values of a function in algebraic, graphic, tabular, and verbal forms.
- A1.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context. (Limit to linear and quadratic in Unit 3)
  - Calculate the average rate of change of a function presented in various forms.
  - Compare the rates of change of two or more functions presented in various forms.
  - Use appropriate units to interpret the meaning of the average rate of change.

- A1.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases. (Limit to linear and quadratic in Unit 3; expands to exponential only in the form $y = a^x + k$ in Unit 4).
  - Explain that the parent function for a quadratic function is the parabola $f(x) = x^2$.
  - Explain that the minimum or maximum of a quadratic is called the vertex.
  - Identify whether the vertex of a quadratic will be a minimum or maximum by looking at the equation.
  - Find the y-intercept of a quadratic by substituting 0 for x and evaluating the function.
  - Estimate the vertex of a quadratic by evaluating different values of x.
  - Use calculated values while looking for a maximum or minimum to decide if the quadratic has x-intercepts.
  - Estimate the x-intercepts of a quadratic by evaluating different values of x.
  - Graph a quadratic using evaluated points.
  - Use technology to graph a quadratic and to find precise values for the x-intercept(s) and the maximum or minimum.
- A1.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; ~~exponential~~.) (Limit to linear and quadratic in Unit 3; expands to exponential with integer exponents in Units 4 and 5.)
  - Compare the relationship between quantities using equations, verbal descriptions, graphs, and tables of the same function.

- A1.FIF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - Explain that there are three forms of quadratic functions: standard form, vertex form, and factored form.
  - Explain that standard form is $f(x) = ax^2 + bx + c$.
  - Explain that the vertex form is $f(x) = a(x-h)^2 + k$.
  - Explain that factored form is $f(x) = a(x-x_1)(x-x_2)$.
  - Explain that the graph of all three forms is a parabola.
  - Explain that all parabolas can be written in standard and vertex form, but that parabolas without x-intercepts can be expressed in factored form using real numbers.
  - Find the x-intercepts of a quadratic written in factored form.
  - Use the x-intercepts of a quadratic function to find the axis of symmetry.
  - Use the axis of symmetry of a quadratic to find the vertex of a parabola.
  - Identify the line of symmetry and vertex of a quadratic written in vertex form.
  - Sketch the graph of a parabola written in vertex form.
  - Determine if a quadratic written in vertex form has x-intercepts by looking at the equation.
  - Use algebra to find the x-intercepts of a quadratic written in vertex form.
  - Convert a standard form quadratic to factored form by factoring.
  - Convert a standard form quadratic to vertex form by completing the square.
  - Demonstrate that the standard, factored, and vertex forms of the same quadratic function produce the same values for the x-intercepts, the y-intercept, and the vertex.
  - Write the function that describes a parabola in all three forms when given a graph with the x-intercepts, y-intercept, and vertex labeled.
- A1.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; ~~exponential~~. Limit to linear and quadratic in Unit 3; expands to exponential with integer exponents in Units 4 and 5.)
  - Provide insight into the properties of linear and quadratic functions by comparing different representations of two functions.

New Academic Vocabulary for This Unit
• Average Rate of Change • Axis of Symmetry • Complete the Square • Factor Completely • Inspection • Intercept Form • Perfect Square Trinomial • Quadratic Equation • Quadratic Expression • Quadratic Formula • Radicand • Square Root Method • Translate • Vertex • Vertex Form • y-Axis Symmetry • Zero of a Function • Zeros

Prior Knowledge Required for this Unit
In earlier grades or units, students have developed conceptual knowledge and been expected to: ● Create one and two variable linear equations (Unit 1: A1.ASE.1 and A1.ASE.2). ● Rewrite simple expressions with radicals (Unit 1: A1.NRNS.1*) ● Find square roots (8.EEI.2) ● Apply fluent variable manipulation to rewrite functions (Unit 1: A1.ACE.4). ● Deepen algebraic conceptual knowledge and terminology regarding polynomial expressions and the properties of operations applied to polynomials (Unit 1: A1.AAPR.1). ● Represent functions in two variables in tabular, equation and graphical forms; compare two different functions using different forms (Unit 1: A1.AREI.10). ● Apply knowledge of linear functions, particularly the function notation of $f(x)$ and how that relates to understanding of function concepts including 1-to-1 domain to range, particularly as communicated within ordered pairs (Unit 2 standards).

Subsequent Knowledge Related to this Unit

• Algebra 1 Unit 3 models that the graph of any quadratic function is a vertical and/or horizontal shift of a vertical stretch or shrink of the basic quadratic function $f(x) = x^2$.
  ○ Will expand to the comparison of functions in Algebra 1 Unit 5 and Algebra 2.
• Algebra 1 Unit 3 explores the vertices of quadratic functions as the relative maximum or minimum. The ordered pair of a vertex includes the output value of the function and the input at which change occurs.
  ○ Will expand to the comparison of functions in Algebra 1 Unit 5 and Algebra 2.
• Algebra 1 Unit 3 demonstrates that the Quadratic Formula can be used to solve any quadratic equation.
  ○ Will expand to the comparison of functions in Algebra 1 Unit 5 and Algebra 2.
• Algebra 1 Unit 3 models and analyzes quadratic functions.
  ○ Will expand to the study of exponential functions in Algebra 1 Unit 4.
• Algebra 1 Unit 3 compares and contrasts linear and quadratic functions.
  ○ Will expand to exponential functions in Algebra 1 Unit 4.

Relationship Among Standards in this Unit
The standards in Unit 3 focus on quadratic functions, equations, and applications. Students will determine if a relation is a quadratic function by analyzing information gathered from tables, graphs, and expressions. They will explore variable rate of change and determine how the rate of change for a quadratic function differs from the rate of change for a linear function. Unit 3 explores critical attributes of quadratic functions, including finding the vertex and axis of symmetry of the graph of any polynomial function; using the graph, table, or factors to identify the zeros; and determining if the vertex is the maximum or minimum value of the function. Access to a variety of technologies, such as graphing utilities, spreadsheets, and computer algebra systems, to solve problems will support mastery of these standards. This unit guides students to recognize the quadratic parent function $f(x) = x^2$, and to explain how a translation of the parent function can stretch the graph, compress it, or move it left, right, up, or down. Students will factor general quadratic expressions and solve general quadratic equations by factoring to find only real solutions. Through contextual examples and models of objects that are thrown in the air and allowed to fall subject to the force of gravity, students will deepen their conceptual understanding to interpret quadratic functions and their solutions. Students will study, apply, and justify the quadratic formula. They will convert the formula for a quadratic function from standard to vertex form and use the vertex form to find real solutions (zeros) that cannot be solved by factoring. Students will develop the quadratic formula by completing the square. They will determine the circumstances when one can take the square root of both sides of an equation. After a thorough analysis of the relative advantages and disadvantages of solving a quadratic function by factoring, completing the square, quadratic formula, or taking the square root of both sides, students will solve equations by choosing the most efficient method.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 3.*
Rewriting Expressions and Equations (A1.NRNS.1; A1.AREI.1*) Better Lesson: Simplify and Rewrite Radicals as Rational Exponents and Vice Versa Better Lesson: Solving Equations
Analysis of Quadratic Functions (FA.FBF.3*; A1.ASE.2; A1.ASE.3(3a); A1.AREI.4; A1.FIF.1*(1a,1b,1c); A1.FIF.8 (8a); A1.FIF.9*) a. Transforming the graph of the quadratic parent function $f(x) = x^2$ b. Graphing Quadratics (FA.FIF.4*, FA.FIF.5*, FA.FIF.6*, FA.FIF.7*) Khan Academy: Quadratics Purplemath: Graph Quadratics CoolMath: Graphing Quadratic Parabolas Virtual Nerd: Graph Quadratics Better Lessons: Completing the Square Day 2 c. Rewriting Quadratics in Different Forms MathBitsNotebook: Vertex Form Khan Academy: Quadratics d. Solving Quadratics Khan Academy: Quadratics Purplemath: Solve Quadratics CoolMath: Solving Quadratics Mathematics Assessment Project – Mathematics Assessment Resource Service: Solving Quadratic Equations
Creating and Solving Quadratic Equations in Terms of Context (A1.ACE.1*; A1.ACE.2*; A1.ACE.4*; A1.AREI.4* (4a and 4b); A1.FIF.2*) Better Lesson: Modeling with Quadratic Functions LearnZillion: Complete the Square in a Quadratic Expression Math Is Fun: Real World Quadratic Equations Math Is Fun: Quadratic Equations Math Is Fun: Solving Formulas for One Variable Khan Academy: How To Interpret Quadratic Models Khan Academy: How To Model a Real-World Quadratic Model

Resources
Dictionaries, Calculators, and Templates (Graphs and Graphic Organizers) a. Explicit Vocabulary Building A Maths Dictionary for Kids: A Maths Dictionary Math Open Reference: Math Open Reference b. Calculator Tools Web 2.0: Calculator c. Graphing Tools and Other Resources A Maths Dictionary for Kids: Math Charts Printable Resources Math Warehouse: Interactive Parabolas TeacherVision: Graphic Organizer Printables
Teaching Strategies a. Creative Educator: Project-Based Learning b. Creative Educator: Authentic Tasks - Write a Great Authentic Task c. Google: Project-Based Learning d. Illustrative Mathematics: Illustrative Mathematics e. Math VIDS: Authentic Contexts f. Power Up What Works: Math Strategies that Work Research

Quadratic Lesson Modules, Teacher Notes, PowerPoints, Videos, Applets, and Activities

a. Cliff Notes – Math: Algebra
b. Emergent Math: Current Education Issues in Mathematics, Blog, and Curriculum Map
c. EngageNY: Algebra 1
d. EngageNY: Introduction to Quadratic Function and Graphing
e. Georgia Performance Standards: Math 9 - 12
f. Howard County Public School System Algebra 1: Algebra 1
g. Khan Academy: Introduction of Algebra
h. Math Open Reference: Math Open Reference
i. MathBitsNotebook: Algebra 1 Online Study Resources
j. Mathematics Assessment Project – Math Assessment Resource Service: Homepage
k. The Concord Consortium: Graphing Quadratic Equations
l. Virtual Nerd: Algebra Skills Videos

Sample Formative Assessment Tasks/Questions
• Better Lesson: Quadratic Functions In Three Forms • Better Lesson: Review Or Move On (To The Quadratic Formula) • Mathematics Assessment Project – Math Assessment Resource Service: Summative Assessment Task: Sorting Functions • Mathematics Assessment Project – Math Assessment Resource Service: Formative Assessment Lesson: Generalizing Patterns: Table Tiles • Mathematics Assessment Project – Math Assessment Resource Service: Formative Assessment Lesson: Representing Quadratic Functions Graphically • Mathematics Assessment Project – Math Assessment Resource Service: Formative Assessment Lesson: Sorting Equations and Identities • Mathematics Assessment Project – Math Assessment Resource Service: Solving Quadratic Equations • Mathematics Assessment Project – Math Assessment Resource Service: Lesson Materials: Solving Quadratic Equations: Lesson Files • Mathematics Assessment Project – Math Assessment Resource Service: Lesson Materials: Formative Assessment Lesson - Sorting Equations and Identities: Lesson Files • Mathematics Assessment Project – Math Assessment Resource Service: Formative Assessment Lesson: Representing Quadratic Functions Graphically: Lesson Files • NCTM: MARS Formative Assessment Lesson - Generalizing Patterns: Lesson Files

Algebra 1 Unit 4 Title
Modeling and Analyzing Exponential Functions

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- A1.FLQE.1* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.
  - Compare slopes between multiple points on of a linear function and slopes between multiple points of an exponential function.
  - Compare situations such as hourly wages (linear) vs. population growth (increasing exponentially).
- A1.FLQE.2* Create symbolic representations of ~~linear and~~ exponential functions, including ~~arithmetic and~~ geometric sequences, given graphs, verbal descriptions, and tables. (Limit to ~~linear~~; exponential.)
  - Write algebraic equations for graphs representing exponential growth (increase) or decay (decrease).
  - Write algebraic equations to represent exponential patterns described in words.
  - Write algebraic equations to represent a given table of values.
- A1.ACE.1* Create and solve equations ~~and inequalities~~ in one variable that model real-world problems involving linear, quadratic, ~~simple rational~~, and exponential relationships. Interpret the solutions and determine whether they are reasonable. (Limit to linear; quadratic; exponential with integer exponents.)
  - Identify the variable and quantities in real-world contexts.
  - Use context to choose the applicable algebraic model (linear equation, linear inequality, quadratic equation, or exponential).
  - Write, solve, and interpret the solution of an equation.
  - Expand what was done in previous units to include exponential functions ($f(x) = b^x$).
- A1.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; exponential with integer exponents; ~~direct and indirect variation~~.)
  - Identify the variables and quantities in real-world contexts.
  - Use context to choose the applicable algebraic model (e.g., linear, quadratic, exponential).
  - Write and graph equations in two or more variables, and interpret relationships among variables.
  - Expand what was done in previous units to include exponential ($f(x) = b^x$).

- A1.FBF.3* Describe the effect of the transformations $kf(x)$, $f(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to ~~linear; quadratic~~; exponential with integer exponents; vertical shift and vertical stretch.)
  - Explain why *f(x)+k* translates the original graph of $f(x)$ up k units and why $f(x)-k$ translates the original graph of $f(x)$ down k units.
  - Explain why $f(x+k)$ translates the original graph of $f(x)$ left k units and why $f(x-k)$ translates the original graph of $f(x)$ right k units.
  - Describe the transformation that changed a graph of *f(x)* into a different graph when given pictures of the pre-image and the image.
  - Determine the value of k given a graph of a transformed function.
  - Graph transformations of the graph of $f(x)$ when given the value(s) of k [$f(x)\pm k$, $f(x\pm k)$ and $f(x\pm k)\pm k$].
  - Generate and compare examples of functions with different k values.
  - Expand what was done in previous units to include exponential functions ($f(x)=b^x$).
- A2.FBF.1* Write a function that describes a relationship between two quantities. (Note that FBF.1a and FBF.1b are Algebra 2 standards.)
  - Determine if a model exhibits an exponential growth (increasing) or decay (decreasing) relationship.
  - Determine the base of growth (increase) or decay (decrease) (e.g., half-life, population doubling).
  - Write a function in the form of $f(x)=b^x$ to represent the exponential relationship between variables.
- A1.FIF.1a Understand that a function from one set (called the domain) to another set (called the "range") assigns to each element of the domain exactly one element of the range.
  - Define a function as a relation in which each input (domain) has exactly one output (range).
  - Determine if a graph, table, or set of ordered pairs represents a function.
  - Determine if stated rules (both numeric and nonnumeric) produce ordered pairs that represent a function.
  - Expand knowledge of domain and range from previous units to exponential functions ($f(x)=b^x$).
- A1.FIF.1b Represent a function using function notation and explain that $f(x)$ denotes the output of function f that corresponds to the input x.
  - When *x* is an element of the input of a function, $f(x)$ represents the corresponding output of the function.
  - Expand knowledge from previous units to exponential functions ($f(x)=b^x$).
- A1.FIF.1c Understand that the graph of a function labeled as f is the set of all ordered pairs (x,y) that satisfy the equation $y=f(x)$.
  - Explain that the graph of f is the graph of the equation $y=f(x)$.
  - Expand knowledge from previous units to exponential functions ($f(x)=b^x$).

- A1.FIF.2* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.
  - Decode function notation and explain how the output of a function is matched to its input. (e.g., The function $f(x) = 3(2)^x + 5$ multiplies the 2 to itself x times, triples that value, and adds five to produce the output).
  - Use order of operations to evaluate a function for a given domain (input) value.
  - Analyze the input and output values of a function based on the context of a problem.
  - Identify the real numbers that are not in the domain of a function.
  - Recognize that the domain might change depending upon the context of problem.
  - Expand knowledge from previous units to exponential functions ($f(x) = b^x$).
- A1.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; ~~relative maximums and minimums; symmetries~~; end behavior and ~~periodicity~~. (Limit to ~~linear; quadratic~~; exponential.)
  - Create a graph that matches the description and shows all of the key features of the function, if applicable, the y-intercept, x-intercept(s), increasing interval, decreasing interval, and end behavior.
  - Explain the meaning of all the key features included in a graph or verbal description.
- A1.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. (Limit to ~~linear; quadratic~~; exponential.)
  - Analyze the input and output values of an exponential function in algebraic, graphic, tabular, and verbal forms.
- A1.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context. (Limit to ~~linear; quadratic~~; exponential.)
  - Calculate the average rate of change of a function presented in various forms.
  - Compare the rates of change of two or more functions presented in various forms.
  - Use appropriate units to interpret the meaning of the average rate of change.

- A1.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; ~~relative maximums and minimums; symmetries;~~ end behavior and ~~periodicity~~. Graph simple cases by hand and use technology for complicated cases. (Limit to ~~linear; quadratic~~; exponential only in the form $y = a^x + k$.)
    - Explain that the parent function for an exponential function is the function $y = a^x$.
    - Find the y-intercept of an exponential by substituting 0 for x and evaluating the function.
    - Explain that the parent function for an exponential function has a horizontal asymptote at $y = 0$, and that this is the asymptote no matter what base is given.
    - Understand that horizontal asymptotes describe the function's end behavior.
    - Explain when a horizontal asymptote is not $y = 0$ based on the value of k from the form $y = a^x + k$.
    - Explore exponential growth (increasing) and exponential decay (decreasing) functions.
    - Understand that exponential functions are strictly increasing or strictly decreasing.
    - Explain the difference between the graphs of exponentials that have a base where $a > 1$ and a base where $0 < a < 1$.
    - Graph an exponential using evaluated points.
    - Use technology to graph an exponential and to find precise values for the y-intercept.
- A1.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to ~~linear; quadratic~~; exponential.)
    - Compare the relationship between quantities using equations, verbal descriptions, graphs, and tables of the same function.
- A1.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; exponential.)
    - Provide insight into the properties of linear, quadratic, and exponential functions by comparing different representations of the functions.

New Academic Vocabulary for This Unit
• Constant percent rate • End behavior • Exponential • Exponential Function • Exponential growth/decay (increase/decrease) • Geometric sequence • Half-life • Horizontal asymptote • Horizontal translation • Parent exponential function ($f(x) = b^x$) • Vertical translation

Algebra 1 Unit 4: Modeling and Analyzing Exponential Functions

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Prior Knowledge Required for this Unit
In earlier grades or units, students have developed conceptual knowledge and been expected to: ● identify variables and what they represent (6.EEI.6). ● explain what a function is, determine if a table, graph or set of ordered pairs is a function, and compare linear and non-linear functions (8.F.1 and 8.F.3). ● apply inverse operations to isolate variables and solve equations, including equations with variables on both sides (6.EEI.7; 7.EEI.4b). ● apply the order of operations to simplify expressions (6.EEI.2). ● apply the properties and laws of exponents (7.EEI.5). ● apply the Pythagorean Theorem (8.GM.7). ● recognize slope as a rate of change (how the change in one quantity affects the change in another quantity) (8.F.4). ● create models to represent and manipulate data (graphs, tables, equations) (6.EEI.9). ● identify linear or quadratic functions in given contexts, create linear and quadratic equations, and interpret their solutions (Algebra 1 Units 1 and 3; A1.ACE.1*) . ● write and interpret inequalities using notation and symbols(Algebra 1 Unit 1; A1.ACE.1*). ● graph points, equations, and inequalities on the coordinate plane (Algebra 1 Unit 1; A1.AREI.10*; Algebra 1 Unit 2; A1.AREI.12*). ● create graphs with appropriate scales and labels for variables (Mathematical Practice 6; 8.F.1). ● solve equations applying the distributive property and combining like terms (7.EEI.4). ● convert units (e.g. dimensional or unit analysis, rounding, etc.) with accuracy and precision to solve problems involving measurement (Mathematical Practice 6; 4.MDA.1; 5.MDA.1; 7.GM.1; Algebra 1 Unit 1; A1.NQ.1* and A1.NQ.3*).

Subsequent Knowledge Related to this Unit

• Unit 4 Algebra 1 applies related knowledge of linear and quadratic equations and the laws of exponents to construct and solve simple exponential equations.
  ○ Will compare and contrast different types of functions in Unit 5 (Comparing and Contrasting Functions). Subsequent high school courses (e.g., Algebra 2) build upon this knowledge to deepen understanding and broaden applications.
• Unit 4 Algebra 1 extends and deepens understanding of functions so that they can recognize and explain how $f(x)$ behaves at different domain intervals (e.g., x approaches 0, $x > 0$, $x < 0$, x approaches positive or negative infinity, growth or decay by a constant percent rate) for any function, and calculate and explain the average rate of change for a given interval.
  ○ Will compare and contrast different types of functions in Unit 5 (Comparing and Contrasting Functions). Subsequent high school courses (e.g., Algebra 2) build upon this knowledge to deepen understanding and broaden applications.
  ○ Will expand the study of content standard A1.FLQE.1* (linear functions change at a constant rate in any interval compared to other functions that change at a constant percent rate for a given interval) in Unit 4 to include A1.FLQE.1a in Unit 5, which requires students to prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
• Unit 4 Algebra 1 expands foundational knowledge for identifying the relationship between variables (domain and range) through contextual conditions as well as algebraic, tabular, or graphic representations of variables.
  ○ Will expand understanding of function parameters in terms of context in Units 5 (Comparing and Contrasting Functions) and Unit 6 (Describing Data) (A1.FLQE.5*).
• Unit 4 Algebra 1 explores data in different representations and encourages students to draw conclusions and to interpret and determine the reasonableness of solutions for linear, quadratic, and exponential functions.
  ○ Will expand the exploration of data through the comparison of different types of functions in Unit 5 (Comparing and Contrasting Functions) and Unit 6 (Describing Data). Subsequent high school courses (e.g., Algebra 2) build upon this knowledge to deepen understanding and broaden applications.
  ○ Will expand understanding of finding solutions for different types of functions and determining the reasonableness of solutions within a given context (A1.FLQE.5*).
• Unit 4 Algebra 1 expands knowledge of sequences to include geometric sequences and how they relate to their algebraic function notation.
  ○ Will expand understanding of sequences in subsequent high school courses (e.g., Algebra 2) to build upon this knowledge, deepen understanding, and broaden applications.

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- Unit 4 Algebra 1 deepens conceptual understanding of key features of functions including intercepts; asymptotes; intervals where functions are increasing, decreasing, constant, positive, or negative; and end behavior.
  - Will expand understanding of function parameters in terms of context in Units 5 (Comparing and Contrasting Functions) and Unit 6 (Describing Data) (A1.FLQE.5*).
- Unit 4 Algebra 1 provides opportunities to use technology appropriately to explore and experiment with variations of a function, specifically exponential models (e.g., the effect of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative), to determine the effects on the graph.
  - Will expand understanding of sequences in subsequent high school courses (e.g., Algebra 2) to build upon this knowledge, deepen understanding, and broaden applications.
- Unit 4 Algebra 1 compares similarities and differences of two different functions represented in different ways (e.g. tables, graphs, verbal descriptions or equations).
  - Will expand the use of graphs and tables in Unit 5 (Comparing and Contrasting Functions) to observe how a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or more generally as a polynomial function (A1.FLQE.3*).
- Unit 4 Algebra 1 provides exponential models to find the value of k given the graphs, and to construct meaning of solutions by comparing different representations.
  - Will expand understanding of sequences in subsequent high school courses (e.g., Algebra 2) to build upon this knowledge, deepen understanding, and broaden applications.

Relationship Among Standards in this Unit
The standards in Unit 4 expand prior knowledge of integer exponents and focus on exponential functions, equations, and their applications. Students graph exponential functions, identify key features (intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; asymptotes; end behavior) and use appropriate technology to investigate multiplicative change in exponential functions and increase conceptual understanding. Applying knowledge of exponential functions and knowledge acquired in Units 2 (linear functions) and 3 (quadratic functions), students determine whether a function is exponential, linear, or quadratic. Students use exponential function notation to model a variety of contexts and use context to interpret functions in specific applications. Analysis of graphs, verbal descriptions, and tables supports students in creating symbolic representations of exponential functions, including geometric sequences, to model a relationship between two quantities. Manipulating the variable k for specific values, students also examine the effects on the graph on the parent function, $f(x) = b^x$, by replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative). As the unit progresses, students compare exponential models, solve exponential equations, interpret the solutions of exponential functions, and find the value of k when given exponential graphs.

Possible Instructional Strategies/Lessons
The order of the topics below illustrates a possible instructional order for Unit 4.

Defining Exponential Relationships (A2.FBF.1*; A1.FLQE.1*; A1.FLQE.2*; A1.FIF.6*; A1.FIF.8*; A1.FIF.9*)

a. Introduction to Exponential Behavior
   Algebra 1 Lessons WCCUSD: Exponential Functions - Writing Exponential Functions Based on Data: An Introductory Lesson
   GoalBook: Equivalent Exponential Expressions Teaching Strategies
   Illustrative Mathematics: Decaying Dice- Investigation
   Illustrative Mathematics: Last Person Standing - Investigation
   Illustrative Mathematics: Paper Folding - Investigation
   Illustrative Mathematics: The Bank Account
   Illustrative Mathematics: Valuable Quarter - Investigation
   Khan Academy: Introduction to Exponential Functions
   Khan Academy: How to Model a Real-World Context with an Exponential Function
   RegentsPrep: Geometric Sequences and Series
   RegentsPrep: Exponential Growth and Decay
   RegentsPrep: Activity for Discovering Exponential Growth and Decay
   VirtualNerd: What's An Exponential Function?
   VirtualNerd: What Is Exponential Growth?
   Zona Land Education: Exponential Functions
b. Distinguish between Linear and Exponential Relationships
   GoalBook: Linear and Exponential Functions Teaching Strategies
   Illustrative Mathematics: Boiling Water - Investigation
   Illustrative Mathematics: Finding Linear and Exponential Models
   Illustrative Mathematics: Do Two Points Always Determine an Exponential Function?
   Khan Academy: How to Construct Linear and Basic Exponential Functions From a Table of Values
   Khan Academy: How to Construct Linear and Exponential Functions From Their Graphs
   Khan Academy: How to Distinguish Between Linear and Exponential Models
   VirtualNerd: How Do You Identify Exponential Behavior From a Pattern in the Data?
c. Compare and Contrast Linear, Quadratic, and Exponential Functions
   Video: How To Tell if a Table is Linear, Quadratic, or Exponential

d. Constant Percent Rate of Change
LearnZillion: Understand the rate of change in exponential growth functions

Solving Exponential Equations and Functions (A1.ACE.1*; A1.ACE.2*; A1.FIF.1a; A1.FIF.1b; A1.FIF.2*;)

a. Evaluating Exponential Functions utilizing Function Notation
Illustrative Mathematics: Carbon 14 Dating
VirtualNerd: What's a Function?
VirtualNerd: How Do You Evaluate an Exponential Function?
VirtualNerd: How Do You Solve a Problem With Exponential Growth?
Zona Land Education: Compound Interest

b. Solving Simple Exponential Equations
Monterey Institute: Introduction to Exponential Functions (Text)
Monterey Institute: Introduction to Exponential Functions (Video & Lesson)
Algebra 1 Lessons WCCUSD: Solving Exponential Equations
Khan Academy: Solving Basic Exponential Models

Graphing Exponential Functions (A1.FBF.3*; A1.FIF.1c; A1.FIF.4*; A1.FIF.5*; A1.FIF.7*)

a. Transformations of the Parent Function
Algebra 1 Lessons WCCUSD: Graphing Exponential Functions
Illustrative Mathematics: Identifying Exponential Graphs - Investigation
MathBitsNotebook: Features of Exponential Functions
RegentsPrep: Transforming Functions
RegentsPrep: Ideas for Working With Transformations
RegentsPrep: Using Transformations to Investigate Functions
Zona Land Education: Transform f(x)=2x

b. Graph Given the Equation
Khan Academy: How to Graph an Exponential Function Given its Formula
VirtualNerd: How Do You Graph an Exponential Function Using a Table?

Resources
Dictionaries, Calculators, and Templates (Graphs, Graphic Organizers) a. A Maths Dictionary for Kids: Explicit Vocabulary Building b. A Maths Dictionary for Kids: Math Charts Printable Resources c. Math Open Reference: Calculator Tools d. Math Warehouse: Interactive Parabola e. Teacher Vision: Graphic Organizer Printables f. Web 2.0: Calculator
Teaching Strategies a. Creative Educator: Success Begins with Effective Design b. Creative Educator: Writing Authentic Tasks c. Google: Problem-Based Learning d. Illustrative Mathematics: Illustrative Mathematics e. Math VIDS: Authentic Contexts f. Power Up What Works: Math Strategies that Work Research
Exponential Lesson Modules, Teacher Notes, PowerPoints, Videos, Applets, and Activities a. Regents Exam Prep Center: Exponential Functions

Defining Exponential Relationships (A1.FLQE.1*; A1.FIF.8*)

a. Introduction to Exponential Behavior
   Algebra 1 Lessons WCCUSD: Exponential Functions - Writing Exponential Functions Based on Data: An Introductory Lesson
   GoalBook: Equivalent Exponential Expressions Teaching Strategies
   Illustrative Mathematics: Decaying Dice- Investigation
   Illustrative Mathematics: Last Person Standing - Investigation
   Illustrative Mathematics: Paper Folding - Investigation
   Illustrative Mathematics: The Bank Account
   Illustrative Mathematics: Valuable Quarter - Investigation
   Khan Academy: Introduction to Exponential Functions
   Khan Academy: How to Model a Real-World Context with an Exponential Function
   RegentsPrep: Exponential Growth and Decay
   RegentsPrep: Activity for Discovering Exponential Growth and Decay
   VirtualNerd: What's An Exponential Function?
   VirtualNerd: What Is Exponential Growth?
   Zona Land Education: Exponential Functions

b. Distinguish between Linear and Exponential Relationships
   GoalBook: Linear and Exponential Functions Teaching Strategies
   Illustrative Mathematics: Boiling Water - Investigation
   Illustrative Mathematics: Finding Linear and Exponential Models
   Illustrative Mathematics: Do Two Points Always Determine an Exponential Function?
   Khan Academy: How to Construct Linear and Basic Exponential Functions From a Table of Values
   Khan Academy: How to Construct Linear and Exponential Functions From Their Graphs
   Khan Academy: How to Distinguish Between Linear and Exponential Models
   VirtualNerd: How Do You Identify Exponential Behavior From a Pattern in the Data?

Solving Exponential Equations and Functions (A1.ACE.1*; A1.ACE.2*; A1.FIF.1a; A1.FIF.1b; A1.FIF.2*)

a. Evaluating Exponential Functions utilizing Function Notation
   Illustrative Mathematics: Carbon 14 Dating
   VirtualNerd: What's a Function?
   VirtualNerd: How Do You Evaluate an Exponential Function?
   VirtualNerd: How Do You Solve a Problem With Exponential Growth?
   Zona Land Education: Compound Interest
b. Solving Simple Exponential Equations
   Monterey Institute: Introduction to Exponential Functions (Text)
   Monterey Institute: Introduction to Exponential Functions (Video & Lesson)
   Algebra 1 Lessons WCCUSD: Solving Exponential Equations
   Khan Academy: Solving Basic Exponential Models

Graphing Exponential Functions (A1.FBF.3*; A1.FIF.1c; A1.FIF.4*; A1.FIF.5*; A1.FIF.7*)

a. Transformations of the Parent Function
   Algebra 1 Lessons WCCUSD: Graphing Exponential Functions
   Illustrative Mathematics: Identifying Exponential Graphs - Investigation
   MathBitsNotebook: Features of Exponential Functions
   RegentsPrep: Transforming Functions
   RegentsPrep: Ideas for Working With Transformations
   RegentsPrep: Using Transformations to Investigate Functions
   Zona Land Education: Transform $f(x) = 2x$
b. Graph Given the Equation
   Khan Academy: How to Graph an Exponential Function Given its Formula
   VirtualNerd: How Do You Graph an Exponential Function Using a Table?

a. Georgia Department of Education: Georgia Performance Standards
b. Mathematics Assessment Project – Mathematics Assessment Resource Service: Math Assessment Resource Service
c. Virtual Nerd: Algebra Skills Videos

Sample Formative Assessment Tasks/Questions
Defining Exponential Relationships (A1.FLQE.1*; A1.FIF.8*) a. GoalBook: Equivalent Exponential Expressions b. GoalBook: Linear and Exponential Functions c. Khan Academy: Construct Basic Exponential Functions d. Khan Academy: Construct Exponential Functions Given the Graph e. Khan Academy: Distinguish Between Linear and Exponential Models f. Illustrative Mathematics: Allergy Medication- Performance Task g. Illustrative Mathematics: Boom Town - Performance Task h. Illustrative Mathematics: Exponential Parameters - Performance Task i. Illustrative Mathematics: Exponential Growth Versus Linear Growth j. Illustrative Mathematics: Linear or Exponential? k. Illustrative Mathematics: Solving Problems with Linear and Exponential Models - Performance Task l. MathBitsNotebook: Exponential Growth and Decay
Writing and Solving Exponential Equations (Includes Geometric Sequences) and Functions (A1.FLQE.2*; A1.ACE.1*; A1.ACE.2*; A2.FBF.1, A1.FIF.1a; A1.FIF.1b; A1.FIF.2*) a. CPALMS: Writing an Exponential Function From a Description b. Illustrative Mathematics: All Your Base Are Belong to Us - Performance Task c. Illustrative Mathematics: DDT-cay- Performance Task d. Illustrative Mathematics: Predicting the Past - Performance Task e. Illustrative Mathematics: Uranium 238 - Performance Task f. Mathematics Vision Project: Arithmetic and Geometric Sequences g. Monterey Institute: Introduction to Exponential Functions (Practice & Review Tabs) h. RegentsPrep: Practice with Applied Exponential Growth/Decay i. RegentsPrep: Practice with Exponential Equations and Graphs

Graphs of Exponential Functions (A1.FBF.3*; A1.FIF.1c; A1.FIF.4*; A1.FIF.5*; A1.FIF.6*; A1.FIF.7*; A1.FIF.9*) a. CPALMS: Air Cannon: Calculating Average Rate of Change; Comparing Linear and Exponential Rates of Change b. Illustrative Mathematics: Exponential Functions c. Khan Academy: Graphs of Basic Exponential Functions d. MathBitsNotebook: Practice Page - Exponential Functions e. Mathematics Assessment Project – Mathematics Assessment Resource Service: Representing Functions of Everyday Situations f. RegentsPrep: Working with Transformations and Functions (Practice)
a. Georgia Department of Education: Mathematics Teacher Support b. Gizmos: Resource Catalog c. IXL Learning: Algebra 1 Practice d. LearnZillion: Algebra Math Video Lessons e. MathBits: Working with Algebra Tiles f. Mathematics Assessment Project – Mathematics Assessment Resource Service: Classroom Challenges (Formative Assessment Lessons) g. Mathematics Assessment Project – Mathematics Assessment Resource Service: Guide for Teachers and Administrators h. National Library of Virtual Manipulatives: Algebra Virtual Manipulatives i. Quia: Mathematics Shared Activities j. Regents Exam Prep Center: Algebra k. SAS Curriculum Pathways: SAS Algebra 1 Course l. Study Zone: Intermediate Test Prep (6-8) m. Virtual Nerd: Algebra 1 n. West Contra Costa Unified School District: Algebra 1 Lessons

Algebra 1 Unit 5 Title
Comparing and Contrasting Functions

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- A1.FLQE.1* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.
  - Compare slopes between multiple points of a linear function and slopes between multiple points of an exponential function.
  - Compare situations such as hourly wages (linear) vs. population growth (exponential).
  - Use context to decide if a rate of change is a linear (constant rate per unit) or an exponential (constant percent rate per unit) model.
  - Compare linear functions ($y = mx + b$) to exponential functions ($y = ax + k$).
- A1.FLQE.1a *(Note: FA.FLQE.1a is not a Graduation Standard.)* Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
  - Determine for linear functions that the differences are the same between consecutive y-values whose x-values are equal distances apart.
  - Determine for exponential functions that the differences are increasing or decreasing by a constant factor between consecutive y-values whose x-values are equal distances apart.
  - Use tabular and graphical representations to show these constant patterns.
  - Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
  - Demonstrate that a linear function has a constant rate of change (slope).
  - Demonstrate that an exponential function has a constant multiplier (common ratio) over equal intervals.
- A1.FLQE.2* Create symbolic representations of linear and exponential functions, including arithmetic and geometric sequences, given graphs, verbal descriptions, and tables. (Limit to linear; exponential.)
  - Write equations given a graph, a verbal description, and/or a table of values.
  - Explain differences in arithmetic and geometric sequences to extend previous knowledge.
  - Write equations for arithmetic or geometric sequences.

- A1.FLQE.3* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or more generally as a polynomial function.
  - Compare the graphs of exponentials, quadratics, and linear functions on the same coordinate plane.
  - Compare the tables of exponentials, quadratics, and linear functions at the same time using the same x-values.
  - Use graphs or tables to compare the output values of linear, quadratic, and exponential functions.
  - Estimate the intervals for which the output of one function is greater than the output of another function when given a table or graph.
  - Use technology to find the point at which the graphs of two functions intersect.
  - Use the points of intersection to precisely describe the intervals for which the output of one function is greater than the output of another function.
  - Explain why exponential functions eventually have greater output values than linear and quadratic functions by comparing simple functions of each type.
- A1.FLQE.5* Interpret the parameters in a linear ~~or exponential~~ function in terms of the context. (Limit to linear.)
  - Use a given situation to Identify limits to possible domain and range values. (e.g., When calculating job wages for a week, there is a limit to the number of hours a worker has in a week to earn wages.)
  - Identify the names and definitions of parameters m and b in the linear function $f(x) = mx + b$.
  - Explain the meaning (using appropriate units) of the slope of a line when the line models a real-world relationship.
  - Explain the meaning (using appropriate units) of the y-intercept of a line when the line models a real-world relationship.
- A1.FBF.3* Describe the effect of the transformations $kf(x)$, $f(x) + k$, $f(x + k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to linear; quadratic; exponential with integer exponents; vertical shift and vertical stretch.)
  - Extend previous knowledge (Unit 4) using all three types of functions.
  - Determine the parent function and its transformation for a given problem.
- A1.FIF.1* Extend previous knowledge of a function to apply to general behavior and features of a function.
  - Extend previous knowledge (Units 2, 3) using linear and quadratic functions.
  - Determine maximum/minimum values, increasing/decreasing, symmetry, end behavior, intercepts, and positive/negative regions of a function.

- A1.FIF.1a Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
  - Extend previous knowledge (Units 2, 3, 4) using all three types of functions.
- A1.FIF.1b Represent a function using function notation and explain that $f(x)$ denotes the output of function that corresponds to the input *x*.
  - Extend previous knowledge (Units 2, 3, 4) using all three types of functions.
  - Recognize that when x is an element of the input of a function, $f(x)$ represents the corresponding output of the function.
- A1.FIF.1c Understand that the graph of a function labeled as $f(x, y)$ is the set of all ordered pairs (x, y) that satisfy the equation $y = f(x)$.
  - Extend previous knowledge (Units 2, 3, 4) using all three types of functions.
  - Explain that the graph of f is the graph of the equation $y = f(x)$.
- A1.FIF.2* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.
  - Extend previous knowledge (Units 2, 3, 4) using all three types of functions.
  - Identify whether the context of the problem involves linear, quadratic, or exponential functions.
- A1.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. (Limit to linear; quadratic; exponential.)
  - Connect this content to the content of A1.AREI.4*.
  - Extend previous knowledge (Units 2, 3, 4) using all three types of functions.
- A1.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. (Limit to linear; quadratic; exponential.)
  - Extend previous knowledge (Units 2, 3, 4) using all three types of functions.
  - Analyze the input and output values of all three types of functions in algebraic, graphic, tabular, and verbal forms.
- A1.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.
  - Extend previous knowledge (Units 2, 3, 4) using all three types of functions.

- A1.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases.
  - Extend previous knowledge (Units 2, 3, 4) using all three types of functions.
  - Identify whether the function given is linear, quadratic, or exponential.
  - Graph a function using evaluated points.
  - Use technology to graph a function and to find precise values for the key features.
  - Determine the domain, range, and end behavior (horizontal asymptote) of an exponential function when examining the graph.
  - Classify exponential functions in function notation as growth or decay.
  - Substitute convenient values for x to create a table and graph for all three types of functions.
  - Use technology to analyze graphs and tables of difficult linear, quadratic, or exponential functions.
- A1.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; exponential.)
  - Extend previous knowledge (Units 2, 3, 4) using all three types of functions.
  - Continue comparisons of different representations of two functions to increase understanding and applications of the properties of linear, quadratic, and exponential functions.

New Academic Vocabulary for This Unit
• Parameters of a Function (Limits) • Polynomial Function

Prior Knowledge Required for this Unit
In earlier grades or units, students have developed conceptual knowledge and been expected to: ● Find the slope of a line given two points. (8.F.4) ● Find the slope of a line given a graph. (8.F.4) ● Find the slope of a line given a table. (8.F.4) ● Explain that slope is a constant rate of change for linear functions while slope is a constant percent rate of change for a given interval for other functions. (8.F.3, 8.F.4, A1.FIF.6; Algebra 1 Units 2-4) ● Find the average constant rate of change or average constant percent rate of change for an interval. (A1.FIF.6; Algebra 1 Units 2-4) ● Explain how $f(x)$behaves at different domain intervals. (A1.FIF.1; Algebra 1 Units 2 – 4) ● Write an equation of a linear function using function notation. (A1.FIF.1; Algebra 1 Unit 2) ● Write an equation for a quadratic function using function notation. (A1.FIF.1; Algebra 1 Unit 3) ● Write an equation for an exponential function using function notation. (A1.FIF.1; Algebra 1 Unit 4) ● Graph linear functions. (A1.ACE.2; Algebra 1 Units 1-2; A1.FIF.7; ; Algebra 1 Units 2) ● Graph quadratic functions. (A1.ACE.2; Algebra 1 Unit 3; A1.FIF.7; Algebra 1 Unit 3) ● Graph exponential functions. (A1.ACE.2; Algebra 1 Unit 4; A1.FIF.7; Algebra 1 Unit 4) ● Construct and solve simple exponential equations. (A1.ACE.1; Algebra 1 Unit 4; A1.AREI.11; Algebra 1 Unit 4) ● Identify and describe the transformations that have been applied to a parent function. (A1.FBF.3; Algebra 1 Units 3 – 4) ● Identify the names and definitions of parameters and key features of a function such as max/min, increasing/decreasing, symmetry, intercepts, asymptotes, and end behavior. (A1.FIF.4; Algebra 1 Units 2 – 4; A1.FIF.7; Algebra 1 Units 2 – 4) ● Identify the domain and range of a function through contextual conditions and multiple representations of variables. (A1.FIF.5; Algebra 1 Units 2 – 4) ● Draw conclusions and interpret the reasonableness of solutions for linear, quadratic, and exponential functions. (A1.ACE.1; Algebra 1 Units 1, 3, and 4) ● Use technology appropriately to explore variations of functions. (A1.FBF.3; Algebra 1 Units 3 – 4)

Subsequent Knowledge Related to this Unit

• Unit 5 Algebra 1 explores different representations of data (input/output) for different types of functions. Students compare properties, draw conclusions, interpret solutions, and determine the reasonableness of answers for linear, quadratic, and exponential functions.
  ○ Will expand the exploration of data in Unit 6 (Describing Data). Subsequent high school courses (e.g., Algebra 2) build upon this knowledge to deepen understanding and broaden applications.
• Unit 5 Algebra 1 continues to compare constant rate per unit intervals and constant percent rate per unit intervals. This unit compares the growth of linear functions over equal intervals and the growth of exponential functions over equal intervals to demonstrate that linear functions grow by equal differences while exponential functions grow by equal factors. Students calculate the average rate of change for given intervals and use context to interpret the meaning.
  ○ Will expand concepts of linear and exponential growth in subsequent high school courses (e.g., Algebra 2) to build upon this knowledge, deepen understanding, and broaden applications.
• Unit 5 Algebra 1 compares, contrasts, and analyzes general behavior, the domain (input), range (output), and other key features of linear, quadratic, other polynomial, and exponential functions. Students sketch graphs and write algebraic functions, including arithmetic and geometric sequences, for linear and exponential data presented in multiple representations.
  ○ Will expand the exploration of data in Unit 6 (Describing Data). Subsequent high school courses (e.g., Algebra 2) build upon this knowledge to deepen understanding and broaden applications.
• Unit 5 Algebra 1 compares quantities increasing exponentially to quantities increasing linearly, quadratically, or more generally as a polynomial function through different representations (tables, graphs) to conclude that exponential functions eventually have greater output values than polynomial functions.
  ○ Will expand understanding of that the output values of exponential functions eventually exceed the output values of polynomial functions in subsequent high school courses (e.g., Algebra 2) to build upon this knowledge and broaden applications.
• Unit 5 Algebra 1 explores the parameters of linear functions in terms of a given context.
  ○ Will increase understanding of the parameters of linear functions when used to describe data for given contexts in Unit 6 (Describing Data).
  ○ Will extend to explore parameters of exponential functions in terms of a given context (A1.FLQE.5*) in subsequent high school courses (e.g., Algebra 2).

- Unit 5 Algebra 1 deepens conceptual knowledge of linear, quadratic, and exponential functions by having students transform parent functions, use function notation to represent real-world contexts, and evaluate functions.
    - Will continue the use of function notation to represent real-world data in Unit 6 (Describing Data). Subsequent high school courses (e.g. Algebra 2) build upon this knowledge to deepen understanding and broaden applications.

Relationship Among Standards in this Unit
Unit 5 standards increase students' conceptual knowledge of linear, quadratic, and exponential functions by describing the domain and range of functions and by using different representations and contexts to compare and contrast key features. All standards except FLQE.1a, 3, and 5 have been taught in Units 1 – 4. The purpose of revisiting these standards is to go deeper into the similarities and differences of each type of function and to connect mathematical ideas to expand students' conceptual understanding. Students continue to use and interpret function notation, write linear and exponential functions to include arithmetic (linear) sequences and geometric (exponential) sequences, evaluate functions using domain values, describe functions' behaviors (e.g. *x* increasing, *x* decreasing, or *x* approaching zero), and identify the effect of one or more transformations on parent functions. Students apply knowledge of these types of functions to solve problems and analyze situations to determine or produce appropriate linear, quadratic, or exponential models for functions. New standards introduced in Unit 5 compare the rate of change per unit for linear functions (constant rate of change) to the rate of change per unit for exponential functions (constant percent rate of change). Students analyze intervals in graphs or tables to demonstrate that linear data changes by equal differences (slope) while exponential data changes by equal factors (common ratio) (A1.FLQE.1a) and that data increasing exponentially will at some point exceed data increasing as a polynomial function, specifically linearly or quadratically (A1.FLQE.3*). Students calculate and interpret the average rate of change over a given interval and interpret its meaning if data is given in terms of context. Unit 5 also introduces the limits (parameters) of linear functions in terms of context (A1.FLQE.5).

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 5.*
Comparing Linear and Exponential Growth (A1.FLQE.1 and 1a, A1.FLQE.2*, A1.FLQE.3) a. Comparing Models Illustrative Mathematics: Exponential Growth Versus Linear Growth I Illustrative Mathematics: Exponential Growth Versus Linear Growth II Khan Academy: Distinguish Between Linear and Exponential Models LearnZillion: Distinguish Between Linear and Exponential Functions by Examining Intervals LearnZillion: Distinguish Between Linear and Exponential Functions Using Tables Algebra 4 All: Comparing Linear and Exponential Functions National Math and Science Initiative: Rate of Change
Evaluating Exponential Equations and Functions (A1.FIF.1a; A1.FIF.1b; A1.FIF.2*) a. Using Functional Notation and Evaluating Functions Regents Prep: Functional Notation and Evaluating Functions b. Evaluating Exponential Functions utilizing Function Notation Illustrative Mathematics: Carbon 14 Dating VirtualNerd: What's a Function? VirtualNerd: How Do You Solve a Problem With Exponential Growth? Zona Land Education: Compound Interest c. Solving Simple Exponential Equations Monterey Institute: Introduction to Exponential Functions (Video & Lesson) Algebra 1 Lessons WCCUSD: Solving Exponential Equations Khan Academy: Solving Basic Exponential Models d. Solving Quadratic Equations Better Lesson: Modeling with Quadratic Functions LearnZillion: Complete the Square in a Quadratic Expression Math Is Fun: Real World Quadratic Equations Math Is Fun: Quadratic Equations Khan Academy: How To Interpret Quadratic Models Khan Academy: How To Model a Real-World Quadratic Model

e. Solving Linear Equations
Algebra-Class: Linear Functions Using Function Notation
Algebra-Class: Solving A Linear Function
Socratic: Function Notation and Linear Function

Writing and Graphing Linear and Exponential Functions (A1.FBF.3*, A1.FIF.1c; A1.FIF.4*; A1.FIF.5*; A1.FIF.7*)

a. Transformations of the Parent Function
Algebra 1 Lessons WCCUSD: Graphing Exponential Functions
Illustrative Mathematics: Identifying Exponential Graphs - Investigation
MathBitsNotebook: Features of Exponential Functions
RegentsPrep: Transforming Functions
RegentsPrep: Ideas for Working With Transformations
RegentsPrep: Using Transformations to Investigate Functions
Zona Land Education: Transform f(x)=2x

b. Graph Given the Equation
Khan Academy: How to Graph an Exponential Function Given its Formula
VirtualNerd: How Do You Graph an Exponential Function Using a Table?

c. Writing Linear and Exponential Equations
LearnZillion: Write and Graph An Exponential Function
LearnZillion: Write and Graph A Linear Equation
RegentsPrep: Geometric Sequences and Series

Comparing Linear, Exponential, and Quadratic Functions (A1.FLQE.5*, A1.FLQE.3, A1.FIF.6*, A1.FIF.9*)

a. Comparing Models
Khan Academy: How To Compare the Growth Rate of an Exponential Model And a Quadratic Model
Khan Academy: Compare the Growth Rates of Exponentials And Polynomials
VirtualNerd: How Do You Determine if a Graph Represents a Linear, Exponential, or Quadratic Function?
Algebra 1 Lessons WCCUSD: Comparing Linear and Quadratic Functions
Algebra 1 Lessons WCCUSD: Graphing Family of Function
Illustrative Mathematics: College Fund
Illustrative Mathematics: What Functions Do Two Graph Points Determine?

b. Finding Parameters
   Math Insight: The Parameters of A Linear Function
c. Finding Average Rate of Change
   LearnZillion (Constant Percent Rate of Change): Understand the rate of change in exponential growth functions
   Khan Academy: Average Rate of Change Video Series
   Math Bits: Average Rate of Change For Linear and Non-linear Functions
d. Comparing Two Functions
   LearnZillion: Identify Function Properties
   Online Math Learning: Compare Features and Properties of Two Functions
   PBS Learning Media: Comparing Exponential, Quadratic, and Linear Functions

Resources
Dictionaries, Calculators, and Templates (Graphs, Graphic Organizers) a. A Math Dictionary for Kids: Explicit Vocabulary Building b. A Math Dictionary for Kids: Math Charts c. Math Open Reference: Math Tools d. Web 2.0: Calculator e. Math Warehouse: Interactive Parabola f. TeacherVision: Graphic Organizers
Teaching Strategies a. Power Up What Works: Math Strategies that Work Research: b. Creative Educator: Write a Great Authentic Task c. Math Vids: Authentic Contexts d. Illustrative Mathematics: Homepage e. Creative Educator: Project-Based Learning f. Problem-Based Workshop: Problem-Based Learning g. Graphing Stories: Homepage
Comparing Linear and Exponential Growth (A1.FLQE.1 and 1a, A1.FLQE.2*, A1.FLQE.3) a. CPalms: Exponential Growth Versus Linear Growth b. Khan Academy: How to Distinguish Between Linear and Exponential Models c. My Math Universe: Comparing Linear and Exponential Functions
Evaluating Exponential Equations and Functions (A1.FIF.1a; A1.FIF.1b; A1.FIF.2*) a. Monterey Institute: Introduction to Exponential Functions b. VirtualNerd: Evaluating An Exponential Function
Writing and Graphing Linear and Exponential Functions (A1.FBF.3*, A1.FIF.1c; A1.FIF.4*; A1.FIF.5*; A1.FIF.7*) a. Khan Academy: Constructing Linear and Exponential Functions from Data b. Purplemath: Graphing Linear Functions c. Purplemath: Graphing Exponential Functions
Comparing Linear, Exponential, and Quadratic Functions (A1.FLQE.5*, A1.FLQE.3, A1.FIF.6*, A1.FIF.9*) a. Virtual Nerd: Identifying Linear, Quadratic, and Exponential Functions

Function Lesson Modules, Teacher Notes, PowerPoints, Videos, Applets, and Activities

a. Regents Exam Prep Center: Exponential Functions
b. Regents Exam Prep Center: Function Activity
c. Mathematics Assessment Project – Math Assessment Resource Service: Homepage
d. Georgia Department of Education: Georgia Performance Standards
e. Virtual Nerd: Algebra Skills Videos

Sample Formative Assessment Tasks/Questions
Comparing Linear and Exponential Growth (A1.FLQE.1 and 1a; A1.FLQE.2*; A1.FLQE.3) a. Illustrative Mathematics: Choosing an Appropriate Growth Model b. CPalms: Comparing Linear and Exponential Functions Formative Task c. CPalms: Comparing Quadratic and Exponential Functions
Evaluating Exponential Equations and Functions (A1.FIF.1a; A1.FIF.1b; A1.FIF.2*) a. Georgia Department of Education: Algebra 1 Comparing and Contrasting Functions
Writing and Graphing of Linear and Exponential Functions (A1.FBF.3*; A1.FIF.1c; A1.FIF.4*; A1.FIF.5*; A1.FIF.6*; A1.FIF.7*; A1.FIF.9*) b. Mathematics Assessment Project: Representing Functions of Everyday Situations c. CPalms: Air Cannon: Calculating Average Rate of Change d. CPalms: Comparing Linear and Exponential Rates of Change e. Illustrative Mathematics: Exponential Functions f. Khan Academy: Graphs of Basic Exponential Functions g. MathBitsNotebook: Practice Page - Exponential Functions h. RegentsPrep: Working with Transformations and Functions (Practice)

Comparing Linear, Exponential, and Quadratic Functions (A1.FLQE.5*; A1.FLQE.3; A1.FIF.6*; A1.FIF.9*)

a. Illustrative Mathematics: Comparing Graphs of Functions - Performance Task
b. IXL: Sample Questions For Linear, Quadratic, and Exponential Functions
c. Illustrative Mathematics: Identifying Functions Performance Task
d. William Mueller, Ph.D. Duke University (Families of Functions): http://wmueller.com/precalculus/families/splash.html
e. Georgia Department of Education: Mathematics Teacher Support
f. Gizmos: Resource Catalog
g. IXL Learning: Algebra 1 Practice
h. LearnZillion: Algebra Math Video Lessons
i. MathBits: Working with Algebra Tiles
j. Mathematics Assessment Project – Mathematics Assessment Resource Service: Classroom Challenges (Formative Assessment Lessons)
k. Mathematics Assessment Project – Mathematics Assessment Resource Service: Guide for Teachers and Administrators
l. National Library of Virtual Manipulatives: Algebra Virtual Manipulatives
m. Quia: Mathematics Shared Activities
n. Regents Exam Prep Center: Algebra
o. SAS Curriculum Pathways: SAS Algebra 1 Course
p. Study Zone: Intermediate Test Prep (6-8)
q. Virtual Nerd: Algebra 1
r. West Contra Costa Unified School District: Algebra 1 Lessons

Algebra 1 Unit 6 Title
Describing Data

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- A1.FLQE.5* Interpret the parameters in a linear ~~or exponential~~ function in terms of the context. (Limit to linear.)
    - Identify limits to possible results based on the context of the problem. (e.g., When calculating job wages for a week, there is a limit to the number of hours a worker has in a week to earn wages.)
- A1.SPID.6* Using technology, create scatterplots and analyze those plots to compare the fit of linear, quadratic, or exponential models to a given data set. Select the appropriate model, fit a function to the data set, and use the function to solve problems in the context of the data.
    - Use technology to input data into a table.
    - Create a scatterplot using technology.
    - Determine which type of function fits the scatterplot.
    - Use technology to create a function that fits the scatterplot.
    - Solve problems based on functions created with technology.
- A1.SPID.7* Create a linear function to graphically model data from a real-world problem and interpret the meaning of the slope and intercept(s) in the context of the given problem.
    - Determine the line of best fit for given data.
    - Choose points from the data to find the slope of the line of best fit.
    - Write a linear function for the line of best fit.
    - Identify the coordinate values for the intercepts along the line of best fit.
    - Use a given situation to describe what the slope and intercepts mean in terms of context .
- A1.SPID.8* Using technology, compute and interpret the correlation coefficient of a linear fit.
    - Use technology to input data into a table and calculate the correlation coefficient (the quantitative relationship between two or variables) of a linear fit.
    - Describe the strength of the correlation between the data and line of linear fit in terms of the correlation coefficient (r).
    - Explain that the strength of the correlation depends on how well the changes to one variable predict changes for the other variable.

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New Academic Vocabulary for This Unit
• Bivariate data • Correlation • Correlation coefficient (r) • Line of best fit • Scatterplot • Univariate data

Algebra 1 Unit 6: Describing Data

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Prior Knowledge Required for this Unit
In earlier grades or Algebra 1 units, students have developed conceptual knowledge and have had the opportunity to learn how to: ● Describe numerical data sets in relation to their real-world context (6.DS.5). ● Investigate bivariate data (8.DSP.1). ● Draw an approximate line of best fit on a scatter plot that appears to have a linear association and informally assess the fit of the line to the data points (8.DSP.2). ● Find an approximate equation for the line of best fit using two appropriate data points, interpret the slope and intercept, solve problems using the equation (8.DSP.3). ● Investigate bivariate categorical data in two-way tables, and interpret data in two-way tables using relative frequencies (8.DSP.4). ● Apply concepts of slope and y-intercept to graphs, equations, and proportional relationships (8.EEI.6). ● Investigate the differences between linear and nonlinear functions (8.F.3). ● Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; exponential with integer exponents; direct and indirect variation.) (A1.ACE.2*; Algebra 1 Units 1 - 4). ● Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; exponential.) (A1.FIF.9*; Algebra 1 Units 2 – 5). ● Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval (A1.FLQE.1*; Algebra 1 Units 4 – 5).

Subsequent Knowledge Related to this Unit
● Algebra 1 Unit 6 interprets the parameters of a linear function in terms of a real-world context (A1.FLQE.5). ○ Will extend to explore parameters of exponential functions in terms of a given context in subsequent high school courses (e.g. Algebra 2 - A2.FLQE.5 or Intermediate Algebra - IA.FLQE.5). ● Algebra 1 Unit 6 analyzes bivariate categorical data using marginal, joint and conditional frequencies (A1.SPID.5*). ○ Will build upon this knowledge of bivariate categorical data in subsequent high school courses to deepen understanding and broaden applications (e.g. Probability and Statistics Unit 1 - PS.SPID.5*). ● Algebra 1 Unit 6 uses technology to create scatterplots, fit a function to the data and compute correlation coefficients of a linear fit (A1.SPID.6*; A1.SPID.8*). ○ Will expand conceptual knowledge of regression and correlation coefficients (r) in subsequent high school courses to deepen understanding and broaden applications (e.g. Probability and Statistics Unit 4 - PS.SPID.6*; PS.SPID.8*). ● Algebra 1 Unit 6 creates linear functions to graphically model data from a real-world problem and analyzes the meaning of the slope and intercept(s) (A1.SPID.7*). ○ Will expand conceptual knowledge of applying linear models to represent real-world situations in subsequent high school courses to deepen understanding and broaden applications (e.g. Probability and Statistics Unit 4 - PS.SPID.7*).

Relationship Among Standards in this Unit
Algebra 1 Unit 6 standards focus on data analysis themes of inferences, implications and predictions. This unit requires students to use single variable statistics to make inferences and data in bivariate tables to make predictions and implications. Students review, connect, and apply previously taught Algebra 1 standards involving linear, quadratic and exponential concepts to create regression equations. New learning readdresses important linear concepts of slope and y-intercepts and the limits of a linear functions (A1.FLQE.5*) through connections to real-world regression situations. Using appropriate technology, students will organize, analyze, and model bivariate data with tables, scatterplots, and regression lines (lines of linear fit). This analysis of data and computed correlation coefficients (*r* values) enable students to explain the strength of the correlation between a given set of data and line of linear fit (A1.SPID.6*; A1.SPID.7*; A1.SPID.8*).

Possible Instructional Strategies/Lessons
The order of the topics below illustrates a possible instructional order for Unit 6.

Lines of Fit and Regression (FA.FLQE.5*; FA.SPID.6*; FA.SPID.7*; FA.SPID.8*)

a. Utilize Technology To Create Scatterplots
   Annenberg Learner: Scatterplots
   MathBitsNotebook: Finding Your Way Around the TI-83/84 Calculator - Scatter Plots
   Texas Instruments: Creating a Scatter Plot Using TI-83/84 Family
   Glencoe: Graphing Calculator - Regression Lines
   PurpleMath: Scatterplots and Regressions - Page 1
   SlideShare: Calculating a Correlation Coefficient and Scatter Plot using Excel
   Excel Easy: Excel Scatter Chart
   LearnZillion: Fit a Linear Regression to a Set of Data using Spreadsheets and Graphing Technology
   Math Is Fun: Scatter Plots
   Khan Academy: Constructing a Scatter Plot
   Khan Academy: Studying, Shoe Size, and Test Scores Scatter Plots
   Khan Academy: People Smoking Less Over Time Scatter Plot
   Khan Academy: Constructing Scatter Plot Exercise Example
   Khan Academy: Constructing Scatter Plots
   Math Planet: Scatter Plots and Linear Models
   Bitesize: Interpreting Scatter Diagrams
   NCTM Illuminations: Exploring Linear Data
   NCTM Illuminations: Line of Best Fit tool

b. Select The Appropriate Model Of Best Fit And Use Technology To Create The Function Of Best Fit
   Illustrative Mathematics: Basketball Bounces Collaborative Investigation
   NCTM: Shrinking Candles, Running Water, Folding Boxes
   MathBitsNotebook: Fitting Functions to Data
   Khan Academy: Comparing Models to Fit Data
   PurpleMath: Scatterplots and Regressions - Page 3
   PurpleMath: Scatterplots and Regressions - Page 4
   LearnZillion: Select a Statistical Regression Line

LearnZillion: Model Real-World Bivariate Data By Using A Quadratic Regression Function
LearnZillion: Model Real-World Bivariate Data By Using An Exponential Regression Function
Khan Academy: Fitting Quadratic and Exponential Functions to Scatter Plots
MathBits: Quadratic Regression Model
MathBits: Exponential Regression Model

c. Use Technology To Compute The Correlation Coefficient (*R*) Of A Linear Fit: Use *R* To Determine Significance, Direction And Magnitude Of The Fit
LearnZillion: Find Correlation Coefficient using Technology
University of North Carolina of Wilmington: Correlation and Line of Best Fit
EngageNY: Interpreting Correlation
MathBitsNotebook: Correlations
MathBitsNotebook: Correlation Coefficients
PurpleMath: Scatterplots and Regressions - Page 2
Math Is Fun: Correlation
Algebra 1 Lessons WCCUSD: Correlation and Line of Best Fit
Purplemath: Scatterplots and Regressions
Regents Prep: Scatter Plots and Correlation

d. Interpret The Meaning Of The Slope And Intercept(S) Of Real-World Linear Relationships
LearnZillion: Interpret the Slope and Intercept of a Regression Line
LearnZillion: Understand and Interpret the Slope of a Regression Line
NCTM: Exploring Linear Data
NCTM: Barbie Bungee
Annenberg Learner: Fitting Lines to Data
MathBitsNotebook: Slopes and Intercepts in Linear Models
LearnZillion: Solve Problems Using Linear Regression
Illustrative Mathematics: Hand Span and Height Collaborative Activity

Resources
Dictionaries, Calculators, and Templates (Graphs, Graphic Organizers) a. A Math Dictionary for Kids: Explicit Vocabulary Building b. A Math Dictionary for Kids: Math Charts c. Math Open Reference: Math Tools d. Math Warehouse: Interactive Parabola e. TeacherVision: Graphic Organizers f. Web 2.0: Calculator
Teaching Strategies a. Creative Educator: Project-Based Learning b. Creative Educator: Write a Great Authentic Task c. Graphing Stories: Homepage d. Illustrative Mathematics: Homepage e. Math Vids: Authentic Contexts f. Power Up What Works: Math Strategies that Work Research: g. Problem-Based Workshop: Problem-Based Learning
Bivariate Data (A1.SPID.5*) a. Khan Academy: Analyzing Trends in Categorical Data b. Regents Exam Prep Center: Univariate and Bivariate Data
Lines of Fit and Regression (A1.FLQE.5*; A1.SPID.6*; A1.SPID.7*; A1.SPID.8*) a. Illuminations: Correlation and the Regression Line b. University of Nebraska – Omaha: Linear Regression c. William Mueller, Ph. D.; Duke University: Linear Parameters
Function Lesson Modules, Teacher Notes, PowerPoints, Videos, Applets, and Activities a. Creative Educator: Project-Based Learning b. Georgia Department of Education: Georgia Performance Standards c. Mathematics Assessment Project – Math Assessment Resource Service: Homepage d. Problem-Based Learning: Problem-Based Learning e. Shodor Interactivate: Regression Applet f. Shodor Interactivate: Scatterplot Applet

g. Virtual Nerd: Algebra Skills Videos
Using the TI-83 or TI-84 a. MathBits: Scatterplots b. MathBits: Line of Best Fit c. MathBootCamps: Find the Correlation Coefficient d. Texas Instruments: Calculating and Graphing Linear Regressions

Sample Formative Assessment Tasks/Questions
Bivariate Data (FA.SPID.5*) a. Illustrative Mathematics: Birds' Eggs Performance Task b. Illustrative Mathematics: Texting and Grades 1 Performance Task c. MathBitsNotebook: Practice Two-Way Frequency Tables d. Khan Academy: Trends in Categorical Data
Lines of Fit and Regression (FA.FLQE.5*; FA.SPID.6*; FA.SPID.7*; FA.SPID.8*) a. Khan Academy: Constructing Scatter Plots b. MathBitsNotebook: Practice Fitting Functions to Data c. Khan Academy: Fitting Quadratic and Exponential Functions to Scatter Plots d. MathBitsNotebook: Practice Linear Regression e. IXL: Interpreting a Scatter Plot f. IXL: Calculate Correlation Coefficients g. MathBitsNotebook: Practice with Correlations h. Mathopolis: Correlation Practice Set i. IXL: Match Correlation Coefficients to Scatter Plot j. School Improvement in Maryland: Correlation Coefficient Performance Activity k. MathBitsNotebook: Practice Slope and Intercepts with Linear Regressions l. Khan Academy: Eyeballing the line of Best Fit m. IXL: Scatter Plots - Line of Best Fit n. Khan Academy: Estimating Slope of Line of Best Fit o. Illustrative Mathematics: US Airports Assessment Variation Performance Task p. Illustrative Mathematics: Animal Brains Performance Task q. National Security Agency (nsa.gov): What is a Scatter Plot Saying? r. Radford.edu: Connecing Scatter Plots and Correlation Coefficients Activity s. ENet Learning (enetlearning.org): Scatter Plots and Lines of Best Fit Worksheet t. National Security Agency (nsa.gov): Line of Best Fit u. Denise Kapler: Line of Best Fit Worksheet v. Kamehameha Schools (blogs.ksbe.edu): Line of Best Fit w. Queen Anne High School (maths.qahs.org.uk): Scattergraphs

x. Math Worksheets Land (www.mathworksheetsland.com): Scatter Plots and Line of Best Fit - Worksheet 1
y. North Seattle Community College (facweb.northseattle.edu): Line of Best Fit Classwork

South Carolina College- and Career-Ready Standards for Mathematics
High School Support Document
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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
Relationships Between Quantities and Expressions	Reasoning with Linear Equations and Inequalities	Modeling and Analyzing Quadratic Functions	Modeling and Analyzing Exponential Functions	Comparing and Contrasting Functions	Describing Data
Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
FA.NRNS.1*	FA.ACE.2*	FA.NRNS.1*	FA.FLQE.1*	FA.FLQE.1*	FA.FLQE.5*
FA.NRNS.2*	FA.AREI.3	FA.ACE.1*	FA.ACE.1*	FA.FLQE.1a	FA.SPID.5*
FA.NRNS.3	FA.AREI.5	FA.ACE.2*	FA.ACE.2*	FA.FLQE.3*	FA.SPID.6*
FA.NQ.1*	FA.AREI.6*	FA.ACE.4*	FA.FBF.3*	FA.FLQE.5*	FA.SPID.7*
FA.NQ.2*	FA.AREI.6a	FA.AREI.1*	FA.FIF.1a	FA.FBF.3*	FA.SPID.8*
FA.NQ.3*	FA.AREI.6b	FA.FBF.3*	FA.FIF.1b	FA.FIF.1*	FA.SPMJ.1*
FA.ASE.1*	FA.AREI.10*	FA.FIF.1*	FA.FIF.1c	FA.FIF.1a	FA.SPMJ.2*
FA.ACE.1*	FA.AREI.11*	FA.FIF.1a	FA.FIF.2*	FA.FIF.1b	FA.SPMD.4*
FA.ACE.2*	FA.AREI.12*	FA.FIF.1b	FA.FIF.4*	FA.FIF.1c	FA.SPMD.5*
FA.ACE.4*	FA.FIF.1*	FA.FIF.1c	FA.FIF.5*	FA.FIF.2*	FA.SPMD.6*
FA.AREI.1*	FA.FIF.1a	FA.FIF.2*	FA.FIF.7*	FA.FIF.4*	
FA.AREI.3*	FA.FIF.1b	FA.FIF.4*	FA.FIF.8*	FA.FIF.5*	
FA.AREI.10*	FA.FIF.1c	FA.FIF.5*		FA.FIF.7*	
	FA.FIF.2*	FA.FIF.7*		FA.FIF.9*	
	FA.FIF.4*	FA.FIF.8*			
	FA.FIF.5*	FA.FIF.8a			
	FA.FIF.7*	FA.FIF.9*			
	FA.FIF.8*				
	FA.FIF.9*				

South Carolina College- and Career-Ready Standards for Mathematics
High School Support Document
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Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

1. Make sense of problems and persevere in solving them. a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

Foundations in Algebra Unit 1 Title
Relationships Between Quantities and Expressions

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes

- FA.NRNS.1* Rewrite expressions involving simple radicals and rational exponents in different forms.
  - Apply properties of exponents to write equivalent expressions that include simple radicals (e.g., square roots and cube roots) and integer exponents.
  - Expand properties of exponents to write equivalent expressions that include rational exponents.
- FA.NRNS.2* Use the definition of the meaning of rational exponents to translate between rational exponent and radical forms.
  - Convert expressions with fractional exponents to equivalent radical forms and vice-versa.
- FA.NRNS.3 Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
  - Use the property of set closure to include rational and irrational numbers under addition and multiplication.
- FA.NQ.1* Use units of measurement to guide the solution of multi-step tasks. Choose and interpret appropriate labels, units, and scales when constructing graphs and other data displays.
  - Convert units of measure, as appropriate (e.g., using like units to compare or combine lengths), to solve multi-step performance tasks.
  - Apply dimensional analysis to convert units of measure.
  - Analyze the context of problems to determine the appropriate unit(s) of measure.
  - Select and interpret appropriate units of measure when solving real-world contexts involving formulas.
- FA.NQ.2* Label and define appropriate quantities in descriptive modeling contexts.
  - Identify the variables or quantities from data displayed in a given model (e.g., text, graph, picture, or algebraic formula)
  - Select the appropriate unit of measure for variables or quantities presented in a given model.
- FA.NQ.3* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities in context.
  - Report solutions to problems with the appropriate level of accuracy, and with precision if necessary, for the unit of measure given in the context of the problem and/or the measuring tool used.
- FA.ASE.1* Interpret the meanings of coefficients, factors, terms, and expressions based on their real-world contexts. Interpret complicated expressions as being composed of simpler expressions.
  - Limit to linear expressions for Unit 1; expand to quadratic in Foundations in Algebra Unit 3 and exponential in Unit 4.
  - Rational functions are taught in Intermediate Algebra Unit 5.
  - Recognize that an algebraic expression can be composed of multiple terms and represent unknown real number value(s).
  - Simplify or factor complicated expressions by combining like terms or extracting the Greatest Common Monomial (factor) to show equivalent expressions (e.g., is equivalent to).

- FA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable.
  - Limit to linear in Unit 1
  - Expand to quadratic in Foundations in Algebra Unit 3 and exponential in Unit 4.
  - Rational functions are taught in Intermediate Algebra Unit 5.
  - Use real-world contexts to generate and solve equations and inequalities in one variable.
  - Analyze solutions for their meaning and rationale within the given context.
  - Interpreting solutions includes solutions graphed on a number line.
- FA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
  - Limit to linear and introduce the terms *direct variation* and *indirect variation* in Unit 1.
  - Expand to quadratic in Foundations in Algebra Unit 3 and exponential in Unit 4.
  - Rational functions are not taught in Foundations in Algebra; they are introduced late in the Intermediate Algebra (unit 5) course and extended in Algebra 2.
  - Understand that linear equations define the relationship between two variables.
  - Generate and graph equations to represent the relationship between two variables.
  - Use appropriate labels, units, and scales to represent the relationship of two variables in a given real-world context.
- FA.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
  - Apply the properties and operations of real numbers to solve for a specified variable (e.g., solve a linear equation in standard form and for y; and include other formulas from a variety of disciplines).
- FA.AREI.1* Understand and justify that the steps taken when solving simple equations in one variable create new equations that have the same solution as the original.
  - State the property or operation being applied that explains why each step of solving an equation generates an equivalent equation.
  - Verify by substitution that the variable's solution in the last step solves the equation for the given problem.
- FA.AREI.3* Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
  - Apply the properties and operations of real numbers to equations and inequalities to solve for a specified variable (e.g., solve for x in ax + by = c; solve the slope-intercept equation $y = mx + b$ for b; solve $y = mx + b$ for m).
- FA.AREI.10* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.
  - Use algebraic and graphical formats to justify that the set of solutions is a one-to-one relationship, which can be graphed with ordered pairs (i.e., domain, range).

New Academic Vocabulary for This Unit
• Compound Inequalities (*notation only*) • Direct Variation • Index (Root) • Indirect Variation • Polynomial • Radical • Radicand • Rational Exponents

Prior Knowledge Required for this Unit
In earlier grades, students have developed conceptual knowledge and have had the opportunity to learn how to: ● Understand and apply the constant of proportionality (7.RP.2). ● Determine and apply the constant rate of change (8.F.3; 8.F.4). ● Understand the relationship between independent and dependent variables (6.EEI.9; 8.F.1). ● Be able to evaluate square and cubic roots, and recognize perfect and non-perfect squares as rational and irrational correspondingly (8.EEI.2). ● Transform and apply the Pythagorean Theorem particularly as it relates to rational and irrational squares (8.EEI.2a, b, and d). ● Understand fundamental concepts of functions, including one-to-one relationships, particularly as communicated as domain to range within ordered pairs (8.F.1). ● Understand and apply the properties of exponents (8.EEI.1). ● Graph the solution of one variable inequality on the number line (7.EEI.4c). ● Understand that slope is a rate of change from one quantity in relation to another quantity within real world and mathematical situations (8.EEI.5b). ● Understand the critical attributes of linear and nonlinear functions (8.F.3). ● Represent linear functions, particularly in the form of through table, equation, and graphical form, and identify value and meanings of slope/rate and y-intercept/initial value as found within real-world and mathematical situations (8.F.3, 8.F.4, 8.F.5, 8.EEEI.6). ● Solve for a single variable in a multiple variable equation and inequalities in real-world and mathematical situations (8.EEEI.7a and d). ● Generate and graph linear equations (8.F.1c and e; 8.F.4b; 8.F.5). ● Understand and apply the properties of operations, equality, and inequality (7.EEI.4a. 7.EEI.4b, 7.EEI.4c, 7.EEI.4d).

Subsequent Knowledge Related to this Unit

• Foundations in Algebra Unit 1 develops one- and two-variable linear equation concepts (FA.ASE.1*; FA.ASE.2*). The course will:
  ○ Apply and expand to include quadratic (Foundations in Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions) and exponential equations (Foundations in Algebra 1 Unit 4: Modeling and Analyzing Exponential Functions).
  ○ Recognize an early form of factoring as to identify and extract the greatest common numeric or algebraic term, such as $2x + 4y$ can be represented as $2(x+2y)$ (FA. ACE.4*; FA.ASE.1*; FA.ASE.2*).
  ○ Extend to factor and simplify polynomials, specifically quadratics (Foundations in Algebra Unit 3: Modeling and Analyzing Quadratic Functions).
  ○ Simplify or factor complicated expressions by combining like terms or extracting the Greatest Common Monomial (factor) to show equivalent expressions (FA.ASE.1*).
  ○ Apply primarily to common monomial factoring (Foundations in Algebra Unit 3: Modeling and Analyzing Quadratic Functions).
• Foundations in Algebra 1 Unit 1 graphs one-variable inequalities on a number-line (FA.FIF.5*). The course will:
  ○ Extend as a possible means to express the domain and range in compound inequality notation in future units.
  ○ Compare functions' graphical, symbolic, or tabular forms (FA.NQ.2*; FA.AREI.10*).
  ○ Use the application of the functions' graphical, symbolic, or tabular form, particularly to measure the average rate of change (FA.FIF.6* in Foundations in Algebra 1 Units 2, 3, 4, and 5).
• Foundations in Algebra 1 Unit 1 introduces direct and indirect variation in linear applications (FA.ACE.2*). The course will:
  ○ Expand the study of linear relationships (Intermediate Algebra Unit 2: Reasoning with Linear Equations and Inequalities).
• Foundations in Algebra 1 Unit 1 simplifies radicals and rationalizes denominators involving square roots and extending to cube roots as appropriate (FA.NRNS.1*). The course will:
  ○ Apply simplifying and solving literal equations, and rationalizing will be applied in subsequent course of Geometry, such as applications with triangles (Geometry Unit 5: Right Triangles and Trigonometry).
• Foundations in Algebra 1 Unit 1 defines, rewrites and explores the relationship between rational exponents and simple radicals, and the relationship of irrational numbers as subset of the entire real number systems (FA.NRNS.1*; FA.NRNS.2*; FA.NRNS.3*). The course will:
  ○ Develop skills found within finding square roots, completing the square, applying the quadratic formula and factoring of quadratic functions using other methods (Foundations in Algebra Unit 3: Modeling and Analyzing Quadratic Functions).
• Foundations in Algebra 1 Unit 1 rewrites and simplifies simple radicals (FA.NRNS.1*). The course will:
  ○ Extend, such as simplifying square roots and rationalizing a denominator (Foundations in Algebra Unit 3: Modeling and Analyzing Quadratic Functions).

- Foundations in Algebra 1 Unit 1 develops fluency in variable manipulation (FA.ACE.4*). The course will:
  - Solve for specified variables and substitute equivalent algebraic value(s) to rewrite functions of linear equations (Unit 2; Reasoning with Linear Equations and Inequalities) and quadratic functions (Foundations in Algebra Unit 3: Modeling and Analyzing Quadratic Functions) and to compare such functions (Foundations in Algebra 1 Unit 5: Comparing and Contrasting Functions).
- Foundations in Algebra 1 Unit 1 writes equations of linear functions with two variables (FA.ACE.2*). The course will:
  - Extend the point-slope form and its various applications (Unit 2: Reasoning with Linear Equations and Inequalities);
  - Expand to quadratics (Foundations in Algebra Unit 3: Modeling and Analyzing Quadratic Functions) and exponential functions (Unit 4: Modeling and Analyzing Exponential Functions).
- Foundations in Algebra 1 Unit 1 develops fluency of polynomial operations and foundational knowledge of algebraic terms and polynomial expressions (FA.AAPR.1*). The course will:
  - Promote linear applications (Foundations in Algebra Unit 2: Reasoning with Linear Equations and Inequalities), quadratic applications and factoring (Foundations in Algebra Unit 3: Modeling and Analyzing Quadratic Functions), and other functions in units to follow.
- Foundations in Algebra 1 Unit 1 develops understanding of functions in two variables and the variables' relationship expressed in table, equation and graphical forms (FA.AREI.10*). The course will:
  - Connect linear functions (Foundations in Algebra Unit 2: Reasoning with Linear Equations and Inequalities), quadratic functions (Foundations in Algebra Unit 3: Modeling and Analyzing Quadratic Functions), and exponential functions (Foundations in Algebra Unit 4: Modeling and Analyzing Exponential Functions) as students begin to use function notation.
- Foundations in Algebra 1 Unit 1 (and Grade 8) limits the algebraic form of a linear function to $y = mx + b$. The course will:
  - Apply function notation in subsequent units and courses.

Relationship Among Standards in this Unit
The standards in Unit 1 provide the foundational knowledge for concepts developed in Foundations in Algebra in subsequent units. Students will interpret the structure of expressions, equations, and inequalities involving one or more variables and solve problems related to unit analysis. Students will construct meaning about the relationships among variables, including direct and indirect variations, through real-world contexts and through algebraic, verbal, graphic, and tabular models. Tasks will progress from simple equations and inequalities to complex equations in two or more variables. Real-world contexts, relevant to STEM-related or other career fields, will engage students in mathematical practices while applying properties and performing operations with quantities involving given units of measure. Measurement units in these contexts may necessitate a conversion, which will require students to attend to precision and accuracy. Students will write, graph, and solve linear equations or inequalities to represent the relationship between independent and dependent variables. Graphing relationships will require students to use appropriate labels, units, and scales on the axes. They will employ logic and reasoning to interpret and explain the meaning of a solution or a set of solutions. From solving linear equations or inequalities in one-variable to rewriting or solving a formula involving two or more variables, students will fluently solve for given variables. Investigation of compound inequalities (written, symbolic, and number line graphing) will empower students to express domain and range using compound inequality notation. The properties of rational and irrational numbers and operations with polynomials are included as a preparation for working with quadratic functions later in the Intermediate Algebra course. Students will simplify radicals and rationalize denominators involving square roots and extend to cube roots as appropriate.

Possible Instructional Strategies/Lessons
The order of the topics below illustrates a possible instructional order for Unit 1.

Exponent Foundations (FA.NRNS.1*; FA.NRNS.2*; FA1.NRNS.3)

a. Basic Exponent Properties Review
   Khan Academy: Basic Exponent Properties Review
   Algebra 1 Lessons WCCUSD: Zero and Negative Exponents
   Algebra 1 Lessons WCCUSD: Quotient of Powers
b. Relationship Between Rational Exponents and Simple Radicals
   Algebra 1 Lessons WCCUSD: Fractional Exponents
   Khan Academy: Rational and Irrational
   Math Practices: Rational and Irrational
   MathBitsNotebook: Exponents
   PurpleMath: Relationship Note
   Illustrative Mathematics: Evaluating Exponential Expressions
   Algebra 1 Lessons WCCUSD: Square and Square Roots
   Algebra 1 Lessons WCCUSD: Roots and Fractional Exponents
c. Simplifying Radicals and Rationalizing Denominators
   Algebra 1 Lessons WCCUSD: Simplifying Radicals - Day 1
   Khan Academy: Rationalizing a Denominator Video
   Math Open Reference: Simplifying Radicals
   MathBitsNotebook: Simplifying and Rationalizing Denominators
   NRICH: Tilted Squares
   Virtual Nerd: Simplifying with Square Roots

Understanding and Representing Quantity (FA.NQ.1*; FA.NQ.2*; FA.NQ.3)

a. Understand the Appropriateness of Unit Size in a Real-World Context
   The *Scale of Universe*: Scale of Universe
   Alysion: Dimensional Analysis Notes and Summary
b. Unit Conversion Analysis

Virginia: Choosing Appropriate Unit of Measurement

Interpret the Meanings of Expressions (FA.ASE.1*)

a. Interpreting Expressions
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Interpreting Algebraic Expressions: Online Lesson Plan
b. Translation of Expressions
   MathBitsNotebook: Expression Translation
   MathBitsNotebook: Expression Practice
c. Evaluation of Expressions
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Classroom Challenges (Formative Assessment Lessons)
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Guide for Teachers and Administrators
   MathBitsNotebook: Expression Evaluation
   Khan Academy: Expression Evaluation
   Virtual Nerd: Variable Substitution

Creating and Solving Equations (FA.ACE.1*; FA.ACE.2*; FA.ACE.4*; FA.AREI.1*; FA.AREI.3*)

a. Direct and Indirect Variation Discussion
   Cliffs Notes Notebook: Direct and Inverse Variation Notes and Practice Quiz
   Virtual Nerd: Video Discussion on Direct Variation and How to Write an Equation in a Problem Situation
   Khan Academy: Video Discussion on Direct and Inverse Variation
b. Solving One-Variable Equations
   Georgia Standards of Excellence Frameworks GSE Foundations of Algebra Module 4: Equations & Inequalities
   Algebra 1 Lessons WCCUSD: Equations - Multiple Representations and What We Know
   Algebra 1 Lessons WCCUSD: Solving Equations using Balance Scale, Decomposition and Graphing
   MathBitsNotebook: Solving Multi-Step Equations
   MathBitsNotebook: Solving Multi-Step Equations: Practice
   Virtual Nerd: Solving Multi-Step Equations: With Distributive Property
   Virtual Nerd: Solving Multi-Step Equations: Clearing Fractions
   Virtual Nerd: Solving Multi-Step Equations: With No Solution
   Virtual Nerd: Solving Multi-Step Equations: Word Problem

Khan Academy: Solving Multi-Step Equations: Intuition Exercise
Khan Academy: Exercise Activity

c. Analyzing Multi-Step Equations
Mathematics Assessment Project – Mathematics Assessment Resource Service: Building and Solving Complex Equations: Online Lesson Plan
Algebra 1 Lessons WCCUSD: Simplifying Expressions & Solving Equations with Two Column Proofs
Algebra 1 Lessons WCCUSD: Solving and Equations with Variables on Both Sides

d. Building and Solving Complex Equations

e. Linear Rates
EngageNY: Linear Rates
Mathematics Assessment Project – Mathematics Assessment Resource Service: Building and Solving Linear Equations: Online Lesson Plan

f. Solving Literal Equations
MathBitsNotebook: Solving Literal Equations
MathBitsNotebook: Solving Literal Equations: Practice Problems
Virtual Nerd: What is a Literal Equation
Virtual Nerd: Solving Literal Equations
Algebra 1 Lessons WCCUSD: Solving and Using Literal Equations
Khan Academy: Solving Literal Equations
Khan Academy: Solving Literal Equations: Practice
Khan Academy: Celsius and Fahrenheit

Solving, Interpreting, and Graphing Inequalities in One-Variable (FA.ACE.1*; FA.AREI.3*)

a. Inequality Concepts
Algebra 1 Lessons WCCUSD: Solving Inequalities
MathBitsNotebook: MathBitsNotebook: Basic Inequalities Information
MathBitsNotebook: MathBitsNotebook: Compound Inequalities

b. Solving One-Variable Inequalities
  MathBitsNotebook: Solving One-Variable Inequalities
  MathBitsNotebook: Solving One-Variable Inequalities Practice
  Virtual Nerd: Solving One-Variable Inequalities: Two Steps
  Virtual Nerd: Solving One-Variable Inequalities: Multiple Steps
  Algebra 1 Lessons WCCUSD: Solving Inequalities
  Algebra 1 Lessons WCCUSD: Graphing Linear Inequalities Sort
c. Inequality Applications
  MathBitsNotebook: Inequalities Word Problem: Practice

Graphing Linear Equations (FA.ACE.2*; FA.AREI.10*)

a. Linear Equations
  Virtual Nerd: What Is A Linear Equation?
b. Table Method
  Virtual Nerd: Graphing Table Method
  MathBitsNotebook: Graphing Linear Equations Using Table Method
c. Slope-Intercept Method
  Virtual Nerd: Graphing By Slope-Intercept
  MathBitsNotebook: Graphing Linear Equations Using Slope Intercept Method
  MathBitsNotebook: Graphing Linear Equations Using Slope Intercept Method - Practice Problems
  Math Open Reference: Lines On Coordinate Plane
  Math Open Reference: Slope-Intercept
  Math Open Reference: Slope-Intercept Applet
  Shodor: Slope-Intercept Slider
d. Intercepts Method
  Virtual Nerd: Graphing By Intercepts Method
  MathBitsNotebook: Graphing Linear Equations Using Intercept Methods (including calculator guide)

e. Modeling and Applications
   MathBitsNotebook: Graphing Linear Equations - Mixed Practice Problems
   Linear Equation Modeling
   NCTM Illuminations: Bathtub Water Levels (Slope-Intercept: Negative Slope)
f. Rate of Change
   Virtual Nerd: Rate Of Change
   Inside Mathematics: Performance Assessment Tasks

Resources
Application Resources (Downloadable Lessons, Video, Applets, and Online Algebra Notes) a. Algebra 1 Skills: Algebra 1 Skills b. Cliff Notes: Algebra 1 c. Emergent Math: Emergent Math d. EngageNY: Algebra 1 Module 1 e. Georgia Standards of Excellence Frameworks GSE Foundations of Algebra Module 5 - Quantitative Reasoning with Functions f. Georgia Standards of Excellence Frameworks GSE Foundations of Algebra Module 4 - Equations & Inequalities g. Khan Academy: Introduction to Algebra h. Math Open Reference Math Open Reference i. MathBitsNotebook: Algebra 1 j. Mathematics Assessment Project – Mathematics Assessment Resource Service: Assessing 21st Century Mathematics k. Virtual Nerd: Algebra 1
Dictionaries, Calculators, and Templates (Graphs, Graphic Organizers) a. A Maths Dictionary for Kids: Math Charts b. A Maths Dictionary for Kids: Math Dictionary c. Math Open Reference: Calculator d. Math Open Reference: Full-Size Calculator e. North Central Regional Educational Laboratory: Graphic Organizers f. University of Georgia Mathematics Education Program: Interactive Mathematics Dictionary g. Web 2.0: Calculator
Teaching Strategies a. Creative Educator: Authentic Tasks - Write a Great Authentic Task b. Creative Educator: Project-Based Learning c. Illustrative Mathematics: Illustrative Mathematics Homepage d. Math Video Instructional Development Source: Authentic Contexts

e. Power Up What Works: Math Strategies that Work Research

Sample Formative Assessment Tasks/Questions
Arithmetic with Polynomials and Expressions (FA.AAPR.1*) a. Illustrative Mathematics Task: Powers of 11 b. NRICH: Quadratic Patterns c. NRICH: Square Number Surprises d. Choose a very big number. *Describe* it in as many ways as you can (Small 67). e. Why might it be useful to write $5^4 * 20^5$ in a different form to perform the calculation? Can you think of another pair of numbers that you might rename for a similar purpose (Small 75)?
Building Functions, Solving Equations and Inequalities, and Describing Relationships (FA.NQ.2*; FA.ACE.1*; FA.ACE.2*; FA.AREI.10*) a. Illustrative Mathematics: Cash Box b. Illustrative Mathematics: Equations and Formulas c. Illustrative Mathematics: Reasoning with Linear Inequalities d. Illustrative Mathematics: Rewriting Equations e. Illustrative Mathematics: Same Solutions f. Illustrative Mathematics: Traffic Jam g. MathBitsNotebook: Solving One-Variable Equations - Summary Practice
Graphing Equations (FA.ACE.2*; FA.AREI.10*) a. MathBitsNotebook: MathBitsNotebook - Practice Graphing Linear Equations
Interpret the Meanings of Expressions (FA.ASE.1*) a. Illustrative Mathematics: Animal Populations b. Illustrative Mathematics: Delivery Trucks c. Illustrative Mathematics: Delivery Trucks (this is a different approach) d. Illustrative Mathematics: Equivalent Expressions e. Illustrative Mathematics: Mixing Candies f. Illustrative Mathematics: Seeing Dots g. MathBitsNotebook: Basic Algebraic Expression Assessment

h. What makes all of the algebraic expressions below similar (Small 22)?

$5x \qquad 4x+5 \qquad x^2 \qquad 30-x$

i. You describe a situation with the expression 5x. What might the situation be (Small 23)?
j. A rectangle has a length of "x" and a width of "4." What algebraic expressions can describe features of the rectangle (Small 23)?

Real Number System (FA.NRNS.1*; FA.NRNS.2*; FA.NRNS.3)

a. Illustrative Mathematics: Calculating the Square Root of 2
b. Illustrative Mathematics: Checking a Calculation of a Decimal Point
c. Illustrative Mathematics: Evaluating a Special Exponential Expression
d. Illustrative Mathematics: Evaluating Exponential Expressions
e. Illustrative Mathematics: Operations with Rational and Irrational Numbers
f. Illustrative Mathematics: Rational or Irrational?
g. Illustrative Mathematics: Sums of Rational and Irrational Numbers
h. New Zealand Maths: It Sounds Like Mah Jong (FA.NRNS.1*)
i. A number is written as a ***root*** of another number, but it's really easy to figure out the standard value without a calculator. What might the number be (Small 75)?
j. Draw a picture that would help someone compare the given pair of numbers (Small 81).

Option 1: $\sqrt{64}$ and $\sqrt{256}$ **Option 2**: $\sqrt{64}$ and $\sqrt{66}$

k. **Option 1:** The answer is 5^{20}. What might the question be? **Option 2:** The answer is $4^{\frac{2}{3}}$. What might the question be (Small 86)?
l. Order the given set of numbers from least to greatest. Do not use a calculator (Small 87).

Option 1: $\sqrt{200}, 2.5^3, 4^2, \sqrt{260}$ **Option 2:** $\left(\frac{1}{2}\right)^4, 4^{\frac{1}{2}}, 3^{\frac{1}{3}}, 1^8$

Foundations in Algebra Unit 1 Bibliography

Small, M. & Lin, A. (2010). *More good questions: great ways to differentiate secondary mathematics instruction*. New York: Teachers College Press.

Foundations in Algebra Unit 2 Title
Reasoning with Linear Equations and Inequalities

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes

- FA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
  - Limit to linear in Unit 2. Expands to quadratic in Foundations in Algebra Unit 3 and exponential in Unit 4. Rational functions are not taught in Intermediate Algebra Unit 5.
  - Understand that linear equations define the relationship between two variables, and graph equations to represent that.
  - Write an equation of a line given a point and slope, both algebraic in model and application.
  - Write an equation of a line given at least two points, both algebraic in model and application.
- F A.AREI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
  - Apply the properties and operations of real numbers to equations and inequalities to solve for a specified variable (e.g., to solve a slope-intercept equation or point-slope equation for x or another specified variable; or $y = mx + b$ for b).
  - Apply the properties and operations of real numbers to the standard form of a linear equation to find the x-intercept, the y-intercept, or the slope of the function.
- FA.AREI.5 Justify that the solution to a system of linear equations is not changed when one of the equations is replaced by a linear combination of the other equation.
  - Define *system of equations* and *solution of a system*.
  - Multiply by the same number on both sides of the equal sign to produce equivalent equations.
  - Replace one equation with the sum of that equation and a multiple of the other to create a system with the same solutions as the original equation.
  - Substitute the common solution (if there is one) into a system to validate every equation.

- FA.AREI.6* Solve systems of linear equations algebraically and graphically focusing on pairs of linear equations in two variables.
  - Determine the approximate solution to a system of linear equations by graphing both equations and estimating the point of intersection.
  - Solve a system of linear equations algebraically (by substitution or elimination/linear combinations) to find an exact solution.
  - Explain why some linear systems have no solutions and identify linear systems that have no solutions.
  - Explain why some linear systems have infinitely many solutions and identify linear systems that have infinitely many solutions.
  - Understand that linear systems can be solved multiple ways and that one method might be more efficient than others. (e.g., and suggests the graphing or substitution method, and suggests the substitution method).
  - Graph the linear equations of a system to determine if the system has one, none, or infinitely many common solutions (points of intersection).
  - Manipulate the equations within a linear system algebraically (through substitution or elimination) to determine the common solution, if any exists.
  - Verify by substitution that the variables' solutions solve the original equations.
- F A.AREI.6a Solve systems of linear equations using the substitution method.
  - Solve and verify the exact solution of a system of equations using substitution.
- FA.AREI.6b Solve systems of linear equations using linear combination.
  - Eliminate a variable algebraically to find an exact solution for a system of linear equations.
  - Verify by substitution that the variable's solutions solve the original equations.
- F A.AREI.10* Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.
  - Verify that any point on a graph will result in a true equation when their coordinates are substituted into the equation.
- FA.AREI.11* Solve an equation of the form $f(x) = g(x)$ graphically by identifying the coordinate(s) of the point(s) of intersection.
  - Understand that the point of intersection, (x, y), on the graph of a system of equations, $y = f(x)$ and $y = g(x)$, represents a solution to both equations.
  - Infer that since$y = f(x)$ and $= g(x)$, $f(x) = g(x)$ by the substitution property.
  - Verify that the x-coordinate of the points of intersection for $y = f(x)$ and $y = g(x)$ are also the solutions for $f(x) = g(x)$.
  - Use a graphing calculator to determine the approximate solutions to a system of equations $f(x)$ and $g(x)$.
- FA.AREI.12* Graph the solutions to a linear inequality in two variables.
  - Graph a linear inequality on a coordinate plane, resulting in a boundary line (solid or dashed) and a shaded half-plane.

- FA.FIF.1a Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
  - Define a function as a relation in which each input (domain) has exactly one output (range).
  - Determine whether a graph, table, or set of ordered pairs represents a function.
  - Determine whether stated rules (both numeric and nonnumeric) produce ordered pairs that represent a function.
- FA.FIF.1b Represent a function using function notation and explain that $f(x)$ denotes the output of function f that corresponds to the input x.
  - Introduce the function notation $f(x)$ to represent the output or range values of a function.
  - Understand that $f(x)$ represents the corresponding output of the function when x is an element of the input of a function.
- FA.FIF.1c Understand that the graph of a function labeled as f is the set of all ordered pairs (x, y) that satisfy the equation $y = f(x)$.
  - Explain the relationship between the graph of f and the graph of the equation $= f(x)$.
- FA.FIF.2* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.
  - Decode function notation and explain how the output of a function is matched to its input. (e.g., the function $f(x) = 3x^2 + 5$ squares the input, triples the square, and adds five to produce the output).
  - Use order of operations to evaluate a function for a given domain (input) value.
  - Analyze the input and output values of a function based on a problem situation.
  - Identify the real numbers that are not in the domain of a function.
  - Recognize that the domain may change depending upon the context of problem.
- FA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.
  - Limit to linear in Unit 2. Expand to quadratic in Foundations in Algebra Unit 3 and exponential in Unit 4.
  - Convert a table, graph, set of ordered pairs, or description into function notation by identifying the rule used to turn inputs into outputs and writing the rule.

- FA.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
    - Limit to linear in Unit 2. Expand to quadratic in Unit 3 and exponential in Unit 4.
    - Analyze the input and output values of a function based on a problem situation.
    - Identify the numbers that are not in the domain of a function recognizing that the domain may change depending upon context of problem.
    - Write the domain and range in various formats (e.g., compound inequalities and interval notation).
- FA.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.
    - Limit to linear in Unit 2. Expand to quadratic in Intermediate Algebra Unit 4 and exponential in Unit 6.
    - Explain the relationship between the average rate of change and the slope formula $m = \frac{\Delta y}{\Delta x}$.
    - Calculate the average rate of change of a function.
    - Compare the rates of change of two or more functions.
- FA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases.
    - Limit to linear in Unit 2. Expand to quadratic in Foundations in Algebra Unit 3 and exponential in Unit 4.
    - Identify that the parent function for linear functions is the line $y = x$.
    - Identify the point-slope form of a linear function as $y - y_1 = m(x - x_2)$.
    - Graph a line in point-slope form and use the graph to show where the starting point $(x_1, y_1)$and the slope (m) are represented on the graph.
    - Identify the slope-intercept form of a linear function as $f(x) = mx + b$.
    - Graph a line in slope-intercept form and use the graph to show where the y-intercept and the slope are represented on the graph.
    - Explain the effects of change of slope *m* and y -intercept on linear functions.

- FA.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function.
  - Limit to linear in Unit 2. Expand to quadratic in Foundations in Algebra Unit 3 and exponential in Unit 4.
  - Identify the point-slope form of a linear function as $-y_1 = m(x - x_1)$.
  - Identify the slope-intercept form of a linear function as $(x) = mx + b$.
  - Identify the standard form of a linear function as $+By = C$.
  - Use definitions of$x$-intercept and$y$ -intercept to find the intercepts of a standard form line.
  - Relate the constants A, B, and C to the values of the x-intercept, y-intercept, and m, slope.
- FA.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal.
  - Limit to linear in Unit 2. Expand to quadratic in Foundations in Algebra Unit 3 and exponential in Unit 4.
  - Use equations, verbal descriptions, graphs, and tables to analyze the relationship between quantities or the properties of two functions.

New Academic Vocabulary for This Unit
• Average Rate Of Change • Boundary • Combinations Method • Function Notation (*notation for*) • Half-Plane • Interval • Linear Inequality • Point-Slope Form • Relation • Standard Equation Form (*for linear functions*) • Substitution Property

Prior Knowledge Required for this Unit
In earlier grades/units, students have developed conceptual knowledge and have had the opportunity to learn how to: ● Extend previous understandings of Order of Operations (7.EEI.3). ● Expand foundational knowledge of inequality solution sets, including compound equalities (6). ● Understand function concepts, including one-to-one domain to range, particularly as communicated within ordered pairs (8.F.1). ● Apply linear functions (particularly in the form of $y = mx + b$through table, equation, and graphical form, and identify value and meanings of slope/rate and y-intercept/initial value as found within real-world and mathematical situations (8.F.3; 8.F.4; 8.F.5; 8.EEEI.6). ● Extend Grade 8's conceptual knowledge of linear functions and expand the definition of linear function, which was limited to, to include function notation. ● Solve for a variable in a multiple variable equation or inequality in real-world and mathematical situations (Foundations in Algebra 1 Unit 1 - FA.ACE.1*; FA.ACE.2*; FA.ACE.4*). ● Investigate the concept of linear systems through graphs or algebraic (substitution and elimination) methods. ● Understand when one, none, and infinitely many solutions arise through application or inspection of linear systems (8.EEI.7b and c; 8.EEI.8). Unit 2 in Foundations in Algebra will scaffold this specific concept to improve mastery and extend understanding of linear systems and their solution sets. For example, in Foundations in Algebra, the difficulty of the solution of a linear system (e.g. non-integer answers) may increase, such as expected in standard FA.AREI.11. The additional standards in Unit 2 of Foundations in Algebra makes linear systems more rigorous than when introduced in Grade 8. ● Understand and write the concept of inequalities from and within real-world and mathematical situations (7.NS.4b).

Subsequent Knowledge Related to this Unit

• Foundations in Algebra Unit 3 extends the notion of a function to a new function family - quadratics. Unit 3 includes creating one and two variable quadratic equations extended from Foundations in Algebra Unit 1, which explored the relationships among variables (Foundations in Algebra Unit 1: Relationships Between Quantities and Expressions) (FA.ASE.1; FA.ASE.2). Functional relationships will be extended to a third function family, exponential (Foundations in Algebra Unit 4: Modeling and Analyzing Exponential Functions).
• Foundations in Algebra 1 Unit 3 deepens knowledge of linear functions through graphical, symbolic, or tabular forms and how to measure the average rate of change (FA.FIF.6). This unit will:
    ○ Apply the functions' graphical, symbolic, or tabular form, particularly to measure the average rate of change (FA.FIF.6* in Algebra 1 Units 3, 4, and 5).
• Foundations in Algebra Unit 1 (FA.ACE.4) requires fluent variable manipulation and empowers students in Foundations in Algebra Unit 2 to rewrite functions of equations (FA.ACE.2*; FA.AREI.3). This unit will:
    ○ Extend to rewrite quadratic functions (Foundations in Algebra 1 Unit 3: Modeling and Analyzing Quadratic Functions) and to compare such functions (Foundations in Algebra Unit 5: Comparing and Contrasting Functions) by solving for and substituting equivalent algebraic value(s).
• Foundations in Algebra Unit 1 (FA.AAPR.1) involves the development of foundational knowledge regarding algebraic terms and polynomial expressions and the properties of operations applied to polynomials. In Foundations in Algebra Unit 2, algebraic fluency of these concepts deepens and broadens while studying linear applications. This unit will:
    ○ Extend to quadratic applications and factoring (Foundations in Algebra Unit 3: Modeling and Analyzing Quadratic Functions) and other functions in subsequent units.
• Foundations in Algebra Unit 2 explores linear function relationships in two variables expressed in table, equation and graphical forms (FA.AREI.10). This unit will:
    ○ Evolve as students explore quadratic (Foundations in Algebra Unit 3: Modeling and Analyzing Quadratic Functions), exponential (Foundations in Algebra 1 Unit 4: Modeling and Analyzing Exponential Functions), and other function relationships expressed in table, equation and graphical forms.
• Foundations in Algebra Unit 2 applies function notation (FA1.FIF.1*). This unit will:
    ○ Extend function notation in all subsequent units and courses.
• Foundations in Algebra Unit 2 integrates linear systems and linear inequality graphing, lending towards extension of linear inequality systems (FA.AREI.12*). This unit will:
    ○ Develop linear inequality systems formally in Intermediate Algebra and linear programming in Algebra 2.

Return to High School Overview, High School Table of Contents, or Foundations in Algebra Coversheet.

- Foundations in Algebra Unit 2 develops skills with writing equations of lines (FA.FIF.7*; FA.FIF.8*). This unit will:
    - Extended in Geometry to build on this foundation, particularly when writing equations of lines that are parallel and perpendicular (G.GGPE.5*) and proving properties of quadrilaterals using coordinates.

Relationship Among Standards in this Unit
The standards of Unit 2 expand students' prior knowledge of functions, specifically linear functions. Students learn function notation, analyze concepts of domain and range, and explore linear relationships through graphic, tabular, and algebraic representations. Students will identify the critical attributes of linear functions (e.g., rate of change, intercepts, domain, and range) and understand that arithmetic sequences are linear functions. Determining an output value for a particular input involves evaluating expressions; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs (Bainbridge). As students further extend their prior knowledge of systems of linear functions and whether or not they have one, none, or infinitely many common solutions (8.EEI.7), they transfer conceptual understandings to apply other algebraic methods (i.e., substitution, combination, and elimination). Students study the relationships between variables and linear functions and linear inequalities in tabular, graphing, and algebraic formats and in real world and mathematical situations. Students also graph two-variable linear inequalities in preparation for subsequent courses. They communicate their understanding of the solution sets of inequalities in algebraic, verbal, and graphic representations. Unit 2 limits standards FA.ACE.2* and FA.FIF.4* through FA.FIF.9 to their linear function applications. This conceptual knowledge, however, is foundational for studies in subsequent units applied to quadratic functions (Foundations in Algebra Unit 3) and exponential functions (Foundations in Algebra Unit 4).

Possible Instructional Strategies/Lessons
The order of the topics below illustrates a possible instructional order for Unit 2.

Function Notation and Evaluative Meanings (FA.FIF.1*; FA.FIF.2*; FA.FIF.5*)

a. Instructional Materials
   Algebra 1 Lessons WCCUSD: Evaluating Linear Functions
   Algebra 1 Lessons WCCUSD: Evaluating Functions Graphically and Algebraically
   Georgia Standards of Excellence Frameworks GSE Foundations of Algebra Module 5 - Quantitative Reasoning with Functions
b. Algebra 1 Lessons WCCUSD: Evaluating Linear Functions
c. Algebra 1 Lessons WCCUSD: Evaluating Functions Graphically and Algebraically

Linear Modeling, Creating Equations with Two Variables, and Graphing Linear Functions (FA.ACE.2*; FA.FIF.5*; FA.FIF.6*)

a. Average Rate of Change
   Algebra 1 Lessons WCCUSD: Average Rate of Change
   Virtual Nerd: Average Rate of Change Review, Slope, and Table-Method
b. Creating Equations and Graphing Linear Functions
   Georgia Standards of Excellence Frameworks GSE Foundations of Algebra Module 4 - Equations & Inequalities
   Algebra 1 Lessons WCCUSD: Discovering Slope
   Khan Academy: Linear Function Graphing Videos on Relationships Discussion, Slope Discussion, Equations Example 1, and Equations Example 2
   Algebra 1 Lessons WCCUSD: Slope-Intercept Sort
c. Linear Functions Relationships
   Algebra 1 Lessons WCCUSD: Discovering Slope

Writing Equations of Lines (A1.FIF.7*; A1.FIF.8*)

a. Point-Slope Form
   Algebra 1 Lessons WCCUSD: Point-Slope Application Problems
   Khan Academy: Introduction To Point-Slope Video
   Virtual Nerd: Rewriting From Slope-Intercept Into Standard Or Point-Slope Formats
   Virtual Nerd: Writing Equations Of Lines In Point-Slope Format, Finding “B” When Given Point And Slope, and Given A Parallel Line And Point
b. Two-Points

Khan Academy: Writing Equation Of A Line Given In Point-Slope Given Two Points
Algebra 1 Lessons WCCUSD: Three Forms of an Equation of a Line

c. Applications
Open Middle: Write A Linear Function
Algebra 1 Lessons WCCUSD: Point-Slope Application Problems
Algebra 1 Lessons WCCUSD: Shifting Linear Equations in Function Notation

d. Standard Form
Algebra 1 Lessons WCCUSD: Discovering Slope of a Line in Standard Form

Linear Systems (FA.FIF.9*; FA.AREI.5; FA.AREI.6*; FA.AREI.6a*; FA.AREI.6b*; FA.AREI.10*; FA.AREI.11*; FA.FIF.4*)

a. Linear Systems - General Understanding
Cliffs Notes: Linear Systems General Information
EngageNY: Creating Systems Of Equations
MathEdPage: Systems With Teacher Notes And Applications

b. Linear Systems - Graphically
Algebra 1 Lessons WCCUSD: Graphing Systems
Georgia Department of Education: Georgia Standards Solving Systems By Graphing – "Cara's Candles; page 64" – Georgia Standards Of Excellence Unit 2
Georgia Department of Education: Solving Systems By Graphing – "Talk Is Cheap; page 143" – Georgia Standards Of Excellence Unit 2
Mathematics Assessment Project – Mathematics Assessment Resource Service: Maximizing Profit - Online Lesson Plan
Mathematics Assessment Project – Mathematics Assessment Resource Service: Maximizing Profit

c. Linear Systems - Algebraically
Algebra 1 Lessons WCCUSD: Solving a System by Substitution
Georgia Department of Education: Solve Systems Of Linear Equations By Elimination – Graphic Organizer Georgia Department of Education: Solving Systems By Graphing – "Family Outing; page 131" – Georgia Standards Of Excellence Unit 2
Mathematics Assessment Project – Mathematics Assessment Resource Service: Solving Linear Equations In Two Variables - Online Lesson Plan
Mathematics Assessment Project – Mathematics Assessment Resource Service: Solving Linear Equations In Two Variables
Georgia Department of Education: Methods For Solving Systems Of Equations – Georgia Standards Of Excellence Unit 2 – Graphic Organizer

Graphing Inequalities with Two-Variables (FA.AREI.12*)

a. Graphing Two-Variable Inequalities
   EngageNY: Graphing Inequalities with Two-Variables
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Representing Inequalities Graphically - Online Lesson Plan
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Representing Inequalities Graphically
   Virtual Nerd: Graphing Two-Variable Inequalities Videos - Determining Boundary Line and Graphing Two-Variable Inequalities
   Georgia Department of Education: Graphing Inequalities with Two-Variables (Extension)– “Family Outing; page 136” – Georgia Standards Of Excellence Unit 2

Sample Formative Assessment Tasks/Questions
Solving Linear Equations and Systems of Equations (FA.ACE.2*; FA.AREI.1*; FA.AREI.3*; FA.AREI.6*; FA.AREI.11*) a. Illustrative Mathematics: Accurately Weighing Pennies I b. Illustrative Mathematics: Collinear Points (FA.AREI.10*) c. Illustrative Mathematics: Estimating a Solution Via Graphs d. Illustrative Mathematics: Find a System e. Illustrative Mathematics: Fishing Adventures 3 (systems of linear inequalities) f. Illustrative Mathematics: Pairs of Whole Numbers g. Illustrative Mathematics: Solution Sets (systems of linear inequalities) h. Illustrative Mathematics: Solving Two Equations in Two Unknowns i. Illustrative Mathematics: Taxi (FA.AREI.10*) j. New Zealand Maths: Renting a Car k. NYC Department of Education: The Cycle Shop l. How are these two equations alike? How are they different (Small 22)? $y = 3x - 2 \quad and \quad y = 6x - 4$ m. Consider only situations where $x > 0$ (Small 59). **Option 1**: Graph the following lines on the same grid and identify the coordinates of the point of intersection: $y = x \;\; and \;\; y = x + 3$. **Option 2**: Graph the following lines on the same grid and identify the coordinates of the point of intersection: $y = 3x \;\; and \;\; y = \frac{1}{2}x$.
Meaning of Functions (FA.FIF.1a) a. Function Finder- Which of these relationships are functions (Ronau 24)? ● Facebook user →password ● Student →hair color ● Students in our class →planet the student lives on ● State →letters in name ● Month →days in the month ● Days in the month →month (inverse) ● Date →temperature outside

- Password →Facebook user
- Any integer →Double the integer

Interpreting Functions (FA.FIF.1a; FA.FIF.1b; FA.FIF.1c; FA.FIF.2*; FA.FIF.4*; FA.FIF.5*; FA.FIF.6*; FA.FIF.7*; FA.FIF.8*; FA.FIF.9*)

a. Georgia Department of Education: Functioning Well (FA.FIF.1)
b. Illustrative Mathematics: 10000 is Half of 2000 (FA.FIF.6*)
c. Illustrative Mathematics: Cell Phones (FA.FIF.2*)
d. Illustrative Mathematics: Domains
e. Illustrative Mathematics: Function Notation I (FA.FIF.1b)
f. Illustrative Mathematics: Hoisting the Flag I
g. Illustrative Mathematics: Hoisting the Flag II
h. Illustrative Mathematics: How is the Weather? (FA.FIF.5)
i. Illustrative Mathematics: Interpreting the Graph
j. Illustrative Mathematics: Laptop Battery Charge (FA.FIF.6*)
k. Illustrative Mathematics: Mathemafish Population (FA.FIF.6*)
l. Illustrative Mathematics: Pizza Place Promotion
m. Illustrative Mathematics: Playing Catch
n. Illustrative Mathematics: Points on a Graph
o. Illustrative Mathematics: Random Walk I (FA.FIF.2*)
p. Illustrative Mathematics: Random Walk II (FA.FIF.2*)
q. Illustrative Mathematics: Temperature Change (FA.FIF.6*)
r. Illustrative Mathematics: The Customers (FA.FIF.1a)
s. Illustrative Mathematics: The High School Gym (FA.FIF.6*)
t. Illustrative Mathematics: The Parking Lot (FA.FIF.1a)
u. Illustrative Mathematics: Warming and Cooling
v. Illustrative Mathematics: Words-Tables-Graphs (FA.FIF.9*)
w. Illustrative Mathematics: Yam in the Oven (FA.FIF.2*)
x. Illustrative Mathematics: Your Father (FA.FIF.1a)
y. The slope of a line is $\frac{2}{3}$. Provide the coordinates of two points on the line (Small 40).

Symbolic Representation of Linear Function (FA.ACE.2*) a. Illustrative Mathematics Task: Do Two Points Always Determine a Linear Function? b. Open Middle problem - Write a Linear Function c. You know that a line goes through the point (4, 2) and that is slants up and to the right. Name at least one other thing that you are sure is NOT true about the line (Small 41). d. **Option 1**: A line of slope $\frac{2}{3}$ goes through (-4, -1). Name two or more points on the line. **Option 2**: A line of slope $\frac{-2}{3}$ goes through (-4, -1). Name two or more points on the line (Small 59).
Inside Mathematics: Performance Assessment Tasks

Foundations in Algebra Unit 2 Bibliography

Bainbridge, K., Holman, B., Baker, S., & Yuu, W. (2011). *The common core: clarifying expectations for teachers & students: math high school - modeling, number & quantity, algebra, functions, geometry, statistics & probability*. Columbus, OH: McGraw-Hill Education.

Ronau, R. N., Meyer, D., Crites, T., & Dougherty, B. J. *Putting essential understanding of functions into practice in grades 9-12*.

Small, M. & Lin, A. (2010). *More good questions: great ways to differentiate secondary mathematics instruction*. New York: Teachers College Press.

Foundations in Algebra Unit 3 Title
Modeling and Analyzing Exponential Functions

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- FA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable. (Limit to linear; quadratic; ~~exponential with integer exponents~~.)
    - Identify the variable quantities in real-world contexts.
    - Use context to choose the applicable algebraic model (linear equation, linear inequality, or quadratic equation).
    - Write, solve, and interpret the solution of an equation or inequality.
- FA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; exponential with integer exponents; direct and indirect variation.)
    - Identify the variables and quantities in real-world contexts.
    - Use context to choose the applicable algebraic model (e.g., linear, quadratic).
    - Write and graph equations in two or more variables, and interpret relationships among variables.
- FA.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
    - Isolate variables to rewrite equations and formulas in equivalent forms.
- A1.AREI.1* Understand and justify that the steps taken when solving simple equations in one variable create new equations that have the same solution as the original.
    - Apply order of operations and inverse operations to solve equations.
    - Construct an argument to justify a solution process.
    - Demonstrate that the solution of the equation solves the equations created for each step of the process.
- FA.FBF.3* Describe the effect of the transformations $f(x)$, $f(x) + k$, $f(x + k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to linear; quadratic; ~~exponential with integer exponents;~~ vertical shift and vertical stretch.) (Limit to linear and quadratic in unit 3; expands to exponential with integer exponents in units 4 and 5).
    - Explain why $f(x) + k$ translates the original graph of $f(x)$ up k units and why $f(x) - k$ translates the original graph of $f(x)$ down k units.
    - Explain why $f(x + k)$ translates the original graph of $f(x)$ left k units and why $f(x - k)$ translates the original graph of $f(x)$ right k units.
    - Describe the transformation that changed a graph of $f(x)$ into a different graph when given pictures of the pre-image and the image.
    - Determine the value of k given a graph of a transformed function.
    - Graph the listed transformations when given a graph of f(x) and a value of k [$f(x) \pm k$, $f(x \pm k)$ and $f(x \pm k) \pm k$].
    - Generate and compare examples of functions with different k values.

- FA.FIF.1* Extend previous knowledge of a function to apply to general behavior and features of a function.
- FA.FIF.1a Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
    - Define a function as a relation in which each input (domain) has exactly one output (range).
    - Determine if a graph, table, or set of ordered pairs represents a function.
    - Determine if stated rules (both numeric and nonnumeric) produce ordered pairs that represent a function.
- FA.FIF.1b Represent a function using function notation and explain that *f(x)* denotes the output of function *f* that corresponds to the input *x*.
    - Recognize that when *x* is an element of the input of a function, *f(x)* represents the corresponding output of the function.
- FA.FIF.1c Understand that the graph of a function labeled as *f* is the set of all ordered pairs *(x, y)* that satisfy the equation *y = f(x)*.
    - Explain that the graph of *f* is the graph of the equation *y = f(x)*.
- FA.FIF.2* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.
    - Decode function notation and explain how the output of a function is matched to its input. (e.g., The function $f(x) = 3x^2 - 5$ squares the input, triples the square, and subtracts five to produce the output).
    - Use order of operations to evaluate a function for a given domain (input) value.
    - Analyze the input and output values of a function based on the context of a problem.
    - Identify the real numbers that are not in the domain of a function.
    - Recognize that the domain might change depending upon the context of problem.
- FA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries;~~ end behavior and periodicity~~. (Limit to linear; quadratic;~~ exponential.~~) (Limit to linear and quadratic in unit 3; expands to exponential with integer exponents in units 4 and 5.)
    - Connect this content to the content of A1.AREI.4*.
    - Create a graph that matches the description and shows all of the key features of the function, if applicable, the y-intercept, x-intercept(s), increasing interval, decreasing interval, relative minimum, relative maximum, and symmetry.
    - Explain the meaning of all the key features included in a graph or verbal description.
- FA.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. (Limit to linear; quadratic; ~~exponential.~~) (Limit to linear and quadratic in unit 3; expands to exponential with integer exponents in units 4 and 5).
    - Analyze the input and output values of a function in algebraic, graphic, tabular, and verbal forms.

- FA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases. (Limit to linear; quadratic; ~~exponential only in the form $y = a^x + k$.)~~
    - Explain that the parent function for a quadratic function is the parabola $f(x) = x^2$.
    - Explain that the minimum or maximum of a quadratic is called the vertex.
    - Identify whether the vertex of a quadratic will be a minimum or maximum by looking at the equation.
    - Find the y-intercept of a quadratic by substituting 0 for *x* and evaluating the function.
    - Estimate the vertex of a quadratic by evaluating different values of *x*.
    - Use calculated values while looking for a maximum or minimum to decide if the quadratic has x-intercepts.
    - Estimate the x-intercepts of a quadratic by evaluating different values of *x*.
    - Graph a quadratic using evaluated points.
    - Use technology to graph a quadratic and to find precise values for the x-intercept(s) and the maximum or minimum.
- FA.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.)
    - Equations, verbal descriptions, graphs, and tables provide insight into the relationship between quantities.
    - Explain that there are three forms of quadratic functions: standard form and vertex form.
    - Explain that standard form is $f(x) = ax^2 + bx + c$.
    - Explain that the vertex form is $f(x) = a(x-h)^2 + k$.
    - Explain that the graph of all these forms is a parabola.
    - Use the x-intercepts of a quadratic function to find the axis of symmetry.
    - Use the axis of symmetry of a quadratic to find the vertex of a parabola.
    - Identify the line of symmetry and vertex of a quadratic written in vertex form.
    - Sketch the graph of a parabola written in vertex form.
    - Determine if a quadratic written in vertex form has x-intercepts by looking at the equation.
    - Use algebra to find the x-intercepts of a quadratic written in vertex form.
    - Demonstrate that the standard and vertex forms of the same quadratic function produce the same values for the x-intercepts, the y-intercept, and the vertex.
    - Write the function that describes a parabola in all three forms when given a graph with the x-intercepts, y-intercept, and vertex labeled.

- FA.FIF.8a Use ~~the process of~~ factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (Note: FA.FIF.8a is not a Graduation Standard.)
  - Use the given factored form to find the zeros of a quadratic function.
  - Use the given vertex form of a quadratic to find the extreme values and symmetry.
  - Emphasis should be given to the real-world context of a problem situation and to the meaning of the zeros and extreme values in the real-world.
  - The processes of factoring and completing the square will be addressed in Intermediate Algebra Unit 4.
- FA.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; ~~exponential.~~) (Limit to linear and quadratic in unit 3; expands to exponential with integer exponents in units 4 and 5.)
  - Provide insight into the properties of linear and quadratic functions by comparing different representations of two functions.

New Academic Vocabulary for This Unit
• Axis of symmetry • Inspection • Parabola • Quadratic equation • Quadratic expression • Radicand • Roots of a quadratic • Square root method • Standard form of a quadratic equation • Translate • Vertex • Vertex form • y-axis symmetry • Zero of a function • Zeros

Prior Knowledge Required for this Unit
• FA.ASE.1 included creating one and two variable linear equations in Unit 1; this standard is now applied and extended to a new function family, quadratic, in this (Unit 3). • In Unit 1, rewriting simple expressions with radicals provided foundation for quadratic solutions with rational and irrational square roots. • FA.ACE.4 requires fluent variable manipulation and empowers students to rewrite functions as equations. • Student understanding of function relationships, as seen in FA.AREI.10 for functions in two variables, can be expressed in tabular, algebraic (equation), and graphical forms. • Previous student exposure to linear functions was established in Units 1 and 2, particularly the function notation of f(x) and how that relates to understanding of function concepts including 1-to-1 domain to range, particularly as communicated within ordered pairs. • In grade 8, students examined the properties of rigid transformations (rotations, reflections, and translations); reflections and translations will be re-examined with the parent function, $f(x) = x^2$, a parabola rather than geometric figures. (8.GM.1) • Student understanding of the geometric concept of symmetry from middle school will be applied to parabolas; symmetry appears in the graph of a parabola as well as the numerical/tabular representation of a quadratic function. A parabola has a vertical line of symmetry that can be expressed as a vertical line (non-function), and the maximum/minimum of the parabola is the vertex and is a point on the line of symmetry.

Subsequent Knowledge Related to this Unit
● Unit 5 compares and contrasts the graphs of linear, quadratic and exponential functions. ● Unit 5 compares and contrasts the rates of change of linear, quadratic and exponential functions. ● Unit 5 compares and contrasts the key features of the graphs of linear, quadratic, and exponential functions (domain, range, y-intercept, zeros, intervals of increase/decrease, axis of symmetry, vertex, end behavior, etc.).

Relationship Among Standards in this Unit
The standards in Unit 3 focus on quadratic functions, equations, and applications of quadratic functions. Students will determine if a relation is a quadratic function by analyzing information gathered from tables, graphs, and expressions. They will explore variable rate of change and determine how the rate of change for a quadratic function differs from the rate of change for a linear function. Unit 3 explores critical attributes of quadratic functions, including finding the vertex and axis of symmetry of the graph of any polynomial function; using the graph, table, or solutions to the quadratic expressed in vertex form to identify the zeros; and determining if the vertex is the maximum or minimum value of the function. Access to a variety of technologies, such as graphing utilities, spreadsheets, and computer algebra systems, to solve problems will support mastery of these standards. This unit guides students to recognize the quadratic parent function $f(x) = x^2$, and to explain how a translation of the parent function can stretch the graph, compress it, or move it left, right, up, or down. Through contextual examples and models of objects that are thrown in the air and allowed to fall subject to the force of gravity, students will deepen their conceptual understanding to interpret quadratic functions and their solutions. They will determine the circumstances when one can take the square root of both sides of an equation. In this unit, students will: ● Focus on quadratic functions, equations, and applications, ● Model quadratic equations through quadratic functions using a vertical motion model, ● Find the vertex of the graph of a quadratic function, ● Apply the vertex form of a quadratic function to find real solutions of quadratic equations, and ● Explain why the graph of every quadratic function is a translation of the graph of the basic function f (x) = x^2.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 3.*
Rewriting Expressions and Equations (FA.AREI.1*) Better Lesson: Simplify and Rewrite Radicals as Rational Exponents and Vice Versa Better Lesson: Solving Equations
Analysis of Quadratic Functions and Parabolas (FA.FBF.3*, FA.AREI.4, FA.FIF.1*(1a,1b,1c), FA.FIF.8 (8a), FA.FIF.9*) a. Transforming the graph of the quadratic parent function $f(x) = x^2$ Introduction to Quadratics Algebra 1 Lessons WCCUSD: Exploring Quadratic Graphs Khan Academy: Transforming the Graphs of Quadratic Functions VirtualNerd: Transforming Parent Functions of Quadratics Hippocampus: Simulations – Algebra 1: Quadratic Functions: Shape Shifter b. Graphing Quadratics Khan Academy: Graphing Quadratic Functions EngageNY: Introduction to Quadratic Function and Graphing Purplemath: Graphing Quadratic Functions CoolMath: Graphing Quadratics (Parabolas) Virtual Nerd: Graphs of Quadratic Functions Algebra 1 Lessons WCCUSD: Key Features of Graphs Hippocampus: Presentations – Algebra 1: Graphing Quadratic Functions Average Rate of Change *c.* Rewriting Quadratics in Different Forms *(only parts of these resources will apply)* MathBitsNotebook: Vertex Form of Quadratic Functions Khan Academy: Different Forms of Quadratic Functions and the Features They Reveal Algebra 1 Lessons WCCUSD: Three Forms of a Quadratic Function: Matching Quadratic Functions Khan Academy: Features of Quadratic Functions d. Solving Quadratics Khan Academy: Quadratic Equations and Functions

Creating and Solving Quadratic Equations in Terms of Context (FA.ACE.1*, FA.ACE.2*, FA.ACE.4*, FA.AREI.4* (4a and 4b), FA.FIF.2*)

Better Lessons: Modeling With Quadratic Functions
Algebra 1 Lessons WCCUSD: Quadratics - Matching Game
Algebra 1 Lessons WCCUSD: Quadratic Equations - What We Know

Resources
Dictionaries, Calculators, and Templates (Graphs, Graphic Organizers) a. A Maths Dictionary for Kids: Explicit Vocabulary Building b. A Maths Dictionary for Kids: Math Charts Printable Resources c. Desmos: Online Graphing Calculator with Many Pre-Made Activities d. Interactive Mathematics Dictionary: Descriptions, Related Terms, Everyday Examples, Interactive Checkpoints, More Info & Challenge e. Math Open Reference: Calculator f. My HRW Classroom: Graphing Calculator Online g. Teacher Vision: Graphic Organizer Printables h. Texas Instruments: Texas Instruments Algebra 2 Graphing Calculator Activities i. Wabbit: Online TI-84 Silver Edition Graphing Calculator Emulator j. Web 2.0: Scientific Calculator
Teaching Strategies a. Creative Educator: Authentic Tasks - Write a Great Authentic Task b. Creative Educator: Project-Based Learning c. Illustrative Mathematics: Illustrative Mathematics Homepage d. Math Video Instructional Development Source: Authentic Contexts e. Power Up What Works: Math Strategies that Work Research f. PrBL: Project-Based Learning

Quadratic Lesson Modules, Teacher Notes, PowerPoints, Videos, Applets, and Activities

a. Cliff Notes: Algebra 1
b. Concord Consortium: Graphing Quadratic Equations
c. EngageNY: Algebra 1
d. Emergent Math: Current Education Issues in Mathematics, Blog, and Curriculum Map
e. Georgia Department of Education: Mathematics Teacher Support
f. Gizmos: Resource
g. Hippocampus: Algebra & Geometry - Presentations, Worked Examples, Test Prep & Simulations
h. Howard County Public School System: Algebra 1
i. IXL Learning: Algebra 1 Practice
j. Illustrative Mathematics: High School Content Lesson Index
k. Khan Academy: Algebra 1
l. LearnZillion: Algebra Math Video Lessons
m. MathBitsNotebook: Algebra 1
n. Math Open Reference: Math Subject Topic List
o. Mathematics Assessment Project – Mathematics Assessment Resource Service: Classroom Challenges (Formative Assessment Lessons)
p. Mathematics Assessment Project – Mathematics Assessment Resource Service: Guide for Teachers and Administrators
q. Monterey Institute: Algebra 1: An Open Course (Part 2)
r. Monterey Institute: Algebra 1: An Open Course - Unit 10 – Quadratic Functions
s. National Library of Virtual Manipulatives: Algebra Virtual Manipulatives
t. Quia: Mathematics Shared Activities
u. Regents Exam Prep Center: Algebra
v. Study Zone: Intermediate Test Prep (6-8)
w. Virtual Nerd: Algebra 1
x. West Contra Costa Unified School District: Algebra 1 Lessons

Sample Formative Assessment Tasks/Questions
a. Mathematics Assessment Project – Mathematics Assessment Resource Service: Building Functions b. Mathematics Assessment Project – Mathematics Assessment Resource Service: Formative Assessment Lesson - Generalizing Patterns: Table Tiles c. Mathematics Assessment Project – Mathematics Assessment Resource Service: Formative Assessment Lesson - Representing Quadratic Functions Graphically d. Mathematics Assessment Project – Mathematics Assessment Resource Service: Formative Assessment Lesson - Sorting Equations and Identities e. Mathematics Assessment Project – Mathematics Assessment Resource Service: Summative Assessment Task - Sorting Functions Two parabolas have the same x-intercepts $(-2, 0)$ and $(4{,}0)$. The maximum or minimum value of the first parabola is two times the maximum or minimum value of the other parabola. Sketch these parabolas on the same coordinate grid (Small 32). f. You graph $y = 6x^2 + 5x + 1$. Does the graph change more if you increase the 6 by 1, the 5 by 1, or the 1 by 1 (Small 33). g. Draw a graph of a parabola that grows quickly and the graph of a parabola that grows slowly. What are the equations of these parabolas (Small 33)? h. Compare the roots of these three equations. What do you notice? • $4x^2 - 17x + 4 = 0$ $6x^2 - 37x + 6 = 0$ $8x^2 - 65x + 8 = 0$ Add another equation that acts the same way (Small 32). i. Graph $y = x^2 + 4x + 4$ and $y = 4(x - 4)^2 - 1$. Tell everything you notice about both equations and both graphs (Small 31). j. Depth of Knowledge and assessment items: • DOK 1 - Find the roots and maximum of the quadratic equation: $y = 3(x - 4)^2 - 3$ • DOK 2 - Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values. • DOK 3 - Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9, no more than one time each. $y = -[\ ]\,(x - [\ ])^2 + [\ ]$. (Kaplinsky, Robert. (2015). DOK Distinguishing Between Depth of Knowledge Levels in Mathematics.

Foundations in Algebra Unit 3 Bibliography

Bainbridge, K., Holman, B., Baker, S., & Yuu, W. (2011). *The common core: clarifying expectations for teachers & students: math high school - modeling, number & quantity, algebra, functions, geometry, statistics & probability*. Columbus, OH: McGraw-Hill Education.

Robert Kaplinsky - Glenrock Consulting. Retrieved December 15, 2015, from http://robertkaplinsky.com/

Foundations in Algebra Unit 4 Title
Modeling and Analyzing Exponential Functions

Content Standards with Clarifying Notes

- FA.FLQE.1* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.
  - Identify situations that display equal ratios of change over equal intervals and can be modeled with exponential functions.
  - Distinguish between situations modeled with linear functions and with exponential functions when presented with a real-world problem.
- FA.FLQE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. *(Note: FA.FLQE.1a is not a Graduation Standard.)*
  - Prior knowledge of linear functions should be utilized to activate learning of exponential functions, however the focus of this unit should be on exponential relationships that grow by multiples.
- FA.ACE.1* Create and solve equations ~~and inequalities~~ in one variable that model real-world problems involving linear, quadratic, ~~simple rational,~~ and exponential relationships. Interpret the solutions and determine whether they are reasonable. (Limit to ~~linear; quadratic;~~ exponential with integer exponents.)
  - Identify the variables and quantities represented in a real-world problem.
  - Decide which model best represents a real-world problem (linear model, quadratic or exponential model).
  - Write the equation that best models the problem.
  - Solve the equation.
  - Interpret the solution in the context of the problem.
- FA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; exponential with integer exponents; direct and indirect variation.)
  - Identify the variable and quantities represented in a real-world problem.
  - Decide which model best represents a real-world problem (linear model, quadratic, or exponential model).
  - Write the equation that best models the problem.
  - Establish coordinate axes using appropriate scale and label the axes.
  - Graph equations on coordinate axes with appropriate labels and scales.
- FA.FBF.3* Describe the effect of the transformations $kf(x)$, $f(x) + k$, $f(x + k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to ~~linear; quadratic;~~ exponential with integer exponents; vertical shift and vertical stretch.)
  - Explain why $f(x) + k$ translates the original graph of $f(x)$ up k units and why $f(x) - k$ translates the original graph of $f(x)$ down k

units.
  - Explain why $f(x+k)$ translates the original graph of $f(x)$ left k units and why $f(x-k)$translates the original graph of $f(x)$ right k units.
  - Describe the transformation that changed a graph of $f(x)$ into a different graph when given pictures of the pre-image and the image.
  - Determine the value of k given a graph of a transformed function.
  - Graph the listed transformations when given a graph of $f(x)$ and a value of k $[f(x) \pm k, f(x \pm k)\ and\ f(x \pm k) \pm k]$.
  - Generate and compare examples of functions with different k values.
- FA.FIF.1 Extend previous knowledge of a function to apply to general behavior and features of a function.
- FA.FIF.1a Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
  - Define a function as a relation in which each input (domain) has exactly one output (range).
  - Determine if a graph, table, or set of ordered pairs represents a function.
  - Determine if stated rules (both numeric and nonnumeric) produce ordered pairs that represent a function.
- FA.FIF.1b Represent a function using function notation and explain that $f(x)$denotes the output of function *f* that corresponds to the input x.
  - Recognize that when *x* is an element of the input of a function, $f(x)$ represents the corresponding output of the function.
- FA.FIF.1c Understand that the graph of a function labeled as *f* is the set of all ordered pairs (*x, y*) that satisfy the equation $y = f(x)$.
  - Explain that the graph of *f* is the graph of the equation $y = f(x)$.
- FA.FIF.2* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.
  - Decode function notation and explain how the output of a function is matched to its input. (e.g., The function $f(x) = 2^x + 5$ multiplies 2 input number of times and adds five to produce the output).
  - Use order of operations to evaluate a function for a given domain (input) value.
  - Analyze the input and output values of a function based on the context of a problem.
  - Identify the real numbers that are not in the domain of a function.
  - Recognize that the domain might change depending upon the context of problem.
- FA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. (Limit to linear; quadratic; exponential.)

- Create a graph that matches the description and shows all of the key features of the function, if applicable, the y-intercept, x-intercept(s), increasing interval, decreasing interval, relative minimum, relative maximum, and symmetry.
- Explain the meaning of all the key features included in a graph or verbal description.

- FA.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. (Limit to linear; quadratic; exponential.)
  - Analyze the input and output values of a function in algebraic, graphic, tabular, and verbal forms.
- FA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative~~; relative maximums and minimums; symmetries~~; and end behavior ~~and periodicity~~. Graph simple cases by hand and use technology for complicated cases. (Limit to ~~linear; quadratic~~; exponential only in the form $y = a^x + k$.)
  - Explain that the parent function for exponentials is $f(x) = b^x$ where *b* is a positive number.
  - Determine the domain, range, and end behavior (horizontal asymptote) of an exponential function when viewing its graph.
  - Classify exponential functions in function notations as growth or decay.
  - Substitute convenient values for *x* to generate a table and graph of an exponential function.
- FA.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to ~~linear; quadratic~~; exponential.)
  - Distinguish between exponential functions that model exponential growth and exponential decay.
  - Interpret the components of an exponential function in the context of a problem ($y = 5 * 1.225^{x/3}$ describes a quantity that was initially 5 and increases 22.5% every three years).

New Academic Vocabulary for This Unit
• Common ratio • Compound interest • Constant percent rate • End-behavior • Exponential decay • Exponential function • Exponential growth • Half-life • Horizontal asymptote • Labels • Percent rate of change • Principal • Properties of exponents • Scale

Prior Knowledge Required for this Unit
In earlier grades, students have developed conceptual knowledge and have had the opportunity to learn how to: ● Represent any point on the coordinate plane by its coordinates (5.G.1b). ● Write and evaluate numerical expressions involving whole-number exponents and positive rational number bases using the Order of Operations (6.EEI.1). ● Extend previous understanding of Order of Operations to solve multi-step real-world and mathematical problems involving rational numbers (7.EEI.3). ● Understand and apply the laws of exponents (i.e. product rule, quotient rule, power to a power, product to a power, quotient to a power, zero power property, negative exponents) to simplify numerical expressions that include integer exponents (8.EEI.1). ● Extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations (8.EEI.7). ● Solve linear equations and inequalities with rational number coefficients that include the use of the distributive property, combining like terms, and variables on both sides (8.EEI.7a). ● Understand that a function assigns to each input exactly one output (8.F.1a). ● Relate inputs (x-values or domain) and outputs (y-values or range) to independent and dependent variables (8.F.1b). ● Translate among the multiple representations of a function, including mappings, tables, graphs, equations, and verbal descriptions (8.F.1c). ● Determine if a relation is a function using multiple representations, including mappings, tables, graphs, equations, and verbal descriptions (8.F.1d). ● Graph a function from a table of values. Understand that the graph and table both represent a set of ordered pairs of that function (8.F.1e). ● Investigate the differences between linear and nonlinear functions using multiple representations (i.e. tables, graphs, equations, and verbal descriptions) (8.F.3). ● Recognize that the graph of a linear function has a constant rate of change (8.F.3b). ● Understand that the slope is the constant rate of change and the y-intercept is the point where $x = 0$ (8.F.4a). ● Construct a function in slope-intercept form that models a linear relationship between two quantities (8.F.4c). ● Analyze and describe attributes of graphs of functions (e.g., constant, increasing/decreasing, linear/nonlinear, maximum/minimum, discrete/continuous) (8.F.5a). ● Sketch the graph of a function from a verbal description (8.F.5b). ● Write a verbal description from the graph of a function with and without scales (8.F.5c). ● Measuring commonly used objects and choosing proper units for measurement are part of the mathematics curriculum prior to high school.

In high school, students experience a broader variety of units through real–world situations and modeling, along with the exploration of the different levels of accuracy and precision of the answers. Note: Students may not realize the importance of unit conversion in conjunction with computation when solving problems involving measurement. Since today's calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than is required.

Earlier during Foundations in Algebra, students developed conceptual knowledge and had the opportunity to learn how to:

- Create and solve equations in one variable that model real-world problems involving linear (Unit 1) and quadratic (Unit 3), students will now extend their knowledge of equations to exponential situations (FA.ACE.1*).
- Create equations in two or more variables to represent linear (Unit 1 and Unit 2) and quadratic (Unit 3) relationships between quantities. As well as, graph the equations on coordinate axes using appropriate labels, units, and scales (FA.ACE.2*).
- Describe the effect of the transformations of a quadratic/parabolic parent graph (Unit 3) and write the equation of a transformed quadratic/parabolic parent function given its graph (FA.FBF.3*).
- Extend previous knowledge of a linear (Unit 2) and quadratic (Unit 3) function to apply to general behavior and features of a function (FA.FIF.1).
- Evaluate linear (Unit 2) and quadratic (Unit 3) functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation (FA.FIF.2*).
- Interpret key features of a function that models the linear (Unit 2) and quadratic (Unit 3) relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features (FA.FIF.4*).
- Relate the domain and range of a linear (Unit 2) and quadratic (Unit 3) function to its graph and to the quantitative relationship it describes (FA.FIF.5*).
- Graph functions from their linear (Unit 2) and quadratic (Unit 3) symbolic representations. Indicate key features (FA.FIF.7*).
- Translate between different but equivalent forms of a linear (Unit 2) and quadratic (Unit 3) function equation to reveal and explain different properties of the function. (FA.FIF.8*).

Subsequent Knowledge Related to this Unit

• Foundations Unit 4 compares and contrasts linear and exponential relationships (FA.FLQE.1*).
  ○ Will be readdressed in greater depth to include quadratic relationships in Comparing and Contrasting Functions Unit 5 (FA.FLQE.1*).
• Foundations Unit 4 explores data in different representations while encouraging students to draw conclusions and to interpret and determine the reasonableness of solutions for linear, quadratic, and exponential functions (FA.ACE.1*).
  ○ Will be extended into Systems of Equations/Inequalities in Intermediate Algebra Unit 2 (IA.ACE.1*).
• Foundations Unit 4 applies related knowledge of linear and quadratic equations and the laws of exponents to construct and solve simple exponential equations; as well as (FA.ACE.2*).
  ○ Will be extended into Systems of Equations/Inequalities in Intermediate Algebra Unit 2 (IA.ACE.2*).
• Foundations Unit 4 provides opportunities to use technology appropriately to explore and experiment with variations of a function, specifically exponential models (e.g. the effect of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative), to determine the effects on the graph (FA.FBF.3*).
  ○ Will be readdressed in Comparing and Contrasting Functions Unit 5 (FA.FBF.3*) and several more times in Intermediate (IA.FBF.3*) as students deepen their understanding of parent functions.
• Foundations Unit 4 extends previous knowledge of functions and function notation (FA.FIF.1*).
  ○ Will be readdressed in greater depth to include linear and quadratic relationships in Comparing and Contrasting Functions Unit 5 (FA.FIF.1*).
• Foundations Unit 4 employs function notation to solve real-world exponential situations (FA.FIF.2*).
  ○ Will be readdressed in greater depth to include linear and quadratic relationships in Comparing and Contrasting Functions Unit 5 (FA.FIF.2*).
• Foundations Unit 4 deepens conceptual understanding of key features of functions including intercepts; asymptotes; intervals where functions are increasing, decreasing, constant, positive, or negative; and end behavior (FA.FIF.4*).
  ○ Will be readdressed in greater depth to include linear and quadratic relationships in Comparing and Contrasting Functions Unit 5 (FA.FIF.4*).
• Foundations Unit 4 expands foundational knowledge for identifying the relationship between variables (domain and range) through contextual conditions as well as algebraic, tabular, or graphic representations of variables (FA.FIF.5*).
  ○ Will be readdressed in greater depth to include linear and quadratic relationships in Comparing and Contrasting Functions Unit 5 (FA.FIF.5*).
• Foundations Unit 4 extends and deepens understanding of functions so that they can recognize and explain how $f(x)$ behaves at different domain intervals (e. g. x approaches 0, $x > 0$, $x < 0$, x approaches positive or negative infinity, growth or decay by a constant percent rate)

for any function (FA.FIF.7*).
    - Will be readdressed in greater depth to include linear and quadratic relationships in Comparing and Contrasting Functions Unit 5 (FA.FIF.7*).
- Foundations Unit 4 utilizes the multiple forms of function equations to determine and create explanations for properties of the function (FA.FIF.8*).
    - Will be extended into Radical and Rational Functions in Intermediate Algebra Unit 5 (IA.FIF.8*).

Relationship Among Standards in this Unit
The standards in Unit 4 expand prior knowledge of integer exponents and focus on exponential functions, equations, and their applications. Applying knowledge of exponential functions and knowledge acquired in Units 2 (linear functions) and 3 (quadratic functions), students determine whether a function is exponential, linear, or quadratic. Students use exponential function notation to model a variety of contexts and use context to interpret functions in specific applications. Students graph exponential functions, identify key features (intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; asymptotes; end behavior) and use appropriate technology to investigate multiplicative change in exponential functions and increase conceptual understanding. Analysis of graphs, verbal descriptions, and tables supports students in creating symbolic representations of exponential functions to model a relationship between two quantities. Manipulating the variable k for specific values, students also examine the effects on the graph on the parent function, $f(x) = b^x$, by replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative). As the unit progresses, students compare exponential models, solve exponential equations, interpret the solutions of exponential functions, and find the value of k when given exponential graphs.

Possible Instructional Strategies/Lessons
The order of the topics below illustrates a possible instructional order for Unit 4.

Defining Exponential Relationships (FA.FLQE.1*; FA.FIF.8*)

a. Introduction to Exponential Behavior

Algebra 1 Lessons WCCUSD: Exponential Functions - Writing Exponential Functions Based on Data: An Introductory Lesson
Dan Meyer: Domino Skyscraper
Dan Meyer: Incredible Shrinking Dollar
Georgia Department of Education: How Long Does it Take? (page 58)
Gizmos: Drug Doses
Gizmos: Dye Elimination
GoalBook: Equivalent Exponential Expressions Teaching Strategies
Illuminations: One Grain of Rice
Illustrative Mathematics: Decaying Dice- Investigation
Illustrative Mathematics: Last Person Standing - Investigation
Illustrative Mathematics: Paper Folding - Investigation
Illustrative Mathematics: The Bank Account
Illustrative Mathematics: Valuable Quarter - Investigation
Khan Academy: Introduction to Exponential Functions
Khan Academy: How to Model a Real-World Context with an Exponential Function
LearnZillion: Explore Exponential Growth by Folding Paper
LearnZillion: Write an Exponential Decay Function
LearnZillion: Write an Exponential Growth Function
RegentsPrep: Exponential Growth and Decay
RegentsPrep: Activity for Discovering Exponential Growth and Decay
SAS Curriculum Pathways: Applications of Exponential Functions
SAS Curriculum Pathways: Properties of Exponents
VirtualNerd: What's An Exponential Function?
VirtualNerd: What Is Exponential Growth?
Zona Land Education: Exponential Functions

b. Distinguish between Linear and Exponential Relationships
GoalBook: Linear and Exponential Functions Teaching Strategies
Illustrative Mathematics: Boiling Water - Investigation
Illustrative Mathematics: Finding Linear and Exponential Models
Illustrative Mathematics: Do Two Points Always Determine an Exponential Function?
Khan Academy: How to Construct Linear and Basic Exponential Functions From a Table of Values
Khan Academy: How to Construct Linear and Exponential Functions From Their Graphs
Khan Academy: How to Distinguish Between Linear and Exponential Models
LearnZillion: Determining if a Relationship Between Two quantities is Exponential
LearnZillion: Distinguish between Linear and Exponential Functions by Examining Intervals
Mathematics Assessment Project: Representing Linear and Exponential Growth
VirtualNerd: How Do You Identify Exponential Behavior From a Pattern in the Data?

Solving Exponential Equations and Functions (FA.ACE.1*; FA.ACE.2*; FA.FIF.1a; FA.FIF.1b; FA.FIF.2*)

a. Writing and Evaluating Exponential Functions utilizing Function Notation
Georgia Department of Education: The Marvel of Medicine (page 36)
Illustrative Mathematics: Carbon 14 Dating
LearnZillion: Create and Solve Exponentials using Functions
LearnZillion: Determine Population using an Exponential Model
VirtualNerd: What's a Function?
VirtualNerd: How Do You Evaluate an Exponential Function?
VirtualNerd: How Do You Solve a Problem With Exponential Growth?
Zona Land Education: Compound Interest

b. Solving Simple Exponential Equations
Algebra 1 Lessons WCCUSD: Solving Exponential Equations
Illuminations: Predicting Your Financial Future
LearnZillion: Create and Solve Exponentials using a Table of Values
LearnZillion: Solve Exponential Equations using Properties of Exponents
Monterey Institute: Introduction to Exponential Functions (Text)
Monterey Institute: Introduction to Exponential Functions (Video & Lesson)
Khan Academy: Solving Basic Exponential Models

Graphing Exponential Functions (FA.FBF.3*; FA.FIF.1c; FA.FIF.4*; FA.FIF.5*; FA.FIF.7*)

a. Transformations of the Parent Function
Algebra 1 Lessons WCCUSD: Graphing Exponential Functions
Georgia Department of Education: Compare/Contrast: Exponential Functions (page 16)
Georgia Department of Education: Graphic Organizer: Graphing Transformations (page 1)
Georgia Department of Education: High Functioning! (page 46)
Gizmos: Introduction to Exponential Functions
Illustrative Mathematics: Identifying Exponential Graphs - Investigation
LearnZillion: Construct an Exponential Model to Approximate Data by Using Technology
LearnZillion: Shift Exponential and Logarithmic Functions (Great Visual - Stop Lesson Prior to Logarithms)
LearnZillion: Determine the End Behavior of an Exponential
LearnZillion: Model Exponential Growth - Drawing Graphs and Writing Equations
MathBitsNotebook: Features of Exponential Functions
RegentsPrep: Transforming Functions
RegentsPrep: Ideas for Working With Transformations
RegentsPrep: Using Transformations to Investigate Functions
Zona Land Education: Transform $f(x) = 2^x$

b. Graph Given the Equation
Gizmos: Exponential Growth and Decay
Khan Academy: How to Graph an Exponential Function Given its Formula
LearnZillion: Create and Graph Exponentials Using Functions
LearnZillion: Graph an Exponential Function
LearnZillion: Graph Exponential Decay Functions
LearnZillion: Graph Exponential Growth Functions
LearnZillion: Understanding Exponential Functions by Graphing
SAS Curriculum Pathways: Graphing Exponential Functions
VirtualNerd: How Do You Graph an Exponential Function Using a Table?

Resources
Application Resources (Downloadable Lessons, Video, Applets, and Online Algebra Notes) a. Georgia Department of Education: Unit 4: Modeling and Analyzing Exponential Functions b. Georgia Department of Education: Mathematics Teacher Support c. Gizmos: Resource Catalog d. GoalBook: Tool Kit e. Hippocampus: Algebra & Geometry - Presentations, Worked Examples, Test Prep & Simulations f. Illuminations: Resources For Teaching Math g. Illustrative Mathematics: High School Content Lesson Index h. IXL Learning: Algebra 1 Practice i. Khan Academy: Algebra 1 - Introduction to Exponential Functions j. LearnZillion: Algebra Math Video Lessons k. MathBitsNotebook: Algebra 1 l. Mathematics Assessment Project – Mathematics Assessment Resource Service: Classroom Challenges (Formative Assessment Lessons) m. Mathematics Assessment Project – Mathematics Assessment Resource Service: Guide for Teachers and Administrators n. Monterey Institute: Intermediate Algebra o. PBS Learning Media: Mathline p. Quia: Mathematics Shared Activities q. Regents Exam Prep Center: Algebra r. SAS Curriculum Pathways: SAS Algebra 1 Course – Unit 4 s. VirtualNerd: Exponential Functions t. West Contra Costa Unified School District: Algebra 1 Lessons u. Zona Land Education: Index of Zona Land Education Topics
Dictionaries, Calculators, and Templates a. Alcula: Online Linear Regression Calculator b. Zona Land Education: Compound Interest Calculator

Teaching Strategies

a. Creative Educator: Authentic Tasks - Write a Great Authentic Task
b. Creative Educator: Project-Based Learning
c. Illustrative Mathematics: Illustrative Mathematics Homepage
d. Math Video Instructional Development Source: Authentic Contexts
e. Power Up What Works: Math Strategies that Work Research

Sample Formative Assessment Tasks/Questions
Defining Exponential Relationships (FA.FLQE.1*; FA.FIF.8*) a. GoalBook: Equivalent Exponential Expressions b. GoalBook: Linear and Exponential Functions c. IXL: Describe Linear and Exponential Growth and Decay d. IXL: Exponential Functions Over Unit Intervals e. Khan Academy: Construct Basic Exponential Functions f. Khan Academy: Construct Exponential Functions Given the Graph g. Khan Academy: Distinguish Between Linear and Exponential Models h. Illustrative Mathematics: Allergy Medication- Performance Task i. Illustrative Mathematics: Boom Town - Performance Task j. Illustrative Mathematics: Exponential Parameters - Performance Task k. Illustrative Mathematics: Exponential Growth Versus Linear Growth l. Illustrative Mathematics: Linear or Exponential? m. Illustrative Mathematics: Solving Problems with Linear and Exponential Models - Performance Task n. IXL: Evaluate an Exponential Function (Using Function Notation) o. MathBitsNotebook: Exponential Growth and Decay p. Mathematics Assessment Project: Linear and Exponential Models q. Mathematics Assessment Project: Multiplying Cells r. SAS Curriculum Pathways: Applications of Exponential Functions (Practice Tab) s. SAS Curriculum Pathways: Properties of Exponents (Practice Tab)
Solving Exponential Equations and Functions (FA.ACE.1*; FA.ACE.2*; FA.FIF.1a; FA.FIF.1b; FA.FIF.2*) a. Monterey Institute: Introduction to Exponential Functions (Practice & Review Tabs) b. Illustrative Mathematics: All Your Base Are Belong to Us - Performance Task c. Illustrative Mathematics: DDT-cay- Performance Task d. Illustrative Mathematics: Predicting the Past - Performance Task e. Illustrative Mathematics: Uranium 238 - Performance Task f. IXL: Exponential Growth and Decay: Word Problems g. RegentsPrep: Practice with Exponential Equations and Graphs h. RegentsPrep: Practice with Applied Exponential Growth/Decay

Graphs of Exponential Functions (FA.FBF.3*; FA.FIF.1c; FA.FIF.4*; FA.FIF.5*; FA.FIF.7*) a. Illustrative Mathematics: Exponential Functions b. IXL: Match Exponential Functions and Graphs c. Khan Academy: Graphs of Basic Exponential Functions d. MathBitsNotebook: Practice Page - Exponential Functions e. Quia: Exponential Graphs f. RegentsPrep: Working with Transformations and Functions (Practice) g. SAS Curriculum Pathways: Graphing Exponential Functions (Practice Tab)
Exponential Unit a. Georgia Department of Education: Growing by Leaps and Bounds (page 67) b. PBS Mathline: Rhinos and M&Ms c. Quia: Exponential Decay Functions Quiz d. Quia: Exponential Growth Functions Quiz

Foundations in Algebra Unit 5 Title
Comparing and Contrasting Functions

Content Standards with Clarifying Notes

• FA.FLQE.1* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval.
  ○ Define linear function $y = mx + b$ and exponential function $y = a^x + k$.
• FA.FLQE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. *(Note: FA.FLQE.1a is not a Graduation Standard.)*
  ○ Demonstrate that a linear function has a constant rate of change (slope).
  ○ Demonstrate that an exponential function has a constant multiplier (common ratio) over equal intervals.
• FA.FLQE.3* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or more generally as a polynomial function.
  ○ Use graphs or tables to compare the output values of linear, quadratic, and exponential functions.
  ○ Estimate the intervals for which the output of one function is greater than the output of another function when given a table or graph.
  ○ Use technology to find the point at which the graphs of two functions intersect.
  ○ Use the points of intersection to precisely describe the intervals for which the output of one function is greater than the output of another function.
  ○ Use graphs or tables to compare the rates of change of linear, quadratic, and exponential functions.
  ○ Explain why exponential functions eventually have greater output values than linear and quadratic functions by comparing simple functions of each type.
• FA.FLQE.5* Interpret the parameters in a linear ~~or exponential~~ function in terms of the context. (Limit to linear.)
  ○ Identify the names and definitions of parameters m and b in the linear function $f(x) = mx + b$.
  ○ Explain the meaning (using appropriate units) of the slope of a line when the line models a real-world relationship.
  ○ Explain the meaning (using appropriate units) of the y-intercept of a line when the line models a real-world relationship.
• FA.FBF.3* Describe the effect of the transformations $kf(x)$, $f(x) + k$, $\cancel{f(x+k)}$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to linear; quadratic; exponential with integer exponents; vertical shift and vertical stretch.)
  ○ Explain why $f(x) + k$ translates the original graph of $f(x)$ up k units and why $f(x) - k$ translates the original graph of $f(x)$ down k units.
  ○ Describe the transformation that changed a graph of $f(x)$ into a different graph when given pictures of the pre-image and the image.

  - Determine the value of k given a graph of a transformed function.
  - Graph the listed transformations when given a graph of $f(x)$ and a value of k $[f(kx) \pm k, and\ kf(x) \pm k]$.
  - Generate and compare examples of functions with different k values.
- FA.FIF.1* Extend previous knowledge of a function to apply to general behavior and features of a function.
- FA.FIF.1a Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
  - Define a function as a relation in which each input (domain) has exactly one output (range).
  - Determine if a graph, table, or set of ordered pairs represents a function.
  - Determine if stated rules (both numeric and nonnumeric) produce ordered pairs that represent a function.
- FA.FIF.1b Represent a function using function notation and explain that $f(x)$ denotes the output of function f that corresponds to the input x.
  - Recognize that when x is an element of the input of a function, $f(x)$ represents the corresponding output of the function.
- FA.FIF.1c Understand that the graph of a function labeled as f is the set of all ordered pairs (x, y) that satisfy the equation $y = f(x)$.
  - Explain that the graph of f is the graph of the equation $y = f(x)$.
- FA.FIF.2* Evaluate functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation.
  - Use order of operations to evaluate a function for a given domain (input) value.
  - Analyze the input and output values of a function based on the context of a problem.
  - Identify the real numbers that are not in the domain of a function.
  - Recognize that the domain might change depending upon the context of problem.
- FA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior ~~and periodicity~~. (Limit to linear; quadratic; exponential.)
  - Create a graph that matches the description and shows all of the key features of the function, if applicable, the y-intercept, x-intercept(s), increasing interval, decreasing interval, relative minimum, relative maximum, and symmetry.
  - Explain the meaning of all the key features included in a graph or verbal description.
- FA.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. (Limit to linear; quadratic; exponential.)
  - Analyze the input and output values of a function in algebraic, graphic, tabular, and verbal forms.

- FA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior ~~and periodicity~~. Graph simple cases by hand and use technology for complicated cases. (Limit to linear; quadratic; exponential only in the form $y = a^x + k$.)
  - Explain that the parent function for exponentials is $f(x) = a^x$ where a is a positive number.
  - Determine the domain, range, and end behavior (horizontal asymptote) of an exponential function when examining the graph.
  - Classify exponential functions in function notation as growth or decay.
  - Substitute convenient values for x to create a table and graph of an exponential function.
- FA.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; exponential.)
  - Provide insight into the properties of linear, quadratic, and exponential functions by comparing different representations of two functions.

New Academic Vocabulary for This Unit
• Parameter • Parent Function • Transformation • Vertical Shift/Translation

Prior Knowledge Required for this Unit
In earlier grades, students have developed conceptual knowledge and have had the opportunity to learn how to: ● Understand and apply the laws of exponents (i.e. product rule, quotient rule, power to a power, product to a power, quotient to a power, zero power property, negative exponents) to simplify numerical expressions that include integer exponents (8.EEI.1). ● Extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations (8.EEI.7). ● Solve linear equations and inequalities with rational number coefficients that include the use of the distributive property, combining like terms, and variables on both sides (8.EEI.7a). ● Understand that a function assigns to each input exactly one output (8.F.1a). ● Relate inputs (x-values or domain) and outputs (y-values or range) to independent and dependent variables (8.F.1b). ● Translate among the multiple representations of a function, including mappings, tables, graphs, equations, and verbal descriptions (8.F.1c). ● Determine if a relation is a function using multiple representations, including mappings, tables, graphs, equations, and verbal descriptions (8.F.1d). ● Graph a function from a table of values. Understand that the graph and table both represent a set of ordered pairs of that function (8.F.1e). ● Investigate the differences between linear and nonlinear functions using multiple representations (i.e. tables, graphs, equations, and verbal descriptions) (8.F.3). ● Recognize that the graph of a linear function has a constant rate of change (8.F.3b). ● Understand that the slope is the constant rate of change and the y-intercept is the point where x = 0 (8.F.4a). ● Construct a function in slope-intercept form that models a linear relationship between two quantities (8.F.4c). ● Analyze and describe attributes of graphs of functions (e.g., constant, increasing/decreasing, linear/nonlinear, maximum/minimum, discrete/continuous) (8.F.5a). ● Sketch the graph of a function from a verbal description (8.F.5b). ● Write a verbal description from the graph of a function with and without scales (8.F.5c). ● Measuring commonly used objects and choosing proper units for measurement are part of the mathematics curriculum prior to high school. Furthermore, earlier during Foundations in Algebra, students developed conceptual knowledge and had the opportunity to learn how to: ● Distinguish between situations that can be modeled with linear functions or exponential functions (Unit 4) (FA.FLQE.1*).

- Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals (Unit 4) (FA.FLQE.1a).
- Interpret the meanings of coefficients of linear relationships (Unit 1) based on their real-world contexts (FA.ASE.1*).
- Describe the effect of the transformations of a quadratic (Unit 3)/exponential (Unit 4) parent graph and write the equation of a transformed quadratic/exponential parent function given its graph (FA.FBF.3*).
- Extend previous knowledge of a linear (Unit 2), quadratic (Unit 3), and exponential (Unit 4) function to apply to general behavior and features of a function (FA.FIF.1).
- Evaluate linear (Unit 2), quadratic (Unit 3), and exponential (Unit 4) functions and interpret the meaning of expressions involving function notation from a mathematical perspective and in terms of the context when the function describes a real-world situation (FA.FIF.2*).
- Interpret key features of a function that models the linear (Unit 2), quadratic (Unit 3), or exponential (Unit 4) relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features (FA.FIF.4*).
- Relate the domain and range of a linear (Unit 2), quadratic (Unit 3), or exponential (Unit 4) function to its graph and to the quantitative relationship it describes (FA.FIF.5*).
- Graph functions from their linear (Unit 2), quadratic (Unit 3), or exponential (Unit 4) symbolic representations. Indicate key features of the graphs (FA.FIF.7*).
- Compare properties of two linear (Unit 2) or quadratic (Unit 3) functions given in different representations (FA.FIF.9*).

Subsequent Knowledge Related to this Unit

• Foundations Unit 5 compares and contrasts linear and exponential relationships (FA.FLQE.1*).
    ○ Will be needed in order to analyze scatterplots to apply a linear, quadratic or exponential model of best fit next unit (Unit 6) (FA.SPID.6*).
• Foundations Unit 5 creates opportunities for students to utilize tables and graphs to observe linear, quadratic, and exponential growth (FA.FLQE.3*).
    ○ Will be essential to the analysis of bivariate categorical data using two-way tables in the Describing Data Unit 6 (FA.SPID.5*).
• Foundations Unit 5 interprets the parameters of a linear function in terms of the context (FA.FLQE.5*).
    ○ Will be crucial when asked to interpret the meaning of the slope and intercept(s) in the context of a real-world problem during Unit 6 (FA.SPID.7*).
    ○ Will, also, be extended to understanding exponential parameters in Intermediate Algebra Unit 6 (IA.FLQE.5*).
• Foundations Unit 5 explores the appropriate use of technology to experiment with variations of linear, quadratic and exponential functions (e.g. the effect of replacing $f(x)$ by $f(x)+k$, $kf(x)$, and $f(kx)$for specific values of k (both positive and negative), to determine the effects on the graph (FA.FBF.3*).
    ○ Will be readdressed several more times in Intermediate Algebra (Units 1, 4, 5, and 6) as students continue to deepen their understanding of the nature of parent functions and extend into horizontal shifts (IA.FBF.3*).
• Foundations Unit 5 extends previous knowledge of functions and function notation (FA.FIF.1*) and employs function notation to solve real-world linear, quadratic and exponential situations (FA.FIF.2*).
    ○ Will be utilized (almost exclusively) to express a function throughout the remainder of the student's mathematics coursework.
• Foundations Unit 5 connects conceptual understanding of key features of linear, quadratic and exponential functions including intercepts; asymptotes; extrema; intervals where functions are increasing, decreasing, constant, positive, or negative; and end behavior (FA.FIF.4*).
    ○ Will be readdressed in Intermediate Algebra Quadratics (Unit 4) and Exponential (Unit 6) (IA.FIF.4*).
    ○ Will be extended to radical/rational functions in Intermediate Algebra Unit 5 (IA.FIF.4*).
• Foundations Unit 5 expands foundational knowledge for identifying the relationship between variables (domain and range) through contextual conditions as well as algebraic, tabular, or graphic representations of variables (FA.FIF.5*).
    ○ Will be readdressed in Intermediate Algebra Quadratics (Unit 4) and Exponential (Unit 6) (IA.FIF.4*).
    ○ Will be extended to radical/rational functions in Intermediate Algebra Unit 5 (IA.FIF.4*).
• Foundations Unit 5 extends and deepens understanding of functions so that they can recognize and explain how $f(x)$ behaves at different domain intervals for any function (FA.FIF.7*).
    ○ Will be extended to include piecewise/absolute value functions in Intermediate Algebra Unit 1 and radical/rational functions in

Intermediate Algebra Unit 5 (IA.FIF.7*).

- Foundations Unit 5 asks students to compare properties of functions given in different representations (FA.FIF.9*).
    - Will be investigated further during Intermediate Algebra Unit 1 for absolute value/piece-wise functions and again in Unit 4 for quadratic relationships (IA.FIF.9*).

Relationship Among Standards in this Unit
Many of the standards taught in Unit 5 have previously been addressed during Foundations in Algebra Units 1, 2, 3, and 4. The purpose of revisiting these standards is to dig deeper into the similarities and differences of linear, quadratic and exponential functions. Constructing, comparing, distinguishing between, interpreting and analyzing linear, quadratic and exponential situations in context will expand students' conceptual understanding of these relationships. The application of these parent function graphs and utilization of function notation are essential to success in future mathematics coursework. Special emphasis should be given to proving that linear functions have additive change while exponential functions have multiplicative change; observing that exponential growth will ultimately exceed linear/quadratic growth; and interpretation of the real-world parameters of a linear function; as these are the only three standards that have not been previously addressed in Foundations in Algebra.

Possible Instructional Strategies/Lessons
The order of the topics below illustrates a possible instructional order for Unit 5.

Distinguish Between Situations That Can Be Modeled With Linear, Quadratic or Exponential Functions (FA.FLQE.1*; FA.FLQE.1a; FA.FLQE.3*; FA.FLQE.5*; FA.FIF.9*)

a. Interpret Parameters of Linear Functions in Context
BetterLesson: Interpret the Parameters in a Linear Function (52 Lesson Plans)
BetterLesson: Recognize Situations in Which One Quantity Changes at a Constant Rate Per Unit (19 Lesson Plans)
Emergent Math: U-Haul Linear Problem
KhanAcademy: Modeling with Linear Functions and Equations
LearnZillion: Compare Linear and Non-Linear Functions
LearnZillion: Prove that Linear Functions Grow by Equal Differences over Equal Intervals
Math Education Page: Make These Designs
Musing Mathematically: Relation Stations
National Math + Science Initiative: Fill It Up, Please – Part III
National Security Agency: Modeling Linear Relationships
SAS Curriculum Pathways: 3-1: Comparing Rates of Change (Get Ready, Learn & Review)

b. Quadratic Relationships
Algebra 1 Lessons WCCUSD: Quadratics - Matching Game
Algebra 1 Lessons WCCUSD: Quadratic Equations - What We Know
Better Lessons: Modeling With Quadratic Functions
DY/Dan: Will It Hit the Hoop?
LearnZillion: Create Quadratic Functions
Mathalicious: Fall of Javert
Musing Mathematically: Connecting Quadratic Representations
Finding Ways: When I Got Them to Beg!
VirtualNerd: What is a Quadratic Function?

c. Exponential Function Factors
BetterLesson: Recognize Situations In Which a Quantity Grows or Decays by a Percent Rate Per Unit (16 Lesson Plans)
KhanAcademy: How to Construct Linear or Exponential Functions From a Table or a Graph
LearnZillion: Prove Exponential Functions Grow by Equal Factors over Equal Intervals

MathBitsNotebook: Exponential Functions
Monterey Institute: Introduction to Exponential Functions
National Security Agency: Exponents and Exponential Functions
Virtual Nerd: How Do You Identify Exponential Behavior From a Pattern in the Data?

d. Recognizing Differences in Models

Algebra 1 Lessons WCCUSD: Comparing Linear and Quadratic Functions
BetterLesson: Distinguish Between Situations That Can Be Modeled with Linear, Exponential and Quadratic Functions (78 Lesson Plans)
BetterLesson: Linear, Exponential, or Quadratic? (Lesson Plan)
CK-12: Linear, Exponential, and Quadratic Models (by Table Values)
Dan Meyer: Double Sunglasses
Dan Meyer: Pixel Pattern
Georgia Department of Education: Compare/Contrast: Linear, Quadratic, and Exponential Functions (Page 14)
Georgia Department of Education: Comparing Linear, Quadratic, and Exponential Models Graphically Learning Task (page 37)
Illuminations: Modeling Orbital Debris Problems
Illustrative Mathematics: Do Two Points Always Determine a Linear Function?
Illustrative Mathematics: Identifying Functions
Illustrative Mathematics: Population and Food Supply
Illustrative Mathematics: What Functions Do Two Graph Points Determine?
KhanAcademy: How to Compare the Growth of an Exponential Model and a Quadratic Model
KhanAcademy: How to Distinguish Between Linear and Exponential Models
LearnZillion: Determine Which of 2 Growing Quantities Will Eventually Exceed the Other
LearnZillion: Distinguish Between Linear and Exponential Functions by Examining Intervals
LearnZillion: Distinguish Between Linear and Quadratic Expressions
LearnZillion: Distinguish Between Linear Functions and Exponential Functions
Mathematics Assessment Project: Table Tiling
National Security Agency: Discovering Different Types of Functions
National Security Agency: Matchstick Math – Using Manipulatives to Model Linear, Quadratic and Exponential Growth
National Security Agency: Strike a Pose – Modeling In The Real World
PBS Learning Media: Comparing Exponential, Quadratic and Linear Functions
RegentsPrep: Comparing Models

SAS Curriculum Pathways: 9-4: Linear, Exponential, and Quadratic Models (Get Ready, Learn & Review)
Sophia: Comparing Linear, Quadratic, and Exponential Functions
YouTube: Determining if a Table is Linear, Quadratic or Exponential
YummyMath: Done with the leaves…now for the snow
YummyMath: Snow Days

Evaluating Linear, Quadratic and Exponential Functions (FA.FIF.1*; FA.FIF.1a; FA.FIF.1b; FA.FIF.2)

a. Function Notation (FA.FIF.1*)
KhanAcademy: Functions and Function Notation
Illustrative Mathematics: The Customers
Illustrative Mathematics: Points on a Graph (using Function Notation)
Illustrative Mathematics: Using Function Notation
Virtual Nerd: What is Function Notation?
Zona Land Education: The Function Institute

b. Evaluating Functions
Algebra 1 Lessons WCCUSD: Evaluating Functions (Linear Function)
Algebra 1 Lessons WCCUSD: Evaluating Functions Graphically and Algebraically
LearnZillion: Write and Evaluate Linear and Exponential Functions by Modeling
MathBitsNotebook: Function Notation and Evaluation
Mathematics Assessment Project: Interpreting Functions
Monterey Institute: Evaluating Functions
SAS Curriculum Pathways: Applications of Exponential Functions (Learn & Review)
VirtualNerd: How Do You Evaluate an Exponential Function?
VirtualNerd: How Do You Find f(x) If You Have a Value For x?
VirtualNerd: Identifying and Evaluating Exponential Functions

Parent Function Graphs of Linear, Quadratic and Exponential Equations (FA.FBF.3*; FA.FIF.1c; FA.FIF.4*; FA.FIF.5*; FA.FIF.7*)

a. Definitions of Linear, Exponential and Quadratic Parent Functions
Algebra 1 Lessons WCCUSD: Graphing Family of Function
Algebra 1 Lessons WCCUSD: Families of Functions Sort
CK-12: Linear, Exponential, and Quadratic Models: Bernoulli Effect PLIX (Play-Learn-Interactive-Xplore)
Gizmos: Exploring Exponential Graphs

LearnZillion: Graph Quadratic Functions
Monterey Institute: Graphing Types of Functions
RegentsPrep: http://www.regentsprep.org/regents/math/algebra/AC5/TGraphFunc.htmGraphing Functions and Observing Behavior
VirtualNerd: How Do You Determine if a Graph Represents a Linear, Exponential, or Quadratic Function?
VirtualNerd: How Do you Graph The Parent Quadratic Function $y=x^2$?
VirtualNerd: What is a Parent Function?

b. Transformations & Translations (Note: Limit to Vertical Shift and Stretch)
Algebra 1 Lessons WCCUSD: Shifting Linear Equations in Function Notation
Hippocampus: Simulations – Algebra 1 – Quadratic Functions: Shape Shifter
KhanAcademy: Shifting and Reflecting Functions
LearnZillion: Vertically Translate Quadratic Equations
LearnZillion: Understand Vertical Scaling of Quadratic Equations
MathBitsNotebook: The Parabola as a Parent Function
Mathematics Assessment Project: Building Functions
RegentsPrep: Graphing Functions and Examining Coefficients
Zona Land Education: Function Transformations

c. Identifying Key Features of Graphs
Algebra 1 Lessons WCCUSD: Key Features of Graphs
Georgia Department of Education: Paula's Peaches: The Sequel Quadratic Graphs Learning Task (page 46)
Graphing Stories: 15 Second Graphs
Illuminations: How Should I Move?
KhanAcademy: Comparing Features of Functions
KhanAcademy: End Behavior and Graphs of Basic Exponential Functions
KhanAcademy: Features of Quadratic Graphs
KhanAcademy: Introduction to the Domain and Range of a Function
LearnZillion: Graph Quadratic Functions and Show Intercepts, Maxima, Minima, Axis of Symmetry, and Vertex
MathBitsNotebook: Domain and Range
Monterey Institute: Finding Domain and Range

Resources
Application Resources (Downloadable Lessons, Video, Applets, and Online Algebra Notes) a. BetterLesson: Construct and Compare Linear, Quadratic, and Exponential Models and Solve Problems b. CK-12: Linear, Exponential and Quadratic Models c. Emergent Math: Common Core Problem Based Curriculum Maps d. Georgia Department of Education: Unit 5: Comparing and Contrasting Functions e. Georgia Department of Education: Mathematics Teacher Support f. Gizmos: Resource Catalog g. Hippocampus: Algebra & Geometry - Presentations, Worked Examples, Test Prep & Simulations h. Illuminations: Resources For Teaching Math i. Illustrative Mathematics: High School Content Lesson Index j. IXL Learning: Algebra 1 Practice k. KhanAcademy: Algebra 1 l. KhanAcademy: Functions – Linear, Quadratic and Exponential Models m. LearnZillion: Algebra Math Video Lessons n. Math Education Page: Start Page o. MathBitsNotebook: Algebra 1 p. Mathematics Assessment Project – Mathematics Assessment Resource Service: Classroom Challenges (Formative Assessment Lessons) q. Mathematics Assessment Project – Mathematics Assessment Resource Service: Guide for Teachers and Administrators r. Monterey Institute: Intermediate Algebra s. National Security Agency: High School Concept Development Units - Algebra t. PBS Learning Media: Mathline u. Quia: Mathematics Shared Activities v. Regents Exam Prep Center: Algebra w. SAS Curriculum Pathways: SAS Algebra 1 Course – Unit 9 x. Sophia: Algebra I Topics y. VirtualNerd: Algebra 1 Tutorials z. West Contra Costa Unified School District: Algebra 1 Lessons aa. Zona Land Education: Index of Zona Land Education Topics

Dictionaries, Calculators, and Templates

a. Alcula: Online Linear Regression Calculator
b. A Maths Dictionary for Kids: Math Charts
c. A Maths Dictionary for Kids: Math Dictionary
d. Math Open Reference: Calculator
e. Math Open Reference: Full-Size Calculator
f. My HRW Classroom: Graphing Calculator Online
g. University of Georgia Mathematics Education Program: Interactive Mathematics Dictionary
h. Video Math Teacher: Download Free Virtual TI Calculator Online
i. Wabbitemu: TI Calculator Emulator

Teaching Strategies

a. Creative Educator: Authentic Tasks - Write a Great Authentic Task
b. Creative Educator: Project-Based Learning
c. Fawn Nguyen: Finding Ways - Math Taboo Game
d. Illustrative Mathematics: Illustrative Mathematics Homepage
e. Math Education Page: Lab Gear
f. Math Video Instructional Development Source: Authentic Contexts
g. Power Up What Works: Math Strategies that Work Research

Sample Formative Assessment Tasks/Questions
Distinguish Between Situations That Can Be Modeled With Linear, Quadratic or Exponential Functions (FA.FLQE.1*; FA.FLQE.1a; FA.FLQE.3*; FA.FLQE.5*; FA.FIF.9*) a. College Preparatory Mathematics (CPM): Linear, Quadratics, and Exponential Tables b. CK-12: Linear, Exponential and Quadratic Models (Practice) c. Georgia Department of Education: Birthday Gifts and Turtle Formative Assessment Lesson (page 30) d. Illuminations: Modeling Orbital Debris Problems – Assessment + Extensions Tab e. Illustrative Mathematics: Choosing an Appropriate Growth Model f. IXL: Describe Linear and Exponential Growth and Decay g. IXL: Exponential Functions Over Unit Intervals h. IXL: Identify Linear, Quadratic, and Exponential Functions From Tables i. IXL: Linear Functions Over Unit Intervals j. IXL: Linear Function Word Problems k. KhanAcademy: Construct Basic Exponential Functions From a Table or a Graph l. KhanAcademy: Distinguish Between Linear and Exponential Growth From Tables m. KhanAcademy: Distinguish Between Linear and Exponential Models n. KhanAcademy: Find the Linear Function that Models a Real-World Relationship o. LearnZillion: Quiz: Linear and Non-Linear Relationships p. MathBitsNotebook: Exponential Growth and Decay (Practice Page) q. Mathalicious: Xbox Xponential r. Mathematics Assessment Project: Modeling Population Growth – Having Kittens s. Mathematics Assessment Project: Representing Linear & Exponential Growth t. SAS Curriculum Pathways: 3-1: Comparing Rates of Change (Practice & Quiz)

Evaluating Linear, Quadratic and Exponential Functions (FA.FIF.1*; FA.FIF.1a; FA.FIF.1b; FA.FIF.2) a. Illustrative Mathematics: Braking Distance b. IXL: Domain and Range c. IXL: Evaluate an Exponential Function d. IXL: Evaluate Function Rules I e. IXL: Write Linear, Quadratic, and Exponential Functions f. KhanAcademy: Evaluate Functions From their Formula g. KhanAcademy: Write Function Rules From Equations h. MathBitsNotebook: Function Notation Evaluation (Practice Page) i. Mathematics Assessment Project: Functions j. Mathematics Assessment Project: Representing Functions of Everyday Situations k. SAS Curriculum Pathways: Applications of Exponential Functions (Practice & Quiz)
Parent Function Graphs of Linear, Quadratic and Exponential Equations (FA.FBF.3*; FA.FIF.1c; FA.FIF.4*; FA.FIF.5*; FA.FIF.7*) a. Emergent Math: Hot Rod Quadratics: Let's Jump this Jump! b. Finding Ways: Des-man c. Georgia Department of Education: Exploring Paths Formative Assessment Lesson (page 32) d. Illuminations: How Should I Move? – Assessment + Extensions Tab e. Illustrative Mathematics: Comparing Graphs of Functions - Performance Task f. Illustrative Mathematics: Warming and Cooling g. IXL: Characteristics of Quadratic Functions h. IXL: Identify Linear, Quadratic, and Exponential Functions From Graphs i. IXL: Match Exponential Functions and Graphs j. IXL: Transformations of Quadratic Functions k. KhanAcademy: Domain and Range from Graph l. KhanAcademy: Graphs of Basic Exponential Functions m. KhanAcademy: Shift Functions n. LearnZillion: Quiz: Graphing Quadratic Relationships o. MathBitsNotebook: Domain and Range (Practice Page) p. MathBitsNotebook: Exponential Functions (Practice Page) q. Quia: Graphs of Functions (Matching or Concentration Game)

r. RegentsPrep: Practice with Graphing Functions
Comparing and Contrasting Functions Unit a. BetterLesson: Construct and Compare Linear, Quadratic, and Exponential Models and Solve Problems (All Plans Include Assessment Ideas) b. Georgia Department of Education: Fences and Functions - Culminating Task (page 63) c. Illuminations: Modeling Orbital Debris Problems d. SAS Curriculum Pathways: 9-4: Linear, Exponential, and Quadratic Models (Practice & Quiz)

Foundations in Algebra Unit 6 Title
Describing Data

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*

- FA.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context. (Limit to linear.)
  - Identify names and definitions of m and b in the function $f(x) = mx + b$.
  - Describe the meaning of the slope of a line that models a real-world situation (use appropriate units).
  - Describe the meaning of the y-intercept and other points of a line that models a real-world situation (use appropriate units).
  - Identify and model a real-world situation that has a linear relationship.
- FA.SPID.5* Analyze bivariate categorical data using two-way tables and identify possible associations between the two categories using marginal, joint, and conditional frequencies.
  - Read and interpret data displayed in a two-way frequency table.
  - Write a summary of data that is displayed by a two-way frequency table.
  - Use percentages of ratios in two-way table to calculate marginal, joint and conditional relative frequencies.
  - Recognize patterns in the data and explain implications of relative frequencies based on the context of the original problem.
  - Choose and create appropriate diagrams of marginal, joint, and conditional distributions.
  - Compare conditional and marginal percentages to describe the relationship between two variables.
- FA.SPID.6* Using technology, create scatterplots and analyze those plots to compare the fit of linear, quadratic, or exponential models to a given data set. Select the appropriate model, fit a function to the data set, and use the function to solve problems in the context of the data.
  - Identify the correct model (linear, quadratic, and exponential) that should be used to fit a set of data.
  - Sketch the function of best fit onto the scatterplot.
  - Recognize whether a linear or exponential model is increasing or decreasing.
  - Utilize a graphing calculator or appropriate technology to produce a scatterplot and the function of best fit.
  - Use function of best fit to predict values of a real world set of data.
- FA.SPID.7* Create a linear function to graphically model data from a real-world problem and interpret the meaning of the slope and intercept(s) in the context of the given problem.
  - Interpret the meaning of the slope in terms of the unit rate.
  - Interpret the meaning of the y-intercept in terms of the units stated in the data.
- FA.SPID.8* Using technology, compute and interpret the correlation coefficient of a linear fit.
  - Understand that the correlation coefficient only applies to quantitative variables and linear models of best fit.
  - Define the correlation coefficient as a measure of the "goodness of a linear fit" (significance) that is inclusively between -1 and 1.
  - Utilize a graphing calculator or other appropriate technology to compute the correlation coefficient of a linear model.

  - Explain the meaning of the correlation coefficient and determine whether the linear model is a good fit for the data (significance).
  - Determine the direction (sign) and the magnitude (strength) of the linear model using the correlation coefficient.
- FA.SPMJ.1* Understand statistics and sampling distributions as a process for making inferences about population parameters based on a random sample from that population
  - Recognize that the sample distribution is a subset of the population, such that it is sizeable enough to make an inference about ultimate parameters of the population.
- FA.SPMJ.2* Distinguish between experimental and theoretical probabilities. Collect data on a chance event and use the relative frequency to estimate the theoretical probability of that event. Determine whether a given probability model is consistent with experimental results.
  - Experimental probability is the ratio of the number of times an event occurs to the total number of trials or times the activity is performed.
  - Theoretical probability is the number of ways that the event can occur, divided by the total number of outcomes.
  - Collect data on the number of times an event occurs after a series of trials. Calculate the resulting experimental probability.
  - After conducting chance event experiment; compare results to theoretical probability.
  - Calculate basic probabilities, including compound events and events with or without replacement.
- FA.SPMD.4* Use probability to evaluate outcomes of decisions by finding expected values and determine if decisions are fair.
  - Utilize probability to analyze the results of a process and decide if it resulted in a fair decision.
- FA.SPMD.5* Use probability to evaluate outcomes of decisions. Use probabilities to make fair decisions.
  - Utilize probability to create a method for making a fair decision.
- FA.SPMD.6* Analyze decisions and strategies using probability concepts.
  - Evaluate data to determine whether or not the best decision was made.
  - Consider all available strategies to determine and defend a strategy to recommend.

New Academic Vocabulary for This Unit
• Conditional Relative Frequency • Correlation Coefficient • Function of Best Fit • Joint Relative Frequency • Marginal Relative Frequency • Population Parameter • Q-Points • Quantitative Variable • Significance • Statistical Inference

Prior Knowledge Required for this Unit
In earlier grades, students have developed conceptual knowledge and have had the opportunity to learn how to: ● Differentiate between statistical and non-statistical questions (6.DS.1). ● Use center, spread, and shape to describe the distribution of a set of data collected to answer a statistical question (6.DS.2). ● Describe numerical data sets in relation to their real-world context (6.DS.5). ● Investigate concepts of random sampling (7.DSP.1). ● Draw inferences about a population by collecting multiple random samples of the same size to investigate variability in estimates of the characteristic of interest (7.DSP.2). ● Investigate the concept of probability of chance events (7.DSP.5). ● Investigate the relationship between theoretical and experimental probabilities for simple events (7.DSP.6). ● Apply the concepts of theoretical and experimental probabilities for simple events (7.DSP.7). ● Extend the concepts of simple events to investigate compound events (7.DSP.8). ● Investigate bivariate data (8.DSP.1). ● Draw an approximate line of best fit on a scatter plot that appears to have a linear association and informally assess the fit of the line to the data points (8.DSP.2). ● Find an approximate equation for the line of best fit using two appropriate data points, interpret the slope and intercept, solve problems using the equation (8.DSP.3). ● Investigate bivariate categorical data in two-way tables, and interpret data in two-way tables using relative frequencies (8.DSP.4). ● Apply concepts of slope and y-intercept to graphs, equations, and proportional relationships (8.EEI.6). ● Investigate the differences between linear and nonlinear functions (8.F.3). Earlier during Foundations in Algebra, students developed conceptual knowledge and had the opportunity to learn how to: ● Create equations in two or more variables to represent relationships between the quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; exponential with integer exponents; direct and indirect variation.) (FA.ACE.2* Units 1, 2, 3, & 4). ● Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; exponential.)(FA.FIF.9* Unit 5).

- Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval (FA.FLQE.1* Unit 5).

Subsequent Knowledge Related to this Unit

• Foundations Unit 6 interprets the parameters of a linear function in terms of a real-world context (FA.FLQE.5).
  ○ This will be extended to exponential functions in Intermediate Algebra Unit 6 (IA.FLQE.5).
• Foundations Unit 6 analyzes bivariate categorical data using marginal, joint and conditional frequencies (FA.SPID.5*).
  ○ this will also be addressed in Probability and Statistics Unit 1 (PS.SPID.5*)
• Foundations Unit 6 uses technology to create scatterplots, fit a function to the data and compute correlation coefficients of a linear fit (FA.SPID.6*; FA.SPID.8*).
  ○ This will be readdressed in Probability and Statistics Unit 4 (PS.SPID.6*; PS.SPID.8*)
• Foundations Unit 6 creates linear functions to graphically model data from a real-world problem and analyzes the meaning of the slope and intercept(s) (FA.SPID.7*).
  ○ This will also be addressed in Probability and Statistics Unit 4 (PS.SPID.7*).
• Foundations Unit 6 makes inferences about population parameters based on a random sample from that population (FA.SPMJ.1*).
  ○ This will also be addressed in Probability and Statistics Unit 7 (PS.SPMJ.1*).
• Foundations Unit 6 compares data from experimental probabilities to that of the theoretical probability (FA.SPMJ.2*).
  ○ This will also be addressed in Probability and Statistics Unit 6 (PS.SPMJ.2*).
• Foundations Unit 6 uses probabilities to evaluate the fairness of decisions (FA.SPMD.4*; FA.SPMD.5*; FA.SPMD.6*).
  ○ This will also be addressed in Probability and Statistics Unit 7 (PS.SPMD.4*; PS.SPMD.5*; PS.SPMD.6*).

Relationship Among Standards in this Unit
The standards in Foundations in Algebra Unit 6 are connected by the data analysis themes of inferences, implications and predictions. The unit begins by having students use single variable statistics to make inferences. Thus, leading the way for simulations to compare theoretical and experimental probabilities in order to make predictions. Then, using probability to evaluate and make fair decisions. Next, students will be able to make predictions and implications to data using bivariate tables. Students must review, connect and apply many of the standards from the prior units to utilize linear, quadratic and exponential ideas to create regression equations. Finally, the important linear concepts of slope and y-intercepts are readdressed using real-world regression situations.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 6.*
Statistics and Parameters (FA.SPMJ.1*) a. Population versus Sample LearnZillion: Distinguish Between Population and Sample MathBitsNotebook: Population vs Sample Data Stat Trek: Population vs Sample b. Determining Population Parameters Statistics How To: Difference Between a Statistic and a Parameter LearnZillion: Determine Population and Parameter from a Statistical Question c. Using Random Samples to make Inferences LearnZillion: Take a Simple Random Sample LearnZillion: Take a Random Sample Khan Academy: Reasonable Samples University of North Carolina of Wilmington: Aquarius Lesson Plan
Simulations (FA.SPMJ.2*) a. Distinguish Between Experimental and Theoretical Probabilities Khan Academy: Comparing Theoretical to Experimental Probabilities Algebra-Class: Theoretical Regents Prep: Theoretical vs. Empirical Probability Online Math Learning: Theoretical Probability and Experimental Probability b. Estimating Theoretical Probabilities Regents Prep: Hitting the Target Activity c. Comparing Probability Models and Experimental Results Khan Academy: Making Predictions with Probability Khan Academy: Constructing Probability Models from Data

Decisions with Probability (FA.SPMD.4*; FA.SPMD.5*; FA.SPMD.6*)

a. Analyze decisions and using probabilities to evaluate if the decision was fair
NCTM: Stick or Switch
NCTM: Will the Best Candidate Win?

b. Utilize probability to create a tool to help make fair decisions
NCTM: Explorations With Chance: Is It Fair
Khan Academy: Picking Fairly
Mathalicious: Three Shots
HotMath: Using Probabilities to Make Fair Decisions
Annenberg Learner: Probability Models

Bivariate Data (FA.SPID.5*)

a. Interpret Data Displayed in a Two-Way Frequency Table
MathBitsNotebook: Two-Way Frequency Tables
Math Is Fun: Bivariate Data

b. Calculate and Analyze Marginal, Joint and Conditional Relative Frequencies
Study.com: Joint, Marginal & Conditional Frequencies
Holt McDougal: Two-Way Tables PPT

Lines of Fit and Regression (FA.FLQE.5*; FA.SPID.6*; FA.SPID.7*; FA.SPID.8*)

a. Utilize technology to create scatterplots
Annenberg Learner: Scatterplots
MathBitsNotebook: Finding Your Way Around the TI-83/84 Calculator - Scatter Plots
Texas Instruments: Creating a Scatter Plot Using TI-83/84 Family
Glencoe: Graphing Calculator - Regression Lines
PurpleMath: Scatterplots and Regressions - Page 1
SlideShare: Calculating a Correlation Coefficient and Scatter Plot using Excel
Excel Easy: Excel Scatter Chart
LearnZillion: Fit a Linear Regression to a Set of Data using Spreadsheets and Graphing Technology

b. Select the appropriate model of best fit and use technology to create the function of best fit
Illustrative Mathematics: Basketball Bounces Collaborative Investigation
NCTM: Shrinking Candles, Running Water, Folding Boxes

MathBitsNotebook: Fitting Functions to Data
Khan Academy: Comparing Models to Fit Data
PurpleMath: Scatterplots and Regressions - Page 3
PurpleMath: Scatterplots and Regressions - Page 4
LearnZillion: Select a Statistical Regression Line
LearnZillion: Model Real-World Bivariate Data By Using A Quadratic Regression Function
LearnZillion: Model Real-World Bivariate Data By Using An Exponential Regression Function

c. Use technology to compute the correlation coefficient (*r*) of a linear fit: use *r* to determine significance, direction and magnitude of the fit
LearnZillion: Find Correlation Coefficient using Technology
University of North Carolina of Wilmington: Correlation and Line of Best Fit
EngageNY: Interpreting Correlation
MathBitsNotebook: Correlations
MathBitsNotebook: Correlation Coefficients
PurpleMath: Scatterplots and Regressions - Page 2
Math Is Fun: Correlation
Algebra 1 Lessons WCCUSD: Correlation and Line of Best Fit

d. Interpret the meaning of the slope and intercept(s) of real-world linear relationships
LearnZillion: Interpret the Slope and Intercept of a Regression Line
LearnZillion: Understand and Interpret the Slope of a Regression Line
NCTM: Exploring Linear Data
NCTM: Barbie Bungee
Annenberg Learner: Fitting Lines to Data
MathBitsNotebook: Slopes and Intercepts in Linear Models
LearnZillion: Solve Problems Using Linear Regression
Illustrative Mathematics: Hand Span and Height Collaborative Activity

Resources

Application Resources (Downloadable Lessons, Video, Applets, and Online Algebra Notes)

a. Algebra-Class: Algebra Examples Index
b. Annenberg Learner: Against All Odds - Inside Statistics
c. EngageNY: Algebra 1 Module 1
d. Excel Easy: Excel Data Analysis
e. Glencoe: Algebra 1 Online
f. Holt McDougal: Online Textbook
g. Hot Math: Review Topic Index
h. Illustrative Mathematics: High School Content Lesson Index
i. Khan Academy: Introduction to Algebra
j. Khan Academy: Probability and Statistics
k. LearnZillion: High School Statistics and Probability
l. Mathalicious: Lessons
m. MathBitsNotebook: Algebra 1
n. Math Is Fun: Math Resource Index
o. NCTM: Illuminations - Resources for Teaching Math
p. Online Math learning: An Introduction to Mathematical Statistics
q. Online Math Learning: An Introduction to Probability
r. Purple Math: Index of Lessons
s. Regents Exam Prep Center: Algebra
t. SAS Curriculum Pathways: SAS Algebra 1 Course
u. Slide Share: Lesson Search
v. Stat Trek: Teach Yourself Statistics
w. Statistics How To: Probability and Statistics Topic Index
x. Study.com: High School Lesson Index
y. Texas Instruments: Classroom Activities
z. University of North Carolina of Wilmington: Aquarius Lesson Plans

Dictionaries, Calculators, and Templates

j. Alcula: Online Linear Regression Calculator
k. A Maths Dictionary for Kids: Math Charts
l. A Maths Dictionary for Kids: Math Dictionary
m. Math Open Reference: Calculator
n. Math Open Reference: Full-Size Calculator
o. My HRW Classroom: Graphing Calculator Online
p. University of Georgia Mathematics Education Program: Interactive Mathematics Dictionary
q. Video Math Teacher: Download Free Virtual TI Calculator Online
r. Wabbitemu: TI Calculator Emulator

Teaching Strategies

a. Creative Educator: Authentic Tasks - Write a Great Authentic Task
b. Creative Educator: Project-Based Learning
c. Illustrative Mathematics: Illustrative Mathematics Homepage
d. Math Video Instructional Development Source: Authentic Contexts
e. Power Up What Works: Math Strategies that Work Research

Sample Formative Assessment Tasks/Questions
Statistics and Parameters (FA.SPMJ.1*) a. Shmoop: Probability and Statistics Sample Assignments b. Shmoop: Sampling Methods c. Amazing Space: Galaxy Hunter: A Cosmic Photo Safari d. Sophia: CCSS Math Standard S-IC.1 Practice e. LearnZillion: Performance Task S-IC.1 f. LearnZillion: School Dress Code Performance Task g. Illustrative Mathematics: School Advisory Panel Performance Task h. Illustrative Mathematics: Why Randomize? Performance Task i. Illustrative Mathematics: Strict Parents Performance Task j. Illustrative Mathematics: Musical Preferences Performance Task k. Illustrative Mathematics: Mr. Briggs's Class Likes Math Performance Task
Simulations (FA.SPMJ.2*) a. Regents Prep: Practice with Probability: Set 1 b. Regents Prep: Practice with Probability: Set 2 c. Illustrative Mathematics: How Many Buttons? Performance Task d. Illustrative Mathematics: Rolling Dice Performance Task e. Monterey Institute: Project-Based Learning Activity: What are the Chances of That?
Decisions with Probability (FA.SPMD.4*; FA.SPMD.5*; FA.SPMD.6*) a. Khan Academy: Making Predictions with Probability b. Khan Academy: Probability Models c. Khan Academy: Using Probability to make Fair Decisions d. IXL: Choose the Better Bet e. Monterey Institute: Extending and Applying Concepts: Probability Game Design f. How many gray socks and how many white socks are in a drawer if the probability of randomly choosing two socks, one of each color, is close to ⅓ (Small 161)?

Bivariate Data (FA.SPID.5*)

a. Illustrative Mathematics: Birds' Eggs Performance Task
b. Illustrative Mathematics: Texting and Grades 1 Performance Task
c. MathBitsNotebook: Practice Two-Way Frequency Tables
d. Khan Academy: Trends in Categorical Data

Lines of Fit and Regression (FA.FLQE.5*; FA.SPID.6*; FA.SPID.7*; FA.SPID.8*)

a. Khan Academy: Constructing Scatter Plots
b. MathBitsNotebook: Practice Fitting Functions to Data
c. Khan Academy: Fitting Quadratic and Exponential Functions to Scatter Plots
d. MathBitsNotebook: Practice Linear Regression
e. IXL: Interpreting a Scatter Plot
f. IXL: Calculate Correlation Coefficients
g. MathBitsNotebook: Practice with Correlations
h. Mathopolis: Correlation Practice Set
i. IXL: Match Correlation Coefficients to Scatter Plot
j. School Improvement in Maryland: Correlation Coefficient Performance Activity
k. MathBitsNotebook: Practice Slope and Intercepts with Linear Regressions
l. Khan Academy: Eyeballing the line of Best Fit
m. IXL: Scatter Plots - Line of Best Fit
n. Khan Academy: Estimating Slope of Line of Best Fit
o. Illustrative Mathematics: US Airports Assessment Variation Performance Task
p. Illustrative Mathematics: Animal Brains Performance Task

Foundations in Algebra Unit 6 Bibliography

Small, M. & Lin, A. (2010). *More good questions: great ways to differentiate secondary mathematics instruction*. New York: Teachers College Press.

South Carolina College- and Career-Ready Standards for Mathematics
High School Support Document
Intermediate Algebra Coversheet

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
Functions: Arithmetic/Geometric Sequences and Absolute Value, Step, and Piece-Wise Functions	Linear Equations/Inequalities and Systems of Equations/Inequalities	Polynomials	Quadratic Functions, Equations, and Inequalities	Radical and Simple Rational Functions and Equations	Exponential Functions and Equations
Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
IA.FBF.1a IA.FBF.2* IA.FBF.3* IA.FIF.3* IA.FIF.7* IA.FIF.9* IA.FLQE.2* IA.FLQE.5*	IA.ACE.1* IA.ACE.2* IA.ACE.4*	IA.AAPR.1* IA.ASE.1* IA.ASE.2*	IA.ACE.1* IA.ACE.2* IA.ACE.4* IA.AREI.4a* IA.AREI.4b* IA.AREI.11* IA.ASE.3a* IA.ASE.3b* IA.FBF.1a* IA.FBF.1b* IA.FBF.3* IA.FIF.4* IA.FIF.5* IA.FIF.6* IA.FIF.8* IA.FIF.9* IA.NCNS.1* IA.NCNS.7*	IA.ACE.1* IA.ACE.4* IA.AREI.2* IA.AREI.11* IA.FBF.1a* IA.FBF.1b* IA.FBF.3* IA.FIF.4* IA.FIF.5* IA.FIF.6* IA.FIF.7* IA.FIF.8*	IA.ACE.1* IA.ACE.2* IA.ACE.4* IA.AREI.11* IA.FBF.1a* IA.FBF.1b* IA.FBF.2* IA.FBF.3* IA.FIF.3* IA.FIF.4* IA.FIF.5* IA.FIF.6* IA.FIF.8b* IA.FLQE.2* IA.FLQE.5*

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Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

1. **Make sense of problems and persevere in solving them.** a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

Intermediate Algebra Unit 1 Title
Functions: Arithmetic/Geometric Sequences and Absolute Value, Step, and Piecewise Functions

Content Standards with Clarifying Notes
• IA.FBF.1a Write a function that models a relationship between two quantities using both explicit expressions and a recursive process and by combining standard forms using addition, subtraction, multiplication and division to build new functions. *(Note: IA.FBF.1a is not a Graduation Standard.)* ○ Define explicit and recursive expressions of a function. ○ Identify the quantities being compared in a real-world problem. ○ Write an explicit and/or recursive expressions of a function to describe real-world problems. ○ Recall parent functions. ○ Apply transformations of parent functions. ○ Combine different parent functions (adding, subtracting, multiplying, and/or dividing) to write a function that describes a real-world problem. • IA.FBF.2* Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ○ Explain how a recursive formula tells one how a sequence starts and tells one how the previous value(s) to generate the next term of the sequence. ○ Explain that an explicit formula enables one to find any term of a sequence without knowing the term before it. [e.g., if one needs to know the fifteenth term of the sequence, one substitutes the number fifteen into the explicit formula.] ○ Differentiate between the explicit and recursive formulas for sequences. ○ Define an arithmetic sequence as a sequence of numbers that is formed so that the difference between consecutive terms is always the same and is known as the common difference. ○ Determine the common difference for arithmetic sequences. ○ Continue arithmetic sequences. ○ Determine the unknown terms of an arithmetic sequence using the explicit formula. ○ Define a geometric sequence as a sequence of numbers that is formed so that the ratio of consecutive terms is always the same and is known as the common ratio. ○ Determine the common ratio for geometric sequences. ○ Continue geometric sequences. ○ Determine unknown terms of a geometric sequence using the explicit formula. ○ Determine whether sequences are arithmetic, geometric or neither. ○ Convert a term of an arithmetic sequence into the next term and write a recursive formula for the sequence, $a_n = a_{n-1} + d$.

  - Write an explicit formula for an arithmetic sequence, $a_n = a_1 + (n\text{-}1)\ d$.
  - Explain why the recursive formula for an arithmetic sequence uses addition and why the explicit formula uses multiplication.
  - Translate between the recursive and the explicit forms of arithmetic sequences.
  - Determine whether a real-world problem models an arithmetic sequence and write an equation to model the situation.
  - Convert a term of a geometric sequence into the next term and write a recursive formula for the sequence, $a_n = r * a_{n\text{-}1}$.
  - Write an explicit formula for a geometric sequence, $a_n = r * a^{n\text{-}1}$.
  - Explain why the recursive formula for a geometric sequence uses multiplication and why the explicit formula uses exponentiation.
  - Translate between the recursive and the explicit forms of geometric sequences.
  - Determine whether a real-world problem models a geometric sequence and write an equation to model the situation.
- IA.FBF.3* Describe the effect of the transformations $kf(x)$, $f(x) + k$, $f(x + k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to linear; quadratic; exponential with integer exponents; vertical shift and vertical stretch.)
  - Explain why $f(x) + k$ translates the original graph of $f(x)$ up k units and why $f(x) - k$ translates the original graph of $f(x)$ down k units.
  - Explain why $f(x + k)$ translates the original graph of $f(x)$ left k units and why $f(x - k)$ translates the original graph of $f(x)$ right k units.
  - Describe the transformation that changed a graph of $f(x)$ into a different graph when given pictures of the pre-image and the image.
  - Determine the value of k given a graph of a transformed function.
  - Graph the listed transformations when given a graph of $f(x)$ and a value of k $[f(x) \pm k, f(x \pm k)\ and\ f(x \pm k) \pm k]$.
  - Generate and compare examples of functions with different k values.
- IA.FIF.3* Define functions recursively and recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
  - Convert a list of numbers (sequence) into a function by making the whole numbers (0, 1, 2, 3, …) the inputs and the terms (elements) of the sequence the outputs (Bainbridge).
  - Explain that an explicit formula enables one to find any term of a sequence without knowing the term before it. [e.g., if one needs to know the fifteenth term of the sequence, one substitutes the number fifteen into the explicit formula.]
  - Differentiate between explicit and recursive formulas for sequences.

- IA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases. (Limit to linear; ~~quadratic~~; exponential only in the form $y = a^x + k$.)
  - Complete tables and graph sequences on the coordinate plane.
  - Write arithmetic sequences as linear functions then graph the functions.
  - Write geometric sequences as exponential functions then graph the functions.
- IA.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; exponential.)
- IA.FLQE.2* Create symbolic representations of linear and exponential functions, including arithmetic and geometric sequences, given graphs, verbal descriptions, and tables. (Limit to linear; exponential.)
- IA.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context.

Absolute Value, Step, and Piecewise Functions

- Piecewise Functions
  - Define piecewise functions as functions that have different rules for evaluation depending on the value of the input.
  - Determine which evaluation rule to use for a specific input value, x.
  - Represent a piecewise function with a table and a graph by evaluating multiple input values.
  - Write piecewise functions to represent data given in tables or depicted on a graph.
  - Graph a piecewise function given the function rule.
- (Linear) Absolute Value Functions
  - Identify the parent absolute value function as $f(x) = |x|$.
  - Identify the two evaluation rules used for $f(x) = |x|$ and other simple absolute values like $f(x) = |6 - x|$ and $f(x) = 2|x + 7|$.
  - Identify the minimum or maximum of the absolute value function as the vertex.
  - Recognize that the vertex will be a minimum or a maximum by inspecting the equation of the absolute value.
  - Calculate the y-intercept of the absolute value function by evaluating when $x = 0$.
  - $f(x) = a|x = h| + k$ Identify the vertex, the line of symmetry, vertex as maximum if $a < 0$, vertex as minimum if $a > 0$.
  - Determine if x-intercepts exist based upon vertex and whether the vertex is a minimum or a maximum.
  - Determine the x-intercepts of the absolute value algebraically.
  - Graph an absolute value function using evaluated points.
  - Graph and absolute value function using transformations of parent functions and using technology.

- Step functions
  - Graph a step function by substituting values for x and plotting the points.

Return to High School Overview, High School Table of Contents, or Intermediate Algebra Coversheet.

New Academic Vocabulary for This Unit
• Arithmetic Sequence • Common Difference • Common Ratio • Composition of Functions • End Behavior • Explicit Formula • Geometric Sequence • Greatest Integer Function • (Linear) Absolute Value Function • Piecewise Function • Recursive Formula • Sequence • Step Function • Term

Prior Knowledge Required for this Unit
In earlier grades/courses, students have developed conceptual knowledge and have had the opportunity to learn how to: ● Represent functions in multiple ways – mappings, tables, graphs, equations and verbal descriptions (8.F.1 – 5). ● Create symbolic representations of linear functions (FA.FLQE.2). ● Graph functions and indicate key features with quadratic and linear only (FA.FIF.7). ● Effects of transformation with linear, quadratic, and exponential with integer exponents (FA.FBF.3). ● Comparing the properties of two functions given in different forms for linear, quadratic, and exponential with integer exponents (FA.FIF.9). ● Interpret the parameters in a linear function in terms of context (FA.FLQE.5).

Subsequent Knowledge Related to this Unit

• Intermediate Algebra Unit 2 addresses linear functions and equations given graphs, verbal descriptions, and tables (IA.FLQE.2*).
• Intermediate Algebra Unit 6 creates symbolic representations of functions given graphs, verbal descriptions, and tables (IA.FLQE.2*).
• Intermediate Algebra Unit 6 addresses the connection between geometric and exponential functions (IA.FBF.2*, IA.FIF.3*, IA.FIF.7*).
• Intermediate Algebra Unit 1 creates symbolic representations of functions given graphs, verbal descriptions, and tables (IA.FLQE.2*).
  ○ Geometric sequences and connections will also be addressed in Intermediate Algebra Unit 6: Exponential Functions and Equations.
  ○ Intermediate Algebra Unit 1 defines functions recursively and recognize that sequences are functions (IA.FIF.3*).
  ○ Geometric sequences and connections will also be addressed in Intermediate Algebra Unit 6: Exponential Functions and Equations.
• Intermediate Algebra Unit 1 writes a function that models a relationship between two quantities (IA.FBF.1a; *note: IA.FBF.1a is not a Graduation Standard.).*
  ○ Will also be addressed in Intermediate Algebra Unit 4: Quadratic Functions and Equations, Intermediate Algebra Unit 5: Rational and Simple Rational Functions and Equations, and Intermediate Algebra Unit 6: Exponential Functions and Equations.
• Intermediate Algebra Unit 1 graphs functions from their symbolic representations (IA.FIF.7*).
  ○ Will also be addressed in Intermediate Algebra Unit 5: Rational and Simple Rational Functions and Equations.
• Intermediate Algebra Unit 1 describes the effect of the transformations on the graph of $y = f(x)$ and writes the equation of a transformed parent function given its graph (IA.FBF.3*).
  ○ Will also be addressed in Intermediate Algebra Unit 4: Quadratic Functions and Equations, Intermediate Algebra Unit 5: Rational and Simple Rational Functions and Equations, and Intermediate Algebra Unit 6: Exponential Functions and Equations.
• Intermediate Algebra Unit 1 compares properties of two functions given in different representations (IA.FIF.9*).
  ○ Will also be addressed in Intermediate Algebra Unit 4: Quadratic Functions and Equations.
• Intermediate Algebra Unit 1 interprets the parameters in a linear function in terms of the context (IAFLQE.5*).
  ○ Will also be addressed in Intermediate Algebra Unit 6: Exponential Functions and Equations.

Relationship Among Standards in this Unit
The standards in this unit include functions not specifically covered in the subsequent Intermediate Algebra units. Students are expected to write, graph, apply, and interpret these functions. The focus in Unit 1 is on the characteristics, similarities and differences in the following functions: linear, absolute value, arithmetic/geometric sequence, piecewise, and step functions. The goal is for students to develop, understand, and make connections between a variety of function forms: equations, graphs, verbal descriptions, and tables. The terms recursively and explicitly are introduced for the first time, along with arithmetic sequences and geometric sequences. Sequences that are neither arithmetic nor geometric (Fibonacci, quadratic, etc.) should be included for comparison. Focus is placed on real world applications and contextual situations that students can relate to. Emphasis is also placed on the transformation of these functions in the coordinate plane. Determining an output value for a particular input involves evaluating an expression; finding inputs yielding a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology (Bainbridge).

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 1.*
Arithmetic Sequences (IA.FBF.1 and 2; IA.FIF.3) a. Writing Arithmetic Sequences Recursively and Explicitly Algebra Lab: Arithmetic Sequences Algebra 1 Lessons WCCUSD: Arithmetic Sequences Algebra 1 Lessons WCCUSD: Sequences - Search and Rescue Activity LearnZillion: Model Arithmetic Sequences And Situations By Using Both Recursive And Explicit Formulas b. Writing Geometric Sequences Recursively and Explicitly Algebra Lab: Geometric Sequences Algebra 1 Lessons WCCUSD: Geometric Sequences Algebra Lab: Algebra 2 Recipe: Geometric Sequences c. Graphing Calculator Exploration of Arithmetic Sequences TI Education: Arithmetic Sequences & Series d. Instructional Unit Georgia Standards of Excellence Frameworks GSE Coordinate Algebra Unit 3: Linear & Exponential Functions Drop-In Unit developed by South Carolina team of mathematics leaders – Sequences: Arithmetic & Geometric and Explicit & Recursive (2010)
Geometric Sequences (IA.FBF.1 and 2; IA.FIF3) a. Writing Geometric Sequences Recursively And Explicitly Algebra Lab: Geometric Sequences Algebra Lab: Algebra 2 Recipe: Geometric Sequences b. Instructional unit Georgia Standards of Excellence Frameworks GSE Coordinate Algebra Unit 3 - Linear & Exponential Functions
Piece-Wise Functions (IA.FIF.7*) a. Connecting Piece-Wise, Absolute Value, and Step Functions Algebra 1 Lessons WCCUSD: Graphing Piecewise Functions Birdville Schools: Extension Activity For Piece-Wise Functions Math Is Fun: Piece-Wise MathBits Notebook: Piece-Wise

Rowe Math Wiki: Piece-Wise Module
Functions – Linear, Absolute Value, Arithmetic Sequences, Piecewise, and Step (IA.FBF.1a, 2, and 3; IA.FBF.3; IA.FIF3) a. Videos On Graphs And Their Transformations Virtual Nerd: Absolute Value and Piece-Wise Functions Algebra 1 Lessons WCCUSD: Connecting Graphing & Solving Absolute Value Equations and Functions b. Examples For Piece-Wise, Step, Greatest Integer, And Absolute Value Glencoe McGraw-Hill: Special Functions c. Extension Activity For Piece-Wise Functions Birdville Schools: Analyzing Piece-Wise Functions Key Features of Graphs MathBits Notebook: Function Features d. Writing Functions Given A Situation Combining Standard Forms of Functions Using Operations: Kansas Flipbook

Resources
Activity Resources a. Illustrative Mathematics: Activities For All Levels b. Robert Kaplinksy: Problem-based Learning Activities
Algebra Course Content Resources from Other States a. EngageNY: Algebra 2 Resources b. Georgia Department of Education: Algebra 2 Resources c. Monterey Institute: Algebra 1: An Open Course (Part 1) d. Regents Prep: Algebra 2 and Trig e. Virginia Department of Education: Algebra 2 Resources
Graphing Calculator Resources a. Desmos: Online Graphing Calculator with Many Pre-Made Activities b. Texas Instruments: Texas Instruments Algebra 2 Graphing Calculator Activities c. Wabbit: Online TI-84 Silver Edition Graphing Calculator Emulator
Interactive Resources a. Emergent Math: Emergent Math b. ExploreLearning: Gizmo Online Simulations c. Interactive Quizzes: Interactive Quizzes for High School Assessments d. Shodor: Sequencer
Practice Tests and Assessment Resources a. California Department of Education: California Algebra Released Test Questions b. Jefferson Lab: Practice Tests from Virginia for All Levels of Math c. Problem-Attic: Problem-Attic d. XL Math: XL Math for Algebra 2
Video Resources a. HippoCampus: Videos for Algebra 2 b. LearnersTV: Videos for Algebra 2 c. Virtual Nerd: Videos for Algebra 2

Sample Formative Assessment Tasks/Questions

Functions – Linear, Absolute Value, Arithmetic/Geometric Sequences, Piecewise, and Step Functions (IA.FBF.1, 2, and 3; IA.FBF.3; IA.FIF.3)

a. Describe key characteristics of the graph of $f(x) = |x - 1| + 3$.
b. Sketch the graph and identify key characteristics of $f(x) = \{x + 2, x \geq 0; \ -x^2, x < -1 \ \}$.
c. What is the difference between a recursive and an explicit representation of a sequence?
d. Use the linear function $f(x) = 5x - 17$to write an explicit formula for an arithmetic sequence that can be described by the function.
e. Can 3,087 be in the pattern described by the given pattern rules? How do you know?
   - Option 1: The pattern rule is: *Start at 9. Keep adding 3*.
   - Option 2: The pattern rule is: *The term value is 4 times the term number + 3*. (Small 46)
f. Create two patterns that are very much like the given pattern. Explain why the three patterns are alike.
   - Option 1: 3, 4, 6, 9, 13, 18
   - Option 2: 3, 7, 11, 15, 19 (Small 48)
g. The function $f(x) = |x|$is transformed by *translating* to the right and then vertically *stretching.* Draw the new function on the graph. What is the equation of the new function? (Small 44)
h. A number pattern includes both a 3 and a 13 as terms. What might be the *general term* of the pattern (Small 19)?
i. A *pattern rule* includes the following words and numbers (among others), not necessarily in this order: *2, subtract, multiply*
   - Write out a complete pattern rule.
   - List the first 10 terms in your pattern based on your rule.
   - Tell about a number greater than 100 that is NOT in your pattern and how you know it's not there (Small 20).
j. Which of these patterns are most alike? Why (Small 21)?
   - Pattern 1: 2, 5, 8, 11, 14, …
   - Pattern 2: 2, 7, 12, 17, 22, 27, …
   - Pattern 3: 3, 6, 9, 12, 15, …
k. Jamie lists the first five terms of a linear pattern that grows quickly, and Adrienne lists the first five terms of a linear pattern that grows slowly. What could their patterns be? How could the graphs representing the patterns differ (Small 21)?

Transformations: Linear, Absolute Value, Arithmetic Sequences, Piecewise, Step (IA.FBF.3)

a. She Loves Math: Parent Functions and Transformations
b. On the axes, graph$f(x) = |x|$. If $(x) = f(x) - 2$, then how is the graph of $f(x)$ translated to form the graph of $g(x)$? If $h(x) = f(x - 4)$, then how is the graph of $g(x)$ translated to form the graph of $h(x)$?

Piece-Wise Functions (IA.FIF.7) a. CPM Educational Program: Connecting Piecewise Functions to Continuity Extension
Arithmetic/Geometric Sequences (IA.FBF.1 and 2; IA.FIF.3) a. Algebra Lab: Algebra 2 Practice b. Mathematics Vision Project: Arithmetic and Geometric Sequences Module/Activity c. NRICH: Activity on Comparing Two Arithmetic Sequences d. Virginia Department of Education: Sequence Matching Activity with Information on Differentiation and Essential Questions
Inside Mathematics: Performance Assessment Tasks

Intermediate Algebra Unit 1 Bibliography

Bainbridge, K., Holman, B., Baker, S., & Yuu, W. (2011). *The common core: clarifying expectations for teachers & students: math high school - modeling, number & quantity, algebra, functions, geometry, statistics & probability*. Columbus, OH: McGraw-Hill Education.

Small, M. & Lin, A. (2010). *More good questions: great ways to differentiate secondary mathematics instruction*. New York: Teachers College Press.

Intermediate Algebra Unit 2 Title
Linear Equations/Inequalities and Systems of Equations/Inequalities

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
For all standards in this unit, Intermediate Algebra must extend the knowledge and applications of linear equations/inequalities/relationships that were begun in Foundations in Algebra. It is important to *apply* and *extend* skills learned in Foundations in Algebra and not to simply reteach already covered in Foundations in Algebra. Students should experience real-world applications of linear equations/inequalities/relationships in a much more complex contextual situation than they experienced in Foundations in Algebra. Students are introduced to linear inequalities in Foundations in Algebra, and in Intermediate Algebra the concept of linear inequalities is extended to include compound inequalities, absolute value inequalities, and systems of inequalities. Emphasize the constraints on domain and range, particularly when applied to real-world contextual situations. ● IA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, ~~quadratic, simple rational~~, and exponential relationships. Interpret the solutions and determine whether they are reasonable. (Limit to linear; quadratic; exponential with integer exponents.) ○ Use real-world contexts to generate and solve equations and inequalities in one-variable. ○ Analyze solutions for their meaning and rationale within the given context. ○ Interpreting solutions includes solutions graphed on a number line. ○ Extend (not re-teach) what was learned in Foundations in Algebra. Students learned to solve and graph linear equations and inequalities in Foundations in Algebra. In Foundations of Algebra, emphasis is placed on the *application* of these skills to real-world and contextual situation type scenarios. ○ Provide examples that are real world applications and more complex than those begun in Foundations in Algebra. ○ Included in the study of inequalities in Intermediate Algebra are compound inequalities and absolute value inequalities; more complex applications of compound and absolute value inequalities will appear in Algebra 2. ○ Emphasize the meaning of the variables in the situational and real-world applications. ○ Extend knowledge of graphing inequalities learned in Foundations in Algebra to graphing systems of inequalities. ○ Included in this unit are linear systems of equations and inequalities. Unit 4 of Intermediate Algebra (Quadratic Functions/Equations/Inequalities) will address systems that include quadratic equations and inequalities. ○ Emphasize the meaning of the variables in the situational and real-world applications. ○ Ensure understanding of the difference in meaning and appearance of the graph for an inequality or system of inequalities that include $\leq, \geq, <, >$ and $\neq$.

- IA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; exponential with integer exponents; direct and indirect variation.)
  - Understand that linear equations define the relationship between two variables.
  - Generate and graph equations to represent the relationship between two variables.
  - Use appropriate labels, units, and scales to represent the relationship of two variables in a given real-world context.
  - Extend knowledge and applications of linear equations/relationships begun in Foundations in Algebra.
  - Emphasize the importance of appropriate labels, units, and scales. Appropriate labels, units, and scales are essential for accurately modeling the relationship between the two quantities (mathematical process standard #6). Incorrect scales on the graph will distort the visual appearance of the relationship. Labels and units are important to accurately understand and interpret graphs.
  - Emphasize the meaning of the variables in the situational and real-world applications.
- IA.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
  - Apply the properties and operations of real numbers to solve for a specified variable (e.g., Solve a linear equation in standard form for y; solve $A = lw$ for w and include other formulas from a variety of disciplines).

New Academic Vocabulary for This Unit
• Break-Even Point • Systems of Inequalities • Unbounded System

Prior Knowledge Required for this Unit
Working with expressions and equations, including formulas, is an integral part of the curriculum in grades 7 and 8. In high school, students explore in more depth the use and application of equations and inequalities to model real-world problems, including restricting domains and ranges to fit the problem's context, as well as rewriting formulas for a variable of interest. In Foundations in Algebra, the primary focus is on graphing and solving linear equations and inequalities. Foundations in Algebra also includes systems of equations but does not include systems of inequalities. In Intermediate Algebra, these skills are extended to more complex situations and modeling of real-world applications. Below are the linear equations and inequalities standards students had the opportunity to learn in Foundations to Algebra: ● Create and solve equations and inequalities in one variable (A1.ACE.1*). ● Create and graph equations in two or more variables (A1.ACE.2*). ● Solve literal equations and formulas for specified variable (A1.ACE.4*). ● Solve linear equations and inequalities with coefficients represented by letters (A1.AREI.3*). ● Justify the solution to a system of linear equations (A1.AREI.5). ● Solve a system of linear equations graphically and algebraically (A1.AREI.6*). ● Graph the solutions of a linear inequality in two variables (A1.AREI.12*).

Subsequent Knowledge Related to this Unit

• IA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable.
  ○ Will also be addressed in Intermediate Algebra Unit 5: Radical and Simple Rational Functions and Equations and Intermediate Algebra Unit 6: Exponential Functions and Equations.
• IA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
  ○ Will also be addressed in Intermediate Algebra Unit 4: Quadratic Functions, Equations and Inequalities and Intermediate Algebra Unit 6: Exponential Functions and Equations.
• IA.ACE.3 Use systems of equations and inequalities to represent constraints arising in real-world situations. Solve such systems using graphical and analytical methods. Interpret the solution within the context of the situation.
  ○ Will also be addressed in Intermediate Algebra Unit 4: Quadratic Functions, Equations and Inequalities.
• IA.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
  ○ Will also be addressed in Algebra 2 Unit 4: Quadratic Functions, Equations and Inequalities; Intermediate Algebra Unit 5: Rational and Simple Rational Functions and Equations, and Intermediate Algebra Unit 6: Exponential Functions and Equations.

Relationship Among Standards in this Unit
All standards in this unit address linear equations/inequalities and systems of linear equations/inequalities. It is important to note that this unit also includes compound inequalities and absolute value inequalities.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 2.*
Linear Equations (IA.ACE.1*; IA.ACE.2*) a. Activities for Creating an Equation Given a Situation Illustrative Mathematics: Planes and Wheat Illustrative Mathematics: Paying the Rent Illustrative Mathematics: Buying a Car Illustrative Mathematics: Clea on an Escalator
Compound/Absolute Value Inequalities (IA.ACE.1*) a. CK – 12: Overview of Solving Compound Inequalities b. Great Valley School District: Absolute Value Inequality Word Problems for Assessment c. Monterey Institute: Explanation of Compound Inequalities as a Union or Intersection of Inequalities d. Purple Math: Overview Of Solving Absolute Value Inequalities
Systems of Equations and Inequalities (IA.ACE.3) a. Algebra-Class: Systems of Inequalities Practice Problems b. Algebra 1 Lessons WCCUSD: Solving Systems of Inequalities c. Illuminations: Using TI-83/84 to Develop Understanding of Linear Programming: Dirt Bike Dilemma
Literal Equations and Formulas (IA.ACE.4*) a. Illustrative Mathematics: Equations and Formulas *When teaching solving literal equations, educators should make connections to relevant science, business, and CATE applications (STEM). Collaboration with teachers within such disciplines is encouraged.*

Resources
Algebra 2 Course Content Resources from Other States a. EngageNY: Algebra 2 Resources b. Georgia Department of Education: 9 – 12 Resources c. Henrico County Public Schools: Algebra 2 Resources d. Monterey Institute: Algebra 1: An Open Course (Part 1) e. Regents Prep: Algebra 2 and Trig
Activity Resources a. Illustrative Mathematics: Activities For All Levels b. Robert Kaplinksy: Problem-Based Learning Activities
Graphing Calculator Resources a. Desmos: Online Graphing Calculator with Many Pre-Made Activities b. Texas Instruments: Texas Instruments Algebra 2 Graphing Calculator Activities c. Wabbit: Online TI-84 Silver Edition Graphing Calculator Emulator
Interactive Resources a. Emergent Math: Emergent Math b. Explore Learning: Gizmo Online Simulations c. Interactive Quizzes: Interactive Quizzes for High School Assessments
Practice Tests and Assessment Resources a. California Department of Education: California Algebra Released Test Questions b. Practice Tests from Virginia for All Levels of Math - http://education.jlab.org/solquiz/ c. Problem-Attic: Sample Problems d. XL Math for Algebra 2 - XL Math for Algebra 2
Video Resources a. HippoCampus: Videos for Algebra 2 b. LearnersTV: Videos for Algebra 2 c. Virtual Nerd: Videos for Algebra 2

Sample Formative Assessment Tasks/Questions
Linear Equations, Inequalities, and Systems of Equations and Inequalities (A2.ACE.1*; A2.ACE.2*; A2.ACE.3; A2.ACE.4*) a. CK – 12: Multiple Choice Problems for Absolute Value Inequalities b. Illuminations: Dirt Bike Dilemma (*Addresses the development and assesses the conceptual understanding of linear programming; can be used for a resource for teaching the lesson or a formative assessment.*) c. Illustrative Mathematics: Activities For Writing A System Of Inequalities With Constraints d. Illustrative Mathematics: How Much Folate?
Literal Equations and Formulas (A2.ACE.4*) a. Consider the equation $3y + 4x - 12 = 0$. What information does the graphical representation of the equation quickly provide that the algebraic representation does not provide as readily (Small 39)? b. These two equations are equivalent: $2x + 5y = 400$ $\qquad$ $x = \frac{-5}{2}y + \frac{400}{2}$ Liam says that both equations model this situation: *I have some \$5 bills and two identical piles of \$1 bills. Altogether, I have \$400.* Which equation do you think is a better description? Why (Small 39)?
a. Inside Mathematics: Performance Assessment Tasks

Intermediate Algebra Unit 2 Bibliography

Small, M., and Lin, A. (2010). *More good questions: great ways to differentiate secondary mathematics instruction*. New York: Teachers College Press.

Intermediate Algebra Unit 3 Title
Polynomials

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
● IA.AAPR.1* Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations. (Limit to linear; quadratic.) ○ Understand the definition of a polynomial. ○ Simplify polynomial expressions to forms that are linear or quadratic in a positive integer power of x. ○ Emphasize that when adding and subtracting polynomials, we can only combine terms with like terms. ○ Understand that when multiplying polynomials, you must use the distributive property. ○ Describe how polynomials are closed under these three operations (when adding, subtracting or multiplying two polynomials the result is a polynomial). ○ In Intermediate Algebra, the polynomials should have real coefficients. ● IA.ASE.1* Interpret the meanings of coefficients, factors, terms, and expressions based on their real-world contexts. Interpret complicated expressions as being composed of simpler expressions. (Limit to linear; quadratic; exponential.) ○ Explain the real-world meanings behind coefficients, factors, terms and expressions in linear, quadratic and exponential situations. ○ Utilize factoring skills to rewrite complicated expressions as simpler expressions. ○ Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts. ● IA.ASE.2* Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions. ○ Expressions can be written in multiple ways using the rules of algebra; each version of the expression reveals something about the problem it represents. ○ Explain why equivalent expressions are equivalent. ○ Apply models for factoring and multiplying polynomials to rewrite expressions. ○ Understand that factoring is the distributive process in reverse. ○ Recognize that the Greatest Common Factor (GCF), which was first introduced as a 7th grade standard, is the first step to any factoring situation. ○ Examine the terms in a polynomial in order to determine which factoring technique should be used to fully factor the polynomial.

New Academic Vocabulary for This Unit
• binomial • degree of a polynomial • descending order • difference of two squares • factoring by grouping • leading coefficient • perfect square trinomial • polynomial • prime polynomials • standard form of a polynomial • trinomial

Prior Knowledge Required for this Unit
In earlier grades, students have developed conceptual knowledge and have had the opportunity to learn how to: ● Find common factors and multiples using two whole numbers (6.NS.4). ● Investigate and identify parts of algebraic expressions using mathematical terminology, including term, coefficient, constant, and factor (6.EEI.2B). ● Apply mathematical properties to simplify and to factor linear algebraic expressions with rational coefficients (7.EEI.1). ● Recognize that algebraic expressions may have a variety of equivalent forms and determine an appropriate form for a given real-world situation (7.EEI.2). ● Understand and apply the laws of exponents to simplify numerical expressions that include integer exponents (8.EEI.1). ● Interpret the meanings of coefficients, factors, terms, and expressions based on their real-world contexts. Interpret complicated expressions as being composed of simpler expressions (FA.ASE.1*). ● Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function (Limit to linear; quadratic; exponential) (FA.FIF.8*).

Subsequent Knowledge Related to this Unit
- Intermediate Algebra Unit 3 develops the sense of polynomials as a closed system under addition, subtraction and multiplication (IA.AAPR.1*)   - This will be extended to addition, subtraction and multiplication of radicals in Intermediate Algebra Unit 5.   - Recognizing and combining like terms is essential to solving all equations in mathematics.   - Intermediate Algebra Unit 3 develops factoring skills (IA.ASE.1*; IA.ASE.2*)   - Factoring will be utilized in Intermediate Algebra Unit 4 to find the zeros of a quadratic function by rewriting it in equivalent factored form and explain the connection between the zeros of the function, its linear factors, the x-intercepts of its graph, and the solutions to the corresponding quadratic equation.   - Factoring techniques are used extensively in all upper level mathematics courses.   - Intermediate Algebra Unit 3 requires students to explain the real-world meanings behind coefficients, factors, terms and expressions in linear, quadratic and exponential situations (IA.ASE.1*).   - Students will explain the meanings of coefficients, factors, terms and expressions in all future mathematics courses.

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Relationship Among Standards in this Unit
The standards in Intermediate Algebra Unit 3 begin with work concentrated on writing linear, quadratic and exponential expressions to model real world situations. Initially, the student will focus on developing their abilities to read and write algebraic statements. Next, students will create connections between the arithmetic of integers and the arithmetic of polynomials. While studying the operations of polynomials, students should utilize area models of multiplication to better grasp the abstract nature. Students are then, asked to go backwards by factoring polynomials. Factoring is one of the most abstract mathematical ideas in introductory Algebra, and students often struggle to learn the techniques because they don't see the point. Connecting these procedures to the elementary model of partial products for multi-digit multiplication, to the area model of multiplication (fractions, decimals, polynomials), and the middle school concept of prime and composite numbers, as well as, interpreting the structure of expressions in a real world context will help students confidently work with these abstractions..

Possible Instructional Strategies/Lessons
The order of the topics below illustrates a possible instructional order for Unit 3.

Interpret the Meanings of Expressions (IA.ASE.1*)

a. Interpreting Expressions
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Interpreting Algebraic Expressions - Online Lesson Plan
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Interpreting Algebraic Expressions
   LearnZillion: Write and Interpret Real-World Expressions
b. Translation of Expressions
   MathBitsNotebook: Expression Translation
   MathBitsNotebook: Expression Practice
c. Evaluation of Expressions
   MathBitsNotebook: Expression Evaluation
   Virtual Nerd: Variable Substitution
   Khan Academy: Expression Evaluation

Polynomial Operations (FA.AAPR.1*)

a. Polynomial Definition and Concepts
   MathBitsNotebook: Defining and Classifying Polynomials
   Monterey Institute: Topic 1: Polynomials
   Regents Prep: Like Terms
   Regents Prep: Monomials, Binomials and Polynomials
   Illustrative Mathematics: Non-Negative Polynomials
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Generating Polynomials from Patterns : Online Lesson Plan
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Generating Polynomials from Patterns
b. Polynomial Addition and Subtraction
   Monterey Institute: Topic 2: Adding and Subtracting Polynomials
   MathBitsNotebook: Polynomial Addition and Subtraction
   Virtual Nerd: Adding Polynomials
   Virtual Nerd: Subtracting Polynomials
   Virtual Nerd: Subtracting Polynomials (Difference in Length and Width)
c. Monomial by Polynomials

MathBitsNotebook: Monomial by Polynomial Multiplication
Virtual Nerd: Monomial by Polynomial Multiplication
Regents Prep: Multiplying By a Monomial

d. Binomial Multiplication
MathBitsNotebook: Binomial Multiplication
Virtual Nerd: Binomial Multiplication
Virtual Nerd: FOIL Binomial Multiplication
Virtual Nerd: Grid Method Multiplication
Regents Prep: Multiplying Binomials - Distributive, Vertical, Grid & FOIL methods
Regents Prep: Special Binomial Patterns
MathBitsNotebook: Special Binomial Multiplication

e. Polynomial (more) Multiplication
Monterey Institute: Topic 3: Multiplying Polynomials
Monterey Institute: Topic 4: Special Products of Polynomials
Regents Prep: Multiplication Involving Trinomials
MathBitsNotebook: Polynomial Multiplication
Virtual Nerd: Polynomial Multiplication
Virtual Nerd: Polynomial Grid Multiplication Application
Monterey Institute: Polynomial Poke Game
Georgia Department of Education: Mathematics Framework: Polynomials Multiplication Unit
Quia: Polynomial Jeopardy

Factoring Polynomials (IA.ASE.1*; IA.ASE.2*)

a. Greatest Common Factor (GCF)
   StudyZone: Factoring using GCF Lesson
   Khan Academy: Factoring Simple Expressions
   Monterey Institute: Factoring and the Distributive Property
b. Special Factors
   Monterey Institute: Factoring Special Products
   Khan Academy: Factoring Special Products
   Regents Prep: Factoring the Difference of Two Perfect Squares
c. Factoring Trinomials
   West Contra Costa Unified School District: Multiplying Binomials and Factoring Trinomials using Algebra Tiles
   West Contra Costa Unified School District: Factoring Quadratics Lesson
   Monterey Institute: Factoring Trinomials by Grouping 1
   Monterey Institute: Factoring Trinomials by Grouping 2
   Khan Academy: How to Factor Quadratics with a Leading Coefficient of 1
   Regent Prep: Factoring Trinomials where a = 1
   Regent Prep: Factoring Trinomials by Grouping
   Monterey Institute: Match Factors Game
   Quia: Factoring Rags to Riches
   Regent Prep: Factoring Completely
d. Interpreting Coefficients, Factors, and Terms in Expressions
   LearnZillion: Interpret Complex Expressions
   Khan Academy: What are Terms, Factors and Coefficients in Algebraic Expressions?
   Goal Book: Interpreting the Parts of an Expression

Resources
• Georgia Department of Education: Mathematics Teacher Support • Gizmos: Resource Catalog • GoalBook: Tool Kit • Hippocampus: Algebra & Geometry - Presentations, Worked Examples, Test Prep & Simulations • IXL Learning: Algebra 1 Practice • Illustrative Mathematics: High School Content Lesson Index • Khan Academy: Algebra 1 • LearnZillion: Algebra Math Video Lessons • MathBits: Working with Algebra Tiles • MathBitsNotebook: Algebra 1 • Mathematics Assessment Project – Mathematics Assessment Resource Service: Classroom Challenges (Formative Assessment Lessons) • Mathematics Assessment Project – Mathematics Assessment Resource Service: Guide for Teachers and Administrators • Monterey Institute: Algebra 1: An Open Course (Part 1) • Monterey Institute: Algebra 1: An Open Course (Part 2) • Monterey Institute: Algebra 1: An Open Course - Unit 8: Polynomials • Monterey Institute: Algebra 1: An Open Course - Unit 9: Factoring • National Library of Virtual Manipulatives: Algebra Virtual Manipulatives • Quia: Mathematics Shared Activities • Regents Exam Prep Center: Algebra • SAS Curriculum Pathways: SAS Algebra 1 Course • Study Zone: Intermediate Test Prep (6-8) • Virtual Nerd: Algebra 1 • West Contra Costa Unified School District: Algebra 1 Lessons

Sample Formative Assessment Tasks/Questions
Interpret the Meanings of Expressions (IA.ASE.1*) a. IXL: Polynomial Vocabulary - 1 b. IXL: Polynomial Vocabulary - 2 c. Khan Academy: Practice Interpreting Expressions d. Khan Academy: Practice Writing Expressions
Polynomial Operations (FA.AAPR.1*) a. IXL: Model Polynomials with Algebra Tiles b. Gizmos: Addition of Polynomials c. IXL: Add and Subtract Polynomials using Algebra Tiles d. IXL: Add and Subtract Polynomials - 1 e. IXL: Add and Subtract Polynomials - 2 f. Regents Prep: Practice Adding Polynomials g. Regents Prep: Practice Subtracting Polynomials h. IXL: Add Polynomials To Find Perimeter i. Regents Prep: Applied Practice with Adding Polynomials j. IXL: Multiply a Polynomial by a Monomial k. Regents Prep: Practice with Monomials l. IXL: Multiply Two Polynomials Using Algebra Tiles m. IXL: Multiply Two Binomials n. IXL: Multiply Two Binomials: Special Cases o. Regents Prep: Multiplying Binomials Matching Quiz p. IXL: Multiply Polynomials -1 q. IXL: Multiply Polynomials - 2 r. Regents Prep: Practice Multiplying Polynomials s. Regents Prep: Practice Applying Multiplication of Polynomials t. Regents Prep: Grid Game for Multiplying (or Factoring) Polynomials u. Monterey Institute: Project-Based Learning Activity: It's All Fun and Games v. Monterey Institute: Exploring Polynomials Performance Task

Factoring Polynomials (IA.ASE.1*; IA.ASE.2*)

a. StudyZone: Factoring GCF Practice
b. Khan Academy: Factoring Algebraic Expressions using the Distributive Property
c. Regents Prep: Practice Factoring with Common Factors
d. Khan Academy: Factor Special Products
e. Regents Prep: Practice Factoring the Difference of Two Perfect Squares
f. Gizmos: Modeling Factorization
g. Khan Academy: Factoring Quadratics with a leading Coefficient of 1
h. Regents Prep: Online Factoring Quiz
i. Monterey Institute: Perfecting the Long Kick in Soccer Performance Task
j. Regents Prep: Applications of Factoring
k. LearnZillion: Bonita's Bakery - Performance Task for Interpreting Expressions, Factors and Coefficients
l. Khan Academy: Identifying Terms, Factors and Coefficients in Expressions

Polynomials Unit

a. Georgia Department of Education: Mathematics Framework: Polynomials Unit Tasks
b. Monterey Institute: Project-Based Learning Activity: A Cool Million

Intermediate Algebra Unit 4 Title
Quadratic Functions, Equations, and Inequalities

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- IA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable. (Limit to linear; quadratic; ~~exponential with integer exponents.~~)
  - Identify the variable quantities in real-world contexts.
  - Use context to choose the applicable algebraic model (linear equation, linear inequality, or quadratic equation).
  - Write, solve, and interpret the solution of an equation or inequality.
- IA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; ~~exponential with integer exponents; direct and indirect variation~~.)
  - Identify the variables and quantities in real-world contexts.
  - Use context to choose the applicable algebraic model (e.g., linear, quadratic).
  - Write and graph equations in two or more variables, and interpret relationships among variables.
- IA.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
  - Isolate variables to rewrite equations and formulas in equivalent forms.
- IA.AREI.4 Solve mathematical and real-world problems involving quadratic equations in one variable. *(Note: A1.AREI.4a and 4b are not Graduation Standards.)*
- IA.AREI.4a* Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-h)^2=k$ that has the same solutions. Derive the quadratic formula from this form.
  - Identify a quadratic expression, $ax^2 + bx + c$.
  - Identify a perfect-square trinomial by noticing if *a* and *c* are perfect squares and if *b* = 2*ac.*
  - Factor a perfect square trinomial.
  - Complete the square of $ax^2 + bx + c$ to write the quadratic in the form $(x - p)^2 = q$.
  - Derive the quadratic formula by completing the square of $ax^2 + bx + c$.
- IA.AREI.4b* Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a+bi$ for real numbers a and b. (Limit to non-complex roots.)
  - Determine the best method to solve a quadratic equation in one variable (e.g. by inspection, finding square roots, competing the square, applying the quadratic formula, or factoring).
  - Explain that complex solutions result when the radicand is negative in the quadratic formula $(b^2 - 4ac < 0)$.

- IA.AREI.11* Solve an equation of the form (x)=$g(x)$ graphically by identifying the x-coordinate(s) of the point(s) of intersection of the graphs of y=$f(x)$ and y=$g(x)$. (Limit to linear; quadratic;~~ exponential.~~)
    - Describe that a point of intersection on the graph of a system of equations, $y = f(x)$ and $y = g(x)$, represents a solution to both equations.
    - Understand that that since $y = f(x)$ and $y = g(x)$, $f(x) = g(x)$ by the substitution property.
    - Understand that the x-coordinate of the point(s) of intersection for $y = f(x)$ and $y = g(x)$ are also solutions for the $f(x) = g(x)$.
    - Use technology (handheld or computer) to approximate solutions to systems of equations $f(x)$ and $g(x)$.
- IA.ASE.3* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
    - Illustrate properties of a function represented through expressions written in multiple ways.
    - Predict whether a quadratic will have a minimum or a maximum based on the value of *a*.
    - Identify the maximum and minimum of a quadratic written in the form $a(x\text{-}h)^2 + k$
- IA.ASE.3a* Find the zeros of a quadratic function by rewriting it in equivalent factored form and explain the connection between the zeros of the function, its linear factors, the x-intercepts of its graph, and the solutions to the corresponding quadratic equation.
    - Connect the linear factors of a quadratic expression (ax^2+bx+c) to the graphic and algebraic representations of the zeros of the function (Bainbridge).
- IA.FBF.1 Write a function that describes a relationship between two quantities. *(Note: IA.FBF.1a is not a Graduation Standard.)*
- IA.FBF.1a Write a function that models a relationship between two quantities using both explicit expressions and a recursive process and by combining standard forms using addition, subtraction, multiplication and division to build new functions.
    - Define explicit and recursive expressions of a function.
    - Identify the quantities being compared in a real-world problem.
    - Write an explicit and/or recursive expressions of a function to describe real-world problems.
- IA.FBF.1b* Combine functions using the operations addition, subtraction, multiplication, and division to build new functions that describe the relationship between two quantities in mathematical and real-world situations.
    - Recall parent functions.
    - Apply transformations of parent functions.
    - Combine different parent functions (adding, subtracting, multiplying, and/or dividing) to write a function that describes a real-world problem.
    - Explain a multi-step real world problem in terms of function composition and write an equation to describe the composition.

- IA.FBF.3 Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph. (Limit to linear; quadratic; ~~exponential with integer exponents~~; vertical shift and vertical stretch.)
    - Explain why *f(x) + k* translates the original graph of *f(x)* up *k* units and why *f(x) - k* translates the original graph of *f(x)* down *k* units.
    - Explain why *f(x + k)* translates the original graph of *f(x)* left *k* units and why *f(x - k)* translates the original graph of *f(x)* right *k* units.
    - Describe the transformation that changed a graph of *f(x)* into a different graph when given pictures of the pre-image and the image.
    - Determine the value of k given a graph of a transformed function.
    - Graph the listed transformations when given a graph of f(x) and a value of k [f(x) ± k, f(x ± k) and f(x ± k) ± k].
    - Generate and compare examples of functions with different *k* values.
- IA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. (Limit to linear; quadratic;~~ exponential.~~)
    - Connect this content to the content of A1.AREI.4*.
    - Create a graph that matches the description and shows all of the key features of the function, if applicable, the y-intercept, x-intercept(s), increasing interval, decreasing interval, relative minimum, relative maximum, and symmetry.
    - Explain the meaning of all the key features included in a graph or verbal description.
    - Explain why some quadratic functions have more than one x-intercept.
    - Define "increasing interval" as a set of function inputs for which the output increases as the input increases.
    - Define "decreasing interval" as a set of function inputs for which the output decreases as the input increases.
    - Identify and explain the contextual meaning of increasing and decreasing intervals and the relative maximum or minimum of a table or graph.
    - Identify the reflective symmetry in a table or graph.
- IA.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes. (Limit to linear; quadratic; ~~exponential~~.)
    - Analyze the input and output values of a function in algebraic, graphic, tabular, and verbal forms.
- IA.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context. (Limit to linear; quadratic; ~~exponential~~.)
    - Calculate the average rate of change of a function (expressed with function notation, a graph, or a table).

- IA.FIF.7 Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases. (Limit to linear; quadratic; ~~exponential only in the form $y=ax+k$~~.)
    - Explain that the parent function for a quadratic function is the parabola $f(x) = x^2$.
    - Explain that the minimum or maximum of a quadratic is called the vertex.
    - Identify whether the vertex of a quadratic will be a minimum or maximum by looking at the equation.
    - Find the y-intercept of a quadratic by substituting 0 for x and evaluating the function.
    - Estimate the vertex of a quadratic by evaluating different values of x.
    - Use calculated values while looking for a maximum or minimum to decide if the quadratic has x-intercepts.
    - Estimate the x-intercepts of a quadratic by evaluating different values of x.
    - Graph a quadratic function using evaluated points.
    - Use technology to graph a quadratic and to find precise values for the x-intercept(s) and the maximum or minimum.
- IA.FIF.8 Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Limit to linear; quadratic; exponential.) *(Note: A1.FIF.8a is not a Graduation Standard.)*
    - Equations, verbal descriptions, graphs, and tables provide insight into the relationship between quantities.
    - Explain that there are three forms of quadratic functions: standard form, vertex form, and factored form/intercept form.
    - Explain that standard form is $f(x) = ax^2 + bx + c$.
    - Explain that the vertex form is $f(x) = a(x-h)^2 + k$.
    - Explain that factored form/intercept form is $f(x) = a(x - x_1)(x-x_2)$.
    - Explain that the graph of all three forms is a parabola.
    - Explain that all parabolas can be written in standard and vertex form, but that parabolas without x-intercepts can be expressed in factored form using real numbers.
    - Use the x-intercepts of a quadratic function to find the axis of symmetry.
    - Use the axis of symmetry of a quadratic to find the vertex of a parabola.
    - Identify the line of symmetry and vertex of a quadratic written in vertex form.
    - Sketch the graph of a parabola written in vertex form.
    - Determine if a quadratic written in vertex form has x-intercepts by looking at the equation.
    - Use algebra to find the x-intercepts of a quadratic written in vertex form.
    - Demonstrate that the standard and vertex forms of the same quadratic function produce the same values for the x-intercepts, the y-intercept, and the vertex.

- IA.FIF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - Translate a standard form quadratic to factored form by factoring.
  - Translate a standard form quadratic to vertex form by completing the square.
  - Demonstrate that the standard, vertex, and factored/intercept forms of the same quadratic function produce the same values for the x-intercepts, the y-intercept, and the vertex.
  - Write the function that describes a parabola in all three forms (standard, vertex, and factored/intercept) when given a graph with the x-intercepts, y-intercept, and vertex labeled.
- IA.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal. (Limit to linear; quadratic; ~~exponential~~.)
  - Provide insight into the properties of linear and quadratic functions by comparing different representations of two functions.
- IA.NCNS.1* Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a+bi$ with a and b real.
  - Identify that *i* is a complex number where $i^2 = -1$ and $i = \sqrt{-1}$.
  - Identify that a complex number is written of the form $a+bi$ with a and b real.
- IA.NCNS.7* Solve quadratic equations in one variable that have complex solutions.
  - Determine when a quadratic equation in standard form, $ax^2 + bx + c = 0$, has complex roots by examining the graph of $f(x) = ax^2 + bx + c$.
  - Solve quadratic equations with real numbers as coefficients.
  - Express the solution of a quadratic equation as a complex number, $a+bi$.

New Academic Vocabulary for This Unit
• Complete the square • Complex number • Factored form • *i* • Intercept form • Real number • Roots of a quadratic • Standard form of a quadratic equation • Vertex form

Return to High School Overview, High School Table of Contents, or Intermediate Algebra Coversheet.

Prior Knowledge Required for this Unit
• FA.ASE.1 and FA.ASE.2 included creating one and two variable linear equations in Unit 1; these standards are now applied and extended to a new function family, quadratic, in this Unit 3. • During Foundations Unit 1, rewriting simple expressions with radicals provided foundation for quadratic solutions with rational and irrational square roots. • FA.ACE.4 requires fluent variable manipulation and empowers students to rewrite functions as equations. • Student understanding of function relationships, as seen in FA.AREI.10 for functions in two variables, can be expressed in tabular, algebraic (equation), and graphical forms. • Previous student exposure to linear functions was established in Units 1 and 2, particularly the function notation of f(x) and how that relates to understanding of function concepts including 1-to-1 domain to range, particularly as communicated within ordered pairs. • In grade 8, students examined the properties of rigid transformations (rotations, reflections, and translations); reflections and translations will be re-examined with the parent function, $f(x) = x^2$, a parabola rather than geometric figures. (8.GM.1) • Student understanding of the geometric concept of symmetry from middle school will be applied to parabolas; symmetry appears in the graph of a parabola as well as the numerical/tabular representation of a quadratic function. A parabola has a vertical line of symmetry that can be expressed as a vertical line (non-function), and the maximum/minimum of the parabola is the vertex and is a point on the line of symmetry. • Factoring polynomials, from Unit 3, will be applied to discover zeros of a quadratic, as well as creating understanding of completing the square.

Subsequent Knowledge Related to this Unit
• Intermediate Algebra Unit 5 introduces radical and simple rational functions and equations. • Intermediate Algebra Unit 5 extends the key features of the graphs of radical and rational functions (domain, range, y-intercept, zeros, intervals of increase/decrease, axis of symmetry, vertex, end behavior, asymptotes, etc.). • Intermediate Algebra Unit 6 introduces exponential functions and equations. • Intermediate Algebra Unit 6 extends the key features of the graphs of simple exponential functions (domain, range, y-intercept, zeros, intervals of increase/decrease, axis of symmetry, vertex, end behavior, asymptotes, etc.).

Relationship Among Standards in this Unit
The standards in Unit 4 focus on quadratic functions, equations, and converting between forms of quadratic functions. Students will determine if a relation is a quadratic function by analyzing information gathered from tables, graphs, and expressions. They will explore variable rate of change and determine how the rate of change for a quadratic function differs from the rate of change for a linear function. Unit 4 explores critical attributes of quadratic functions, including finding the vertex and axis of symmetry of the graph of a quadratic function; using the graph, table, or factors to identify the zeros; and determining if the vertex is the maximum or minimum value of the function. Access to a variety of technologies, such as graphing utilities, spreadsheets, and computer algebra systems, to solve problems will support mastery of these standards. This unit guides students to recognize the quadratic parent function $f(x) = x^2$, and to explain how a transformation of the parent function can stretch the graph, compress it, or move it left, right, up, or down. Building upon a solid understanding of the features of the graphs of quadratic functions, students will identify the nature and number of solutions (real or complex, and none, one, or two). In this unit, students will: ● Focus on quadratic functions, equations, and applications, ● Model quadratic equations through quadratic functions using a vertical motion model, ● Find the vertex of the graph of a quadratic function and convert a quadratic function from standard to vertex form, ● Apply the vertex form of a quadratic function to find real solutions of quadratic equations, ● Explain why the graph of every quadratic function is a translation of the graph of the basic function $f(x) = x^2$, ● Convert between factored/intercept, vertex, and standard forms of quadratic functions. ● Solve quadratic equations.

Possible Instructional Strategies/Lessons
The order of the topics below illustrates a possible instructional order for Unit 4.

Analysis of Quadratic Functions and Parabolas (IA.FBF.3*; IA.ASE.3(3a); IA.AREI.4; IA.FIF.1*(1a,1b,1c); IA.FIF.8 (8a); IA.FIF.9*)

a. Transforming the graph of the quadratic parent function $f(x) = x^2$
   Introduction to Quadratics
   Algebra 1 Lessons WCCUSD: Exploring Quadratic Graphs
   Khan Academy: Transforming the Graphs of Quadratic Functions
   VirtualNerd: Transforming Parent Functions of Quadratics
   Hippocampus: Simulations – Algebra 1 – Quadratic Functions: Shape Shifter
   You Cubed – Stanford: Squares to Stairs
   You Cubed – Stanford: Squares Upon Squares
b. Graphing Quadratics
   Khan Academy: Graphing Quadratic Functions
   EngageNY: Introduction to Quadratic Function and Graphing
   Purplemath: Graphing Quadratic Functions
   CoolMath: Graphing Quadratics (Parabolas)
   Virtual Nerd: Graphs of Quadratic Functions
   Algebra 1 Lessons WCCUSD: Key Features of Graphs
   Hippocampus: Presentations – Algebra 1 – Graphing Quadratic Functions
   Average Rate of Change
c. Rewriting Quadratics in Different Forms
   MathBitsNotebook: Vertex Form of Quadratic Functions
   Khan Academy: Different Forms of Quadratic Functions and the Features They Reveal
   Algebra 1 Lessons WCCUSD: Three Forms of a Quadratic Function: Matching Quadratic Functions
   Khan Academy: Features of Quadratic Functions
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Representing Quadratic Functions Graphically

d. Solving Quadratics Khan Academy: Quadratic Equations and Functions Purplemath: Solving Quadratic Equations CoolMath: Solving Quadratics CK-12 Text: Solving Quadratic Equations with Complex Roots Math Warehouse: Quadratic Equation Solving
Creating and Solving Quadratic Equations in Terms of Context (IA.ACE.1*; IA.ACE.2*; IA.ACE.4*; IA.AREI.4* (4a, 4b); IA.FIF.2*) Better Lessons: Modeling With Quadratic Functions Algebra 1 Lessons WCCUSD: Factoring Quadratic Expressions Algebra 1 Lessons WCCUSD: Derivation of Quadratic Formula Algebra 1 Lessons WCCUSD: Investigating the Discriminant Algebra 1 Lessons WCCUSD: Quadratics - Matching Game Algebra 1 Lessons WCCUSD: Quadratic Equations - What We Know Algebra 1 Lessons WCCUSD: Three Forms of a Quadratic Function: Matching Quadratic Functions Algebra 1 Lessons WCCUSD: Investigating the Discriminant

Resources
Quadratic Lesson Modules, Teacher Notes, PowerPoints, Videos, Applets, and Activities a. Cliff Notes: Algebra 1 b. Concord Consortium: Graphing Quadratic Equations c. EngageNY: Algebra 1 d. Emergent Math: Current Education Issues in Mathematics, Blog, and Curriculum Map e. Georgia Department of Education: Mathematics Teacher Support f. Gizmos: Resource g. Hippocampus: Algebra & Geometry - Presentations, Worked Examples, Test Prep & Simulations h. Howard County Public School System: Algebra 1 i. IXL Learning: Algebra 1 Practice j. Illustrative Mathematics: High School Content Lesson Index k. Khan Academy: Algebra 1 l. LearnZillion: Algebra Math Video Lessons m. MathBits: Working with Algebra Tiles n. MathBitsNotebook: Algebra 1 o. Math Open Reference: Math Subject Topic List p. Mathematics Assessment Project – Mathematics Assessment Resource Service: Classroom Challenges (Formative Assessment Lessons) q. Mathematics Assessment Project – Mathematics Assessment Resource Service: Guide for Teachers and Administrators r. Monterey Institute: Algebra 1: An Open Course (Part 2) s. Monterey Institute: Algebra 1: An Open Course - Unit 10 – Quadratic Functions t. National Library of Virtual Manipulatives: Algebra Virtual Manipulatives u. Parabola Match - Graph, Equation, Vertex & Axis of Symmetry v. Quia: Mathematics Shared Activities w. Regents Exam Prep Center: Algebra x. SAS Curriculum Pathways: Quadratics y. Study Zone: Intermediate Test Prep (6-8) z. Virtual Nerd: Algebra 1 aa. West Contra Costa Unified School District: Algebra 1 Lessons

Dictionaries, Calculators, and Templates (Graphs, Graphic Organizers) a. A Maths Dictionary for Kids: Explicit Vocabulary Building b. A Maths Dictionary for Kids: Math Charts Printable Resources c. Interactive Mathematics Dictionary: Descriptions, Related Terms, Everyday Examples, Interactive Checkpoints, More Info & Challenge d. Desmos: Online Graphing Calculator with Many Pre-Made Activities e. Math Open Reference: Calculator f. My HRW Classroom: Graphing Calculator Online g. Teacher Vision: Graphic Organizer Printables h. Texas Instruments: Texas Instruments Algebra 2 Graphing Calculator Activities i. Wabbit: Online TI-84 Silver Edition Graphing Calculator Emulator j. Web 2.0: Scientific Calculator
Teaching Strategies a. Creative Educator: Authentic Tasks - Write a Great Authentic Task b. Creative Educator: Project-Based Learning c. Illustrative Mathematics: Illustrative Mathematics Homepage d. Math Video Instructional Development Source: Authentic Contexts e. Power Up What Works: Math Strategies that Work Research f. PrBL: Project-Based Learning

Sample Formative Assessment Tasks/Questions
a. Mathematics Assessment Project – Mathematics Assessment Resource Service: Building Functions b. Mathematics Assessment Project – Mathematics Assessment Resource Service: Formative Assessment Lesson - Generalizing Patterns: Table Tiles c. Mathematics Assessment Project – Mathematics Assessment Resource Service: Formative Assessment Lesson - Representing Quadratic Functions Graphically d. Mathematics Assessment Project – Mathematics Assessment Resource Service: Formative Assessment Lesson - Sorting Equations and Identities e. Mathematics Assessment Project – Mathematics Assessment Resource Service: Summative Assessment Task - Sorting Functions f. Two parabolas have the same x-intercepts $(-2, 0)$ and $(4,0)$. The maximum or minimum value of the first parabola is two times the maximum or minimum value of the other parabola. Sketch these parabolas on the same coordinate grid (Small 32). g. You graph $y = 6x^2 + 5x + 1$. Does the graph change more if you increase the 6 by 1, the 5 by 1, or the 1 by 1 (Small 33). h. Draw a graph of a parabola that grows quickly and the graph of a parabola that grows slowly. What are the equations of these parabolas (Small 33)? i. Compare the roots of these three equations. What do you notice? • $4x^2 - 17x + 4 = 0$ $6x^2 - 37x + 6 = 0$ $8x^2 - 65x + 8 = 0$ Add another equation that acts the same way (Small 32). j. Graph $y = x^2 + 4x + 4$ and $y = 4(x - 4)^2 - 1$. Tell everything you notice about both equations and both graphs (Small 31). k. Depth of Knowledge and assessment items: • DOK 1 - Find the roots and maximum of the quadratic equation: $y = 3(x - 4)^2 - 3$ • DOK 2 - Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values. • DOK 3 - Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9, no more than one time each. $y = -[\ ](x - [\ ])^2 + [\ ]$. (Kaplinsky, Robert. (2015). DOK Distinguishing Between Depth of Knowledge Levels in Mathematics.

Intermediate Algebra Unit 4 Bibliography

Bainbridge, K., Holman, B., Baker, S., and Yuu, W. (2011). *The common core: clarifying expectations for teachers and students: math high school - modeling, number and quantity, algebra, functions, geometry, statistics and probability*. Columbus, OH: McGraw-Hill Education.

Robert Kaplinsky - Glenrock Consulting. Retrieved December 15, 2015, from http://robertkaplinsky.com/

Intermediate Algebra Unit 5 Title
Radical and Simple Rational Functions and Equations

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- IA.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving ~~linear, quadratic~~, simple rational, ~~and exponential~~ relationships. Interpret the solutions and determine whether they are reasonable.
    - Create, solve, and evaluate rational equations with monomial and polynomial denominators.
    - Interpret the solutions, noting if any are extraneous.
    - Create, solve and evaluate radical equations.
- IA.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
    - Use equations and formulas from a variety of subjects and disciplines.
- A2.AREI.2* Solve simple rational and radical equations in one variable and understand how extraneous solutions may arise.
    - Understand that not all solutions generated algebraically are actually solutions to the original equations, therefore, extraneous solutions must be explored.
- IA.AREI.11* Solve an equation of the form $f(x) = g(x)$ graphically by identifying the x- coordinate(s) of the point(s) of intersection of the graphs of $y = f(x)$ and $y = g(x)$.
    - Extend this understanding as the standard was taught in Foundations in Algebra and previous units in Intermediate Algebra.
    - Explain that when we set two equations equal and solve, it is essentially the same thing as solving a system of equations. This standard deals primarily with graphing but can easily be extended to an algebraic method.
    - Limit to rational and radical functions in this unit.
- IA.FBF.1a *(Note: IA.FBF.1a is not a Graduation Standard.)* Write a function that models a relationship between two quantities using both explicit expressions and ~~a recursive process~~ and by combining standard forms using addition, subtraction, multiplication and division to build new functions.
    - Understand that rational functions are the result of the division of two polynomial functions.
- IA.FBF.1b* Combine functions using the operations addition, subtraction, multiplication, and division to build new functions that describe the relationship between two quantities in mathematical and real-world situations.
    - Understand that many rational functions are the result of the combination of other functions. For example, f(x) = 1, g(x) = x, and h(x) = x+2. m(x) = f(x)/[g(x)*h(x)] = 1/[x(x+2)] = 1/x^2+2x
- IA.FBF.3* Describe the effect of the transformations $kf(x)$, $f(x) + k$, $f(x + k)$ and combinations of such transformations on the graph of $y = f(x)$ for any real number *k*. Find the value of *k* given the graphs and write the equation of a transformed parent function given its graph.
    - Understand that many rational functions are the result of a transformation of f(x) = 1/x. For example, g(x) = f(x+a) = 1/(x+a).
    - Understand that many radical functions are the result of a transformation of f(x) = $\sqrt{x}$. For example, g(x) = f(x-2) = $\sqrt{x-2}$

- IA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.
    - Create a graph that matches the description and indicates all of the key features of the function.
    - Describe positive end behavior as the trend of a function's outputs as the input grows increasingly positive.
    - Describe negative end behavior as the trend of the function's outputs as the input grows increasingly negative.
    - Use the problem situation to explain the end behavior of a function.
    - A key feature of rational functions is the possibility of asymptotes and points of discontinuity.
- IA.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
    - Explain how the domain of a function is represented in its graph.
    - Emphasize that the domain of a radical function is restricted due to the requirement that the radicand must be greater than or equal to zero.
    - State the appropriate domain of a function that represents a real-world situation and explain why other numbers might be excluded from the domain.
- IA.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.
    - Calculate the average rate of change of a function presented in various forms.
    - Compare the rates of change of two or more functions presented in various forms.
    - Use appropriate units to interpret the meaning of the average rate of change.

- IA.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases.
  - Explain that the parent function for square root functions is the function $f(x) = \sqrt{x}$.
  - Sketch the graph of a square root function using convenient input values for x (perfect squares).
  - Use technology to graph square root functions and to find intercepts.
  - Define a rational function as the ratio of two polynomials.
  - Describe the end behavior of a rational function when examining the graph of the function.
  - Determine the y-intercept of a rational function.
  - Determine the x-intercept of a rational function.
  - Determine if a rational function has a horizontal asymptote and find the asymptote.
  - Sketch the graph of a rational function based on its domain, x-intercepts, y-intercept, and end behavior.
  - Use technology to graph rational functions and to find precise values for the x-intercept(s) and the maximums and minimums.
  - Emphasize asymptotes for rational functions and restricted domains for radical functions.
- IA.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function.
  - Simplifying (by factoring or other strategies) the rational equations will help explain the discontinuity, asymptotes, and other features of the graph of the function.
  - Include rationalizing any radical monomials by using the exponent properties or binomials that include radicals by multiplying by the conjugate.

New Academic Vocabulary for This Unit
• Asymptotes (vertical and horizontal) • Conjugate • Extraneous Solution • Point of Discontinuity (removable and non-removable) • Rational Expression

Prior Knowledge Required for this Unit
• Notice the arithmetic operations of rational expressions are governed by the same rules as the arithmetic operations of rational numbers (7.NS.3). • Recall and make use of their knowledge of polynomial functions to investigate the characteristics of rational functions (AAPR.1 and AAPR.3).

Subsequent Knowledge Related to this Unit
● IA.ACE.1* Unit 5 addresses solving radical and rational equations. ○ Will also be addressed in Algebra 2 Unit 5 (Radical and Rational Functions and Equations). ● IA.ACE.4* Unit 5 addresses solving radical and rational literal equations and formulas. ○ Will also be addressed in Algebra 2 Unit 5 (Radical and Rational Functions and Equations). ● IA.AREI.11* Unit 5 addresses solving the radical and/or rational equations graphically. ○ Will also be addressed in Algebra 2 Unit 5 (Radical and Rational Functions and Equations). ● IA.FBF.1* Unit 5 writes a function that models a relationship between two quantities. ○ Will also be addressed in Algebra 2 Unit 5 (Radical and Rational Functions and Equations). ● IA.FBF.3*Unit 5 describes the effect of the transformations on the graphs of radical and rational functions and writes the equation of a transformed parent function given its graph. ○ Will also be addressed in Algebra 2 Unit 5 (Radical and Rational Functions and Equations). ● IA.FIF.4* Unit 5 interprets the key features of radical and rational functions. ○ Will also be addressed in Algebra 2 Unit 5 (Radical and Rational Functions and Equations). ● IA.FIF.5* Unit 5 relates the domain and range of radical and rational functions to the relationships described. ○ Will also be addressed in Algebra 2 Unit 5 (Radical and Rational Functions and Equations). ● IA.FIF.8* Unit 5 translates between equivalent forms of rational functions. ○ Will also be addressed in Algebra 2 Unit 5 (Radical and Rational Functions and Equations).

Relationship Among Standards in this Unit
The standards in this unit include functions not specifically covered in previous math courses. Students are expected to write, graph, apply, and interpret these functions. The focus in Unit 5 is on the key features (asymptotes and points of discontinuity) of rational and radical functions. The goal is for students to develop, understand, and make connections between a variety of function forms: equations, graphs, verbal descriptions, and tables. Emphasis is also placed on the transformation of these functions in the coordinate plane. The terms extraneous solutions, asymptotes, and points of discontinuity are introduced for the first time.

Possible Instructional Strategies/Lessons

The order of the topics below illustrates a possible instructional order for Unit 5.

In earlier grades, students should have noted rational numbers extend the arithmetic of integers by allowing division by all numbers except zero. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. Sign charts are recommended when evaluating rational functions to show the change in directions of the graphs of rational functions. For both rational and radical functions, it is essential for students to see and analyze many graphs of both types of functions. During the analysis of the graphs of the functions, the students should look for patterns and notice what features remain the same and what changes with the graphs. After observing several graphs, students should understand that the inverses of the power functions $y = x^2$ (with domains restricted as needed) form parent functions $y = \sqrt[2]{x}$ families of radical functions.

- Illustrative Mathematics: Graphing Rational Functions (IA.FIF.7)
- West Contra School District: Key Features and Vocabulary for Rational Functions (IA.FIF.4)
- West Contra School District: Solving Radical & Equations (IA.AREI.2)
- Brain Genie: Rational Exponents and Radical Functions (IA.REI.2)
- Brain Genie: Solve Rational Equations (IA.REI.2)
- Illustrative Math: Who Wins the Race (I A.AREI.2, IA.ACE.1, IA.AREI.11)
- Geogebra: Graphs of Radical Functions (IA.FBF.3)
- Geogebra: Characteristics of Radical Functions (IA.FBF.3)
- Geogebra: Graph Simple Rational (IA.FBF.3)

Resources
Algebra 2 Course Content Resources from Other States a. Algebra Lesson WCCUSD Resource: Solving Radical Equations b. Algebra Lesson WCCUSD Resource: Solving Rational Equations c. EngageNY: Algebra 2 Resources d. Georgia Department of Education: 9 – 12 Resources e. Georgia Department of Education: Unit 4 – Rational and Radical Relationships f. Henrico County Public Schools: Algebra 2 Resources g. Regents Prep: Algebra 2 and Trig
Activity Resources a. Illustrative Mathematics: Activities for Content Standards b. Robert Kaplinsky: Problem-Based Learning Activities c. Yummy Math: Interpreting Functions Tasks
Interactive Resources a. Emergent Math: Homepage b. Explore Learning: Gizmo Online Simulations
Practice Tests and Assessment Resources a. California Department of Education: Algebra 2 Released Test Questions b. IXL Math: Algebra 2 c. Jefferson Lab: Virginia Standards Practice Tests d. Problem Attic: Homepage
Video Resources a. HippoCampus: Videos for High School Math b. Virtual Nerd: Videos for Algebra 2
Graphing Calculator Resources a. Desmos: Online Graphing Calculator with Many Pre-Made Activities b. Online TI-84 Silver Edition: Graphing Calculator Emulator c. Texas Instruments: Algebra 2 Graphing Calculator Activities

Sample Formative Assessment Tasks/Questions
Solving Equations (A2.ACE.1) a. Math Bits Notebook: Radical Equations #3, 11, 12 b. Math Bits Notebook: Radical Equations #1 and 10 c. Problem-Attic: Homepage
Rational Function that Models Two Quantities (A2.FBF.1, A2.FIF.5) a. Georgia Department of Education: Unit 4 – Rational and Radical Relationships
Functions that Model Two Quantities a. Mathematics Assessment Project - Representing Functions of Everyday Situations
Sample Items (JMU Pivotal Items) a. Draw a graph and give the equation of a function with a hole in the graph and one asymptote. b. Draw a graph of a function that has all of the characteristics below. Explain how your graph satisfies the conditions. • There is a hole in the graph at x = -5. • There is an asymptote at x = 2. • The graph decreases as x goes to -∞ and also decreases as x goes to ∞. • The graph has a zero at x = -3. • The graph decreases throughout the interval [2, ∞]. c. Describe a function whose domain is NOT all real numbers. What is the range of your function? d. Describe a function whose range does not contain any points above the x-axis. Explain how you know your function's range satisfies the condition.

Intermediate Algebra Unit 5 Bibliography

Bainbridge, Katy, Bob Holman, Sandra Baker, and Wesley Yuu. *The Common Core: Clarifying Expectations for Teachers & Students: Math High School - Modeling, Number & Quantity, Algebra, Functions, Geometry, Statistics & Probability*. Columbus, OH: McGraw-Hill Education, 2011. Print.

Intermediate Algebra Unit 6 Title
Exponential Functions and Equations

Return to High School Overview, High School Table of Contents, or Intermediate Algebra Coversheet.

Content Standards with Clarifying Notes

- IA.ACE.1* Create and solve equations and ~~inequalities~~ in one variable that model real-world problems involving linear, quadratic, ~~simple rational~~, and exponential relationships. Interpret the solutions and determine whether they are reasonable.
    - Identify the variable and quantities in real-world contexts.
    - Use context to choose the applicable algebraic model (linear equation, linear inequality, quadratic equation, or exponential).
    - Write, solve, and interpret the solution of an equation.
    - Expand what was done in previous units to include exponential functions ($f(x) = b^x$).
    - Extend beyond integer exponents from Foundations.
- IA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
    - Emphasis should be on the appropriate use of labels, units, and scales when graphing.
    - Identify the variables and quantities in real-world contexts.
    - Use context to choose the applicable algebraic model (e.g., linear, quadratic, exponential).
    - Write and graph equations in two or more variables, and interpret relationships among variables.
    - Expand what was done in previous units to include exponential ($f(x) = b^x$).
    - Extend beyond integer exponents from Foundations.
- IA.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
    - Emphasis should be placed on the compound interest formulas for exponential growth/decay (e.g., solve the compound interest formula to find the principal value).
- IA.AREI.11* Solve an equation of the form $(x) = (x)$ graphically by identifying the x-coordinate(s) of the point(s) of intersection of the graphs of $y = (x)$ and $y = g(x)$.
    - Extend this standard to review quadratic, rational, radical, absolute value and exponential functions.
    - Understand that graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically.
    - Include the use of a graphing calculator for finding points of intersection.

- IA.FBF.1a* Write a function that models a relationship between two quantities using both explicit expressions and a recursive process and by combining standard forms using addition, subtraction, multiplication and division to build new functions. (Note: IA.FBF.1a is not a Graduation Standard.)
    - Review explicit and recursive expressions of a function.
    - Identify the quantities being compared in a real-world problem.
    - Write an explicit and/or recursive expressions of a function to describe real-world problems.
    - Recall parent functions.
    - Apply transformations of parent functions.
    - Combine different parent functions (adding, subtracting, multiplying, and/or dividing) to write a function that describes a real-world problem.
    - Emphasize that there are times when one form (explicit and recursive) to describe the function is preferred over the other.
- IA.FBF.1b* Combine functions using the operations addition, subtraction, multiplication, and division to build new functions that describe the relationship between two quantities in mathematical and real-world situations. (Note: IA.FBF.1b is not a Graduation Standard.)
    - More complex exponential functions should be developed by using the operations addition, subtraction, and multiplication (such as $f(x) = 100 + 6^t$ or build a function that models the temperature of a cooling liquid by adding a constant function to a decaying exponential, and relate these functions to the model.)
    - Emphasis should be on how the operation affects the function and the relationship it is modelling
- IA.FBF.2* Write ~~arithmetic and~~ geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
    - Understand the difference between an explicit and recursive formula. An explicit formula is a function rule that relates each term of a sequence to the term number/position. An explicit formula allows the students to find any element of a sequence without knowing the element before it. Recursive formula is a function rule that relates each term of a sequence to the prior term.
    - Connect geometric sequences to exponential functions.
    - Exponential functions are the explicit form of recursively-defined geometric sequences.
    - The recursive formula for a geometric sequence uses multiplication and the explicit formula uses exponentiation.

- IA.FBF.3* Describe the effect of the transformations (x), $(x)+k$, $f(x+k)$, and combinations of such transformations on the graph of $y=f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.
  - Exploration of the effects of a constant, k, should include comparing the shape and position and analyzing the differences between the graphs of the original equation and transformed equation.
  - Explain why *f(x)+k* translates the original graph of *f(x)* up *k* units and why *f(x)-k* translates the original graph of *f(x)* down *k* units.
  - Explain why *f(x+k)* translates the original graph of *f(x)* left *k* units and why *f(x-k)* translates the original graph of *f(x)* right *k* units.
  - Describe the transformation that changed a graph of *f(x)* into a different graph when given pictures of the pre-image and the image.
  - Determine the value of k given a graph of a transformed function.
  - Graph transformations of the graph of f(x) when given the value(s) of k [f(x)±k, f(x±k) and f(x±k)±k].
  - Generate and compare examples of functions with different *k* values.
  - Expand what was done in previous units to include exponential functions ($f(x) = b^x$).
  - Emphasis should be given to horizontal shifts which were excluded in the Foundations Standard.
- IA.FIF.3* Define functions recursively and recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
  - Explain that a recursive formula tells how a sequence starts and how to use the previous value(s) to generate the next element of the sequence.
- IA.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.
  - Create a graph that matches the description and shows all of the key features of the function, if applicable, the y-intercept, x-intercept(s), increasing interval, decreasing interval, and end behavior.
  - Explain the meaning of all the key features included in a graph or verbal description.
  - A connection to FBF.1b (function compositions) and FBF.3 (parent function transformations) should be made while teaching.
- IA.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - Analyze the input and output values of an exponential function in algebraic, graphic, tabular, and verbal forms.
  - Emphasize the domain includes all real numbers as students may think otherwise when reviewing the visual representation.
  - Explore graphical representations to analyze the variability in the range.

- IA.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.
  - Calculate the average rate of change of a function presented in various forms.
  - Compare the rates of change of two or more functions presented in various forms.
  - Use appropriate units to interpret the meaning of the average rate of change.
  - Due to the rapid increase/decrease of exponential functions, it may be necessary to look at smaller intervals when determining average rate of change.
- IA.FIF.8b* Interpret expressions for exponential functions by using the properties of exponents. (Note: IA.FIF.8b is not a Graduation Standard.)
  - Identify percent rate of change in functions such as $y = 1.02^t$, $y = .97^t$, and classify them as representing exponential growth or decay.
  - Identify the beginning "principle" value of an exponential equation.
  - Identify the number of times an interest equation is compounded based on the exponent.
  - Manipulate exponential equations using the properties of exponents in order to solve a one variable equation connect to ACE.1* while teaching.
- IA.FLQE.2* Create symbolic representations of ~~linear and~~ exponential functions, including ~~arithmetic and~~ geometric sequences, given graphs, verbal descriptions, and tables.
  - Write algebraic equations for graphs representing exponential growth (increase) or decay (decrease).
  - Write algebraic equations to represent exponential patterns described in words.
  - Write algebraic equations to represent a given table of values.
- IA.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context.
  - Use real-world contexts to help students understand how the parameters of exponential functions depend on the context.
  - Give students different parameters of a function to manipulate and compare the results to draw conclusions about the effects of the changes.

New Academic Vocabulary for This Unit
• Compounded Interest • Explicit function • Geometric sequence • Half-life • Horizontal translation • Percent rate of change • Parent exponential function ($f(x) = b^x$) • Principle • Recursive Function • Recursion

Prior Knowledge Required for this Unit
In earlier grades, students have developed conceptual knowledge and have had the opportunity to learn how to: ● Represent any point on the coordinate plane by its coordinates (5.G.1b). ● Write and evaluate numerical expressions involving whole-number exponents and positive rational number bases using the Order of Operations (6.EEI.1). ● Extend previous understanding of Order of Operations to solve multi-step real-world and mathematical problems involving rational numbers (7.EEI.3). ● Understand and apply the laws of exponents (i.e. product rule, quotient rule, power to a power, product to a power, quotient to a power, zero power property, negative exponents) to simplify numerical expressions that include integer exponents (8.EEI.1). ● Extend concepts of linear equations and inequalities in one variable to more complex multi-step equations and inequalities in real-world and mathematical situations (8.EEI.7). ● Solve linear equations and inequalities with rational number coefficients that include the use of the distributive property, combining like terms, and variables on both sides (8.EEI.7a). ● Relate inputs (x-values or domain) and outputs (y-values or range) to independent and dependent variables (8.F.1b). ● Translate among the multiple representations of a function, including mappings, tables, graphs, equations, and verbal descriptions (8.F.1c). ● Graph a function from a table of values. Understand that the graph and table both represent a set of ordered pairs of that function (8.F.1e). ● Investigate the differences between linear and nonlinear functions using multiple representations (i.e. tables, graphs, equations, and verbal descriptions) (8.F.3). ● Recognize that the graph of a linear function has a constant rate of change (8.F.3b). ● Understand that the slope is the constant rate of change and the y-intercept is the point where $x = 0$ (8.F.4a). ● Construct a function in slope-intercept form that models a linear relationship between two quantities (8.F.4c). ● Analyze and describe attributes of graphs of functions (e.g., constant, increasing/decreasing, linear/nonlinear, maximum/minimum, discrete/continuous) (8.F.5a). ● Sketch the graph of a function from a verbal description (8.F.5b). ● Write a verbal description from the graph of a function with and without scales (8.F.5c). ● Convert units (e.g. dimensional or unit analysis, rounding, etc.) with accuracy and precision to solve problems involving measurement (FA.NQ.3*). ● Compare and contrast linear and exponential relationships (FA.FLQE.1*). ● Create and solve equations in one variable that model real-world problems (limited to exponentials with linear exponents) (FA.ACE.1*).

- Create equations in two or more variables to represent linear relationships between quantities. As well as, graph the equations on coordinate axes using appropriate labels, units, and scales (limited to exponentials with linear exponents) (FA.ACE.2*).
- Describe the effect of the transformations of a parent graph and write the equation of a transformed parent function given its graph (limited to vertical shift and vertical stretch) (FA.FBF.3*).
- Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features (FA.FIF.4*).
- Relate the domain and range of a function to its graph and to the quantitative relationship it describes (FA.FIF.5*).
- Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (FA.FIF.8*).

Subsequent Knowledge Related to this Unit
After the Intermediate Algebra class, students have course options that differ in scope and sequence. Students who take SCCCR Algebra 2 will encounter all of the South Carolina College and Career Ready standards addressed in this Unit again in their entirety, while students who opt to take SCCCR Probability and Statistics will utilize knowledge gained from this unit by looking at mathematics from a different perspective.

Relationship Among Standards in this Unit
The standards in Unit 6 expand prior knowledge of integer exponents and focus on exponential functions, equations, and their applications. The major differences between this unit and Foundations in Algebra Unit 4 (Modeling and Analyzing Exponential Functions) is increased complexity of real-world problems with non-integer exponents and answers; the inclusion of horizontal shifts on the graph; and the introduction of geometric sequences and recursion. Students graph exponential functions, identify key features (intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; asymptotes; end behavior) and use appropriate technology to investigate multiplicative change in exponential functions and increase conceptual understanding. Manipulating the variable k for specific values, students also examine the effects on the graph on the parent function, $f(x) = b^x$, by replacing f(x) with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative). As the unit progresses, students compare exponential models, solve exponential equations, interpret the solutions of exponential functions taking into consideration any restrictions to the domain/range and find the value of k when given exponential graphs. Calculating and interpreting the average rate of change over a given interval of a function from a function equation, graph or table, and explaining what that means in terms of the context of the function gives students the opportunity to connect this topic to all other graphs studied. Students use exponential function notation to model a variety of contexts and use context to interpret functions in specific applications. Analysis of graphs, verbal descriptions, and tables supports students in creating symbolic representations of exponential functions, including geometric sequences, to model a relationship between two quantities. The unit culminates by giving students the chance to apply their knowledge of exponential graphs and functions through writing geometric sequence equations in both recursive and explicit form.

Possible Instructional Strategies/Lessons
The order of the topics below illustrates a possible instructional order for Unit 6.

Graphs of Exponential Relationships (IA.ACE.2*; IA.AREI.11*; IA.FBF.3*; IA.FIF.4*; IA.FIF.5*; IA.FIF.6*)

a. Exponential Graph Transformations
   BetterLesson: Shifting Exponential Functions (Lesson 4)
   BetterLesson: Stretching Exponential Functions (and your mind)
   CK-12: Graphs of Exponential Functions
   CK-12: Graphs of Exponential Functions: Studying Algae Blooms (Play-Learn-Interact-Xplore)
   Georgia Standards of Excellence Framework: Graphic Organizer – Graphing Transformations (page 17)
   Georgia Standards of Excellence Framework: High Functioning! (page 46)
   Gizmos: Graphing Exponential Functions (Interactive)
   KhanAcademy: Graphing Exponential Functions
   Sophia: The Graph of an Exponential Equation
   Texas Instruments: Growth and Decay TI Calculator (Lab)
b. Key Features of Graphs (intercepts, increasing/decreasing, end-behavior, domain/range)
   BetterLesson: Graphing Exponential functions (Lesson 3)
   EngageNY: Algebra 1 Module 3, Topic B, Lesson 13 – Interpreting the Graph of a Function
   Georgia Standards of Excellence Framework: Compare/Contrast: Exponential Functions (page 16)
   Georgia Standards of Excellence Framework: Graphs of Exponential Functions (page 18)
   Georgia Standards of Excellence Framework: Zombie Apocalypse Simulation (page 31)
   MathBitsNotebook: Features of Exponential Functions
c. Solve Real-World Exponential Equations by Graphing (restricting domain to problem situation)
   EngageNY: Algebra 1 Module 3, Topic B, Lesson 11 – The Graph of a Function
   Georgia Standards of Excellence Framework: Bacteria in the Swimming Pool (page 35)
   Monterey Institute: Introduction to Exponential Functions (Presentation & Worked Examples)

d. Average Rate of Change
   Algebra 1 Lessons WCCUSD: Average Rate of Change
   BetterLesson: Counting the Change: Linear, Quadratic or Exponential? (Lessons 5 & 6)
   CK-12: Linear, Exponential and Quadratic Models: Bernoulli Effect (Play-Learn-Interact-Xplore)
   EngageNY: Algebra 1 Module 3, Topic A, Lesson 4 – Why do banks pay YOU?
   EngageNY: Algebra 1 Module 3, Topic B, Lesson 14 – Linear and Exponential Models – Comparing Growth Rates
   LearnZillion: Understanding Rate of Change in Exponential Growth Functions
   MathBitsNotebook: Average Rate of Change

Solving Exponential Equations and Formulas (IA.ACE.1*; IA.ACE.4*; IA.FBF.1b; IA.FIF.8b; IA.FLQE.5*)

a. One Variable (with non-integer Exponents)
   BetterLesson: Cat Island – Cats can't add but they do multiply! (Lessons 1 & 2)
   BetterLesson: Sorting out the Change (Lesson 7)
   CK-12: Applications of Exponential Functions
   Georgia Standards of Excellence Framework: The Marvel of Medicine (page 36)
   Georgia Standards of Excellence Framework: Zombie Apocalypse Simulation (page 25)
   Kate Nowak: Exponential Functions and Credit Cards
   Math Warehouse: Exponential Growth Activity

b. Solving Exponential Problems using Formulas
   A Recursive Process: Aaron's (Rent to Buy)
   ALEX: Exponential Growth and Decay
   BetterLesson: Credit Card Investigation (Lessons 9 - 12)
   CK-12: Exponential Decay: Sunglasses Tint (Play-Learn-Interact-Xplore)
   CK-12: Exponential Fractal Snowflake (Play-Learn-Interact-Xplore)
   CK-12: Exponential Growth and Decay
   College Preparatory Mathematics: Exponential Growth and Decay
   Gizmos: Exponential Growth and Decay (Interactive)
   Illuminations: Trout Population Calculator
   KhanAcademy: Exponential Decay Functions
   KhanAcademy: Exponential Growth Functions
   LearnZillion: Manipulate Exponential Models to Reveal Information

RegentsPrep: Exponential Growth and Decay
WeberTube: Exponential Growth and Decay (Paper Folding) (page 7/193)
VirtualNerd: How do You Use the Formula for Compound Interest?
Youtube: Mythbusters – Folding Paper Seven Plus Times

c. Combine Functions using + -*/ to build new functions
EngageNY: Algebra 1, Module 3, Topic B, Lesson 9 - Representing, Naming and Evaluating Functions
Georgia Standards of Excellence Framework: Investigating Exponential Growth and Decay (page 13)
WeberTube: Exponential Function Contexts (build) (page 20/206)

d. Interpret Exponential Expressions and Parameters using properties of exponents
LearnZillion: Solve Exponential Equations Using Properties Of Exponents
LearnZillion: Use The Properties of Exponents To Interpret Expressions and Functions
LearnZillion: Use The Properties Of Exponents To Transform Expressions

Geometric Sequences - Explicit and Recursive Forms (IA.FBF.1a; IA.FBF.2*; IA.FIF.3*; IA.FLQE.2*)

a. Geometric Sequences Written Explicitly
Algebra Lab: Algebra 2 Recipe: Geometric Sequences
CK-12: Geometric Sequences and Exponential Functions
CK-12: Geometric Sequences: Bouncing Ball (Play-Learn-Interact-Xplore)
KhanAcademy: Geometric Sequences Intro
LearnZillion: Represent an Explicit Rule as an Exponential Function
LearnZillion: Writing an Explicit Formula for an Exponential Relationship
Mathematics Vision Project: Chew On This – Solidify Understanding Task (page 21)
Mathematics Vision Project: Don't Break the Chain – Solidify Understanding Task (page 15)
Mathematics Vision Project: Growing, Growing Dots - Develop Understanding Task (page 4)
MathGuide: Geometric Sequences
PurpleMath: Arithmetic and Geometric Sequences
SAS Curriculum Pathways: Geometric Sequences
WeberTube: Linear, Exponential or Neither (page 8/194)
WeberTube: Write sequence formulas from contexts, tables, graphs (page 27/213)

b. Recursive Functions
   Insights into Algebra: Understanding Basic Recursion
   EngageNY: Algebra 1 Module 3, Topic A, Lesson 2 – Recursive Functions
   Illuminations: Using Graphs, Equations and Tables to Investigate Recursion
   LearnZillion: Writing a Recursive Formula for an Exponential Relationship
   Lost In Recursion: The "Lost in Recursion" Recursion
   TeacherWeb: Difference Between Recursive and Explicit Formulas
   WeberTube: Explicit & Recursive Formulas for Arithmetic Sequences (page 14/200)
c. Geometric Sequences Written Recursively
   Algebra 1 Lessons WCCUSD: Geometric Sequences
   Algebra Lab: Geometric Sequences
   Georgia Standards of Excellence Framework: Community Service, Sequences and Functions (page 52)
   Illuminations: Counting the Trains
   Mathematics Vision Project: What Comes Next? What Comes Later? – Solidify Understanding Task (page 25)
   Riverside Unified School District: Geometric Explicit and Recursive Sequences
   WeberTube: Explicit & Recursive Formulas for Geometric Sequences (page 22/208)

Resources

Application Resources (Downloadable Lessons, Video, Applets, and Online Algebra Notes)

a. BetterLesson: Exponential Functions and Equations (14 Lessons)
b. CK-12: Algebra Topics
c. Emergent Math: Common Core Problem Based Curriculum Maps
d. Engage New York: Algebra I Modules
e. Georgia Department of Education: Unit 4: Modeling and Analyzing Exponential Functions
f. Georgia Department of Education: Unit 5: Exponential and Logarithmic Functions
g. Georgia Department of Education: Mathematics Teacher Support
h. Gizmos: Resource Catalog
i. Illuminations: Resources For Teaching Math
j. Illustrative Mathematics: High School Content Lesson Index
k. IXL Learning: Algebra 1 Practice
l. James Madison University: Algebra II – Pivotal Items
m. KhanAcademy: Algebra 1
n. Khan Academy: Algebra 1 - Introduction to Exponential Functions
o. LearnZillion: Algebra Math Video Lessons
p. Math Education Page: Start Page
q. MathBitsNotebook: Algebra 1
r. Mathematics Assessment Project – Mathematics Assessment Resource Service: Classroom Challenges (Formative Assessment Lessons)
s. Mathematics Assessment Project – Mathematics Assessment Resource Service: Guide for Teachers and Administrators
t. Mathematics Vision Project: Arithmetic and Geometric Sequences Module/Activity
u. Monterey Institute: Intermediate Algebra
v. National Security Agency: High School Concept Development Units - Algebra
w. PBS Learning Media: Mathline
x. Regents Exam Prep Center: Algebra
y. SAS Curriculum Pathways: SAS Algebra 1 Course – Unit 4
z. Sophia: Exponential Equations and Functions
aa. VirtualNerd: Exponential Functions
bb. WeberTube: Connecting Sequences and Functions

cc. West Contra Costa Unified School District: Algebra 1 Lessons

Dictionaries, Calculators, and Templates

a. Alcula: Online Linear Regression Calculator
b. A Maths Dictionary for Kids: Math Charts
c. A Maths Dictionary for Kids: Math Dictionary
d. Illuminations: Affine Recurrence Plotter
e. Illuminations: Affine Recurrence Spreadsheet
f. Illuminations: Compound Interest Simulator
g. Math Open Reference: Calculator
h. Math Open Reference: Full-Size Calculator
i. My HRW Classroom: Graphing Calculator Online
j. University of Georgia Mathematics Education Program: Interactive Mathematics Dictionary
k. Video Math Teacher: Download Free Virtual TI Calculator Online
l. Wabbitemu: TI Calculator Emulator

Teaching Strategies

a. Creative Educator: Authentic Tasks - Write a Great Authentic Task
b. Creative Educator: Project-Based Learning
c. Fawn Nguyen: Finding Ways - Math Taboo Game
d. Illustrative Mathematics: Illustrative Mathematics Homepage
e. Math Education Page: Lab Gear
f. Math Video Instructional Development Source: Authentic Contexts
g. Power Up What Works: Math Strategies that Work Research

Sample Formative Assessment Tasks/Questions
Graphs of Exponential Relationships (IA.ACE.2*; IA.AREI.11*; IA.FBF.3*; IA.FIF.4*; IA.FIF.5*; IA.FIF.6*) a. CK-12: Graphs of Exponential Functions (Practice) b. Georgia Standards of Excellence Framework: Compare/Contrast: Exponential Functions (page 16) c. JMU Pivotal Items: AII.6.2 Explain how you determined your new equation. (page 7) d. Illustrative Mathematics: A Saturating Exponential e. Illustrative Mathematics: Exponential Functions f. IXL: Match Exponential Functions and Graphs g. Monterey Institute: Introduction to Exponential Functions (Practice & Review) h. Sophia: Graphing Exponential Equations (Quiz) i. Sophia: The Graph of an Exponential Equation (Quiz) j. WeberTube: Exponential Growth and Decay (apply) (page 11/197)
Solving Exponential Equations and Formulas (IA.ACE.1*; IA.ACE.4*; IA.FBF.1b; IA.FIF.8b; IA.FLQE.5*) a. ALEX: Exponential Growth and Decay Practice b. ALEX: More Practice on Growth and Decay c. ALEX: Practice with Exponential Growth and Decay d. CK-12: Exponential Growth and Decay (Practice) e. CK-12: Applications of Exponential Functions (Practice) f. Create an equation that represents the temperature of a cup of coffee sitting in a room that is maintained at $68°$. What will $68°$ represent on the graph? g. Dartmouth: 3.2 Exponential Growth and Decay - Quiz h. Engage New York: Algebra 1 Module 3 Lesson 5 – The Power of Exponential Growth i. Engage New York: Algebra 1 Module 3 Lesson 22 – Modeling an Invasive Species Population j. Engage New York: Algebra 1 Module 3 Lesson 23 – Newton's Law of Cooling k. Illuminations: National Debt and Wars l. IXL: Evaluate and Exponential Function m. IXL: Exponential Growth and Decay – Word Problem Practice n. JMU Pivotal Items: AII.9.1 Prices of Postage Stamps (page 10) o. Mathematics Assessment Project: Modeling Population Growth: Having Kittens p. Mathematics Assessment Project: Representing Linear and Exponential Growth

q. NRICH: Activity on Comparing Two Arithmetic Sequences
r. RegentsPrep: Practice with Exponential Equations and Graphs

Geometric Sequences - Explicit and Recursive Forms (IA.FBF.1a; IA.FBF.2*; IA.FIF.3*; IA.FLQE.2*)

a. Algebra Lab: Algebra 2 Practice
b. Algebra Lab: Geometric Sequences
c. CK-12: Geometric Sequences and Exponential Functions (Practice)
d. Engage New York: Algebra 1 Module 3 Lesson 3 – Arithmetic and Geometric Sequences
e. Illustrative Mathematics: Kitchen Floor Tiles
f. IXL: Geometric Sequences
g. IXL: Write variable expressions for geometric sequences
h. JMU Pivotal Items: AII.2.1 Write a sequence from a real-world problem. (page 3)
i. Mathematics Assessment Project: Multiplying Cells
j. Mathematics Vision Project: Geometric Meanies – Practice Understanding Task (page 34)
k. Mathematics Vision Project: I Know…What Do You Know? – Practice Understanding Task (page 39)
l. RegentsPrep: Applied Exponential Growth and Decay
m. SAS Curriculum Pathways: Geometric Sequences (Practice Tab)
n. Virginia Department of Education: Sequence Matching Activity with Information on Differentiation and Essential Questions
o. WeberTube: Geometric Sequences & Their Formulas (apply) (page 26/212)
p. WeberTube: Linear, Exponential or Neither 2 (apply) (page 12/198)
q. WeberTube: Mixed Practice: Linear & Exponential Sequences (page 24/210)
r. WeberTube: Practice: Write the explicit and recursive equation for each graph. (page 31/217)
s. WeberTube: Recursive & Explicit Formulas from Sequences, Tables, Contexts, & Graphs (page 33/219)
t. WeberTube: Story Contexts: Functions in Recursive and Explicit Format (apply) (page 35/221)

Exponential Functions and Equations Unit Tasks

a. BetterLesson: Review for Unit (Lesson 13)
b. Monterey Institute: Project-Based Learning Task – Money in the Bank (Note: limit to exponential portions of project)
c. National Security Agency: Exponents and Exponential Functions
d. PBS Mathline: Rhinos and M&Ms

Note to Intermediate Algebra Teachers:

At the conclusion of the Intermediate Algebra course students are required to take the Algebra 1 End-Of-Course Examination. For more information regarding preparation for this Examination Program teachers are encouraged to visit SCDE End-Of-Course Examination Program (EOCEP).

South Carolina College- and Career-Ready Standards for Mathematics
High School Support Document
Algebra 2 Coversheet

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
Functions: Arithmetic/Geometric Sequences and Absolute Value, Step, and Piece-Wise Functions	Linear Equations/Inequalities and Systems of Equations/Inequalities	Polynomials	Quadratic Functions, Equations, and Inequalities	Radical and Simple Rational Functions and Equations	Exponential Functions and Equations
Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
A2.FBF.1a A2.FBF.2* A2.FBF.3* A2.FIF.3* A2.FIF.7* A2.FIF.9* A2.FLQE.2* A2.FLQE.5*	A2.ACE.1* A2.ACE.2* A2.ACE.3 A2.ACE.4*	A2.AAPR.1* A2.AAPR.3 A2.ASE.1* A2.ASE.2*	A2.ACE.1* A2.ACE.2* A2.ACE.3 A2.ACE.4* A2.AREI.4b* A2.AREI.7 A2.AREI.11* A2.ASE.3b* A2.FBF.1a* A2.FBF.1b* A2.FBF.3* A2.FIF.4* A2.FIF.5* A2.FIF.6* A2.FIF.8* A2.FIF.9* A2.NCNS.1* A2.NCNS.7*	A2.ACE.1* A2.ACE.4* A2.AREI.2* A2.AREI.11* A2.FBF.1a* A2.FBF.1b* A2.FBF.3* A2.FIF.4* A2.FIF.5* A2.FIF.6* A2.FIF.7* A2.FIF.8*	A2.ACE.1* A2.ACE.2* A2.ACE.4* A2.ASE.3c* A2.AREI.11* A2.FBF.1a* A2.FBF.1b* A2.FBF.2* A2.FBF.3* A2.FIF.3* A2.FIF.4* A2.FIF.5* A2.FIF.6* A2.FIF.8b* A2.FLQE.1b* A2.FLQE.2* A2.FLQE.5*

South Carolina College- and Career-Ready Standards for Mathematics
High School Support Document
Algebra 2 Coversheet

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Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

1. Make sense of problems and persevere in solving them. a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

Algebra 2 Unit 1 Title
Functions: Arithmetic/Geometric Sequences and Absolute Value, Step, and Piece-Wise Functions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- A2.FLQE.2* Create symbolic representations of linear ~~and exponential~~ functions, including arithmetic ~~s~~equences, given graphs, verbal descriptions, and tables.
    - Representation of exponential functions will be addressed in Unit 6.
    - Focus for arithmetic sequences in Algebra 2 is on the arithmetic sequences and its connection to linear functions.
    - Engage prior knowledge by beginning with linear functions to describe features and representations, making the connection to arithmetic sequences, then extending to geometric, piece-wise and step functions.
- A2.FIF.3* Define functions recursively and recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
    - Emphasize understanding of what a recursive function is.
- A2.FBF.2* Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, translate between the two forms.
    - Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.
    - Understand that linear functions are the explicit form of recursively-defined arithmetic sequences and that exponential functions are the explicit form of recursively-defined geometric sequence.
    - Emphasize understanding of why the recursive formula for an arithmetic sequence uses addition and the explicit form uses multiplication.
    - Emphasize understanding of why the recursive formula for a geometric sequence uses multiplication and why the explicit form uses exponential.
    - Distinguish between explicit and recursive formulas.
        - Recursive formulas exhibit how a sequence starts and how to use the previous value(s) to generate the next element.
        - Explicit formulas allow one to find any element of a sequence without knowing the previous element.

- A2.FBF.1*a Write a function that models a relationship between two quantities using both explicit expressions and a recursive process and by combining standard forms using addition, subtraction, multiplication and division to build new functions.
    - Provide multiple applied contexts and opportunities in which to explore these functions. Use real-world examples, so students can not only describe what they see in a table, equation, or graph, but also relate the key features to real-life meanings.
    - Distinguish between relationships that are functions and are not functions (first introduced in Grade 8) and examine graphs and tables of non-functions versus functions. Often students have the misconception that all relationships having an input and output are functions and therefore misuse the function terminology.
    - A2.FBF.1a is not a Graduation Standard.
- A2.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior ~~and periodicity.~~ Graph simple cases by hand and use technology for complicated cases.
    - Included functions are absolute value, arithmetic sequence, piece-wise, and step.
    - Graph functions, both with and without a calculator. Students could begin by examining graphs, describing the characteristics (intercepts, increasing/decreasing intervals, relative maxima, minima, symmetry, end behavior, and asymptotes) and then transition to using a given set of characteristics to sketch the graph of a function.
    - Understand that absolute value and step functions are part of the piece-wise family of functions
        - Absolute value and step functions have linear pieces
        - General piece-wise functions may have quadratic and exponential pieces.
    - Discussion of periodicity will take place in Pre-Calculus.
- A2.FBF.3* Describe the effect of the transformations (x), $(x) + k$, $f(x + k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.
    - Included functions are absolute value, arithmetic sequence, piece-wise, and step.
- A2.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal.
    - Included functions are absolute value, arithmetic sequence, piece-wise, and step.
- A2.FLQE.5* Interpret the parameters in a linear ~~or exponential~~ function in terms of the context.
    - Included functions are absolute value, arithmetic sequence, piece-wise, and step.
    - Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. For example, if the function $f(x)$ gives the number of person-hours it takes to assemble x engines in a factory, then the positive integers would be an appropriate domain for the function.

New Academic Vocabulary for This Unit
• Arithmetic Sequence • Composition of Functions • End Behavior • Explicit Formula • Geometric Sequence • Greatest Integer Function • Piece-Wise Defined Function • Recursive Formula • Sequence • Step Function

Prior Knowledge Required for this Unit
In earlier grades/courses, students have developed conceptual knowledge and have had the opportunity to learn how to: • Representing functions in multiple ways – mappings, tables, graphs, equations and verbal descriptions (8.F.1 – 5). • Create symbolic representations of linear functions (A1.FLQE.2). • Graphing functions and indicating key features with quadratic and linear only (A1.FIF.7). • Effects of transformation with linear, quadratic, and exponential with integer exponents (A1.FBF.3). • Comparing the properties of two functions given in different forms for linear, quadratic, and exponential with integer exponents (A1.FIF.9). • Interpret the parameters in a linear function in terms of context (A1.FLQE.5).

Subsequent Knowledge Related to this Unit

• Algebra 2 Unit 1 creates symbolic representations of functions given graphs, verbal descriptions, and tables (A2.FLQE.2*).
  ○ Will also be addressed in Algebra 2 Unit 6: Exponential Functions and Equations.
• Algebra 2 Unit 1 defines functions recursively and recognize that sequences are functions (A2.FIF.3*).
  ○ Will also be addressed in Algebra 2 Unit 6: Exponential Functions and Equations.
  ○ Geometric Sequences are included in Algebra 2 Unit 1 but can also be repeated in Algebra 2 Unit 6: Exponential Functions and Equations.
• Algebra 2 Unit 1 writes a function that models a relationship between two quantities (A2.FBF.1a; *note: IA.FBF.1a is not a Graduation Standard.).*
  ○ Will also be addressed in Algebra 2 Unit 4: Quadratic Functions and Equations, Algebra 2 Unit 5: Rational and Simple Rational Functions and Equations, and Algebra 2 Unit 6: Exponential Functions and Equations.
• Algebra 2 Unit 1 graphs functions from their symbolic representations (A2.FIF.7*).
  ○ Will also be addressed in Algebra 2 Unit 5: Rational and Simple Rational Functions and Equations.
• Algebra 2 Unit 1 describes the effect of the transformations on the graph of $y = (x)$ and writes the equation of a transformed parent function given its graph (A2.FBF.3*).
  ○ Will also be addressed in Algebra 2 Unit 4: Quadratic Functions and Equations, Algebra 2 Unit 5: Rational and Simple Rational Functions and Equations, and Algebra 2 Unit 6: Exponential Functions and Equations.
• Algebra 2 Unit 1 compares properties of two functions given in different representations (A2.FIF.9*).
  ○ Will also be addressed in Algebra 2 Unit 4: Quadratic Functions and Equations.
• Algebra 2 Unit 1 interprets the parameters in a linear function in terms of the context (A2.FLQE.5*).
  ○ Will also be addressed in Algebra 2 Unit 6: Exponential Functions and Equations.

Relationship Among Standards in this Unit
The standards in this unit include functions not specifically covered in the subsequent Algebra 2 units. Students are expected to write, graph, apply, and interpret these functions. The focus in Unit 1 is on the characteristics, similarities and differences in the following functions: linear, absolute value, arithmetic/geometric sequence, piece-wise, and step functions. The goal is for students to develop, understand, and make connections between a variety of function forms: equations, graphs, verbal descriptions, and tables. The terms “recursively” and “explicitly” are introduced for the first time, along with arithmetic/geometric sequences. Focus is placed on real world applications and contextual situations that students can relate to. Emphasis is also placed on the transformation of these functions in the coordinate plane.

Possible Instructional Strategies/Lessons
The order of the topics below illustrates a possible instructional order for Unit 1.

Arithmetic Sequences (A.2.FBF.1 and 2; A2.FIF.3)

a. Writing Arithmetic Sequences Recursively and Explicitly
   Algebra Lab: Arithmetic Sequences
   LearnZillion: Model Arithmetic Sequences And Situations By Using Both Recursive And Explicit Formulas
b. Writing Geometric Sequences Recursively and Explicitly
   Algebra Lab: Geometric Sequences
   Algebra Lab: Algebra 2 Recipe: Geometric Sequences
c. Graphing Calculator Exploration of Arithmetic Sequences
   TI Education: Arithmetic Sequences & Series

Geometric Sequences (A.2.FBF.1 and 2; A2.FIF3)

a. Writing Geometric Sequences Recursively And Explicitly
   Algebra Lab: Geometric Sequences
   Algebra Lab: Algebra 2 Recipe: Geometric Sequences

Piece-Wise Functions (A2.FIF.7*)

a. Connecting Piece-Wise, Absolute Value, and Step Functions
   Birdville Schools: Extension Activity For Piece-Wise Functions
   Math Is Fun: Piece-Wise
   MathBits Notebook: Piece-Wise
   Rowe Math Wiki: Piece-Wise Module

Functions – Linear, Absolute Value, Arithmetic Sequences, Piece-Wise, and Step (A.2.FBF.1a, 2, and 3; A.2.FBF.3; A2.FIF3)

a. Videos On Graphs And Their Transformations
   Virtual Nerd: Absolute Value and Piece-Wise Functions
b. Examples For Piece-Wise, Step, Greatest Integer, And Absolute Value
   Glencoe McGraw-Hill: Special Functions
c. Extension Activity For Piece-Wise Functions
   Birdville Schools: Analyzing Piece-Wise Functions
d. Key Features of Graphs
   MathBits Notebook: Function Features

Resources
Activity Resources a. Illustrative Mathematics: Activities For All Levels b. Robert Kaplinksy: Problem-based Learning Activities
Algebra 2 Course Content Resources from Other States a. EngageNY: Algebra 2 Resources b. Georgia Department of Education: Algebra 2 Resources c. Regents Prep: Algebra 2 and Trig d. Virginia Department of Education: Algebra 2 Resources
Graphing Calculator Resources a. Desmos: Online Graphing Calculator with Many Pre-Made Activities b. Texas Instruments: Texas Instruments Algebra 2 Graphing Calculator Activities c. Wabbit: Online TI-84 Silver Edition Graphing Calculator Emulator
Interactive Resources a. Emergent Math: Emergent Math b. ExploreLearning: Gizmo Online Simulations c. Interactive Quizzes: Interactive Quizzes for High School Assessments
Practice Tests and Assessment Resources a. California Department of Education: California Algebra Released Test Questions b. Jefferson Lab: Practice Tests from Virginia for All Levels of Math c. Problem-Attic: Problem-Attic d. XL Math: XL Math for Algebra 2
Video Resources a. Virtual Nerd: Videos for Algebra 2 b. LearnersTV: Videos for Algebra 2 c. HippoCampus: Videos for Algebra 2

Sample Formative Assessment Tasks/Questions
Functions – Linear, Absolute Value, Arithmetic/Geometric Sequences, Piece-Wise, and Step Functions (A2.FBF.1, 2, and 3; A2.FBF.3; A2.FIF.3) a. Describe key characteristics of the graph of $f(x) = \|x - 1\| + 3$ b. Sketch the graph and identify key characteristics of $f(x) = \begin{cases} x + 2, x \geq 0 \\ -x^2, x < -1 \end{cases}$ c. What is the difference between a recursive and an explicit representation of a sequence?
Transformations: Linear, Absolute Value, Arithmetic Sequences, Piece-Wise, Step (A2.FBF.3) a. She Loves Math: Parent Functions and Transformations b. On the axes, graph $f(x) = \|x\|$. If $g(x) = f(x) - 2$, then how is the graph of $f(x)$ translated to form the graph of $g(x)$? If $h(x) = f(x - 4)$, then how is the graph of $f(x)$ translated to form the graph of $h(x)$?
Piece-Wise Functions (A2.FIF.7) a. CPM Educational Program: Connecting Piece-wise Functions to Continuity Extension
Arithmetic/Geometric Sequences (A.2.FBF.1 and 2; A2.FIF.3) a. Algebra Lab: Algebra 2 Practice b. Mathematics Vision Project: Arithmetic and Geometric Sequences Module/Activity c. NRICH: Activity on Comparing Two Arithmetic Sequences d. Virginia Department of Education: Sequence Matching Activity with Information on Differentiation and Essential Questions
a. Inside Mathematics: Performance Assessment Tasks

Algebra 2 Unit 2 Title
Linear Equations/Inequalities and Systems of Equations/Inequalities

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

For all standards in this unit, Algebra 2 must extend the knowledge and applications of linear equations/inequalities/relationships that were begun in Algebra 1. It is important to apply and extend skills learned in Algebra 1 and not to simply reteach already covered in Algebra 1. Students should experience real-world applications of linear equations/inequalities/relationships in a much more complex contextual situation than they experienced in Algebra 1. Students are introduced to linear inequalities in Algebra 1, and in Algebra 2 the concept of linear inequalities is extended to include compound inequalities, absolute value inequalities, and systems of inequalities. Importance should be placed on consideration of the constraints on domain and range, particularly when applied to real-world contextual situations.

- A2.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, ~~quadratic, simple rational, and exponential~~ relationships. Interpret the solutions and determine whether they are reasonable.
    - Extend (not re-teach) what was learned in Algebra 1. Students learned to solve and graph linear equations and inequalities in Algebra 1. In Algebra 2, emphasis is placed on the application of these skills to real-world and contextual situation type scenarios.
    - Provide examples that are real world applications and more complex than those begun in Algebra 1.
    - Included in the study of Inequalities in Algebra 2 are compound inequalities and absolute value inequalities.
    - Emphasize the meaning of the variables in the situational and real-world applications.
- A2.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
    - Extend knowledge and applications of linear equations/relationships begun in Algebra 1.
    - Emphasize the importance of appropriate labels, units, and scales. Appropriate labels, units, and scales are essential for accurately modeling the relationship between the two quantities (mathematical process standard #6). Incorrect scales on the graph will distort the visual appearance of the relationship. Labels and units are important to accurately understand and interpret graphs.
    - Emphasize the meaning of the variables in the situational and real-world applications.

- A2.ACE.3 Use systems of equations and inequalities to represent constraints arising in real-world situations. Solve such systems using graphical and analytical methods, including linear programing. Interpret the solution within the context of the situation. (Limit to linear programming.)
    - Extend knowledge of graphing inequalities learned in Algebra 1 to graphing systems of inequalities.
    - Included in this unit are linear systems of equations and inequalities. Unit 4 (Quadratic Functions/Equations/Inequalities) will address systems that include quadratic equations and inequalities.
    - Include solving and analytical method applications to linear programming.
    - Emphasize the meaning of the variables in the situational and real-world applications.
    - Ensure understanding of the difference in meaning and appearance of the graph for an inequality or system of inequalities that include $\geq, \leq, >, <$, and $\neq$.
- A2.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
    - Extend knowledge and applications of linear equations/relationships begun in Algebra 1.
    - Provide applications that allow students to see the usefulness/purpose of rewriting a formula by solving for one of the variables in the formula.

New Academic Vocabulary for This Unit
• Break-Even Point • Constraints • Feasible Region • Linear Programming • Optimization • Systems of Inequalities • Unbounded System

Prior Knowledge Required for this Unit
Working with expressions and equations, including formulas, is an integral part of the curriculum in Grades 7 and 8. In high school, students explore in more depth the use and application of equations and inequalities to model real-world problems, including restricting domains and ranges to fit the problem's context, as well as rewriting formulas for a variable of interest. In Algebra 1, the primary focus is on graphing and solving linear equations and inequalities. Algebra 1 also includes systems of equations, but does not include systems of inequalities. In Algebra 2, these skills are extended to more complex situations and modeling of real-world applications to include linear programming (not to be re-taught). Below are the linear equations and inequalities standards students had the opportunity to learn in Algebra 1: • Create and solve equations and inequalities in one variable (A1.ACE.1*). • Create and graph equations in two or more variables (A1.ACE.2*). • Solve literal equations and formulas for specified variable (A1.ACE.4*). • Solve linear equations and inequalities with coefficients represented by letters (A1.AREI.3*). • Justify the solution to a system of linear equations (A1.AREI.5). • Solve a system of linear equations graphically and algebraically (A1.AREI.6*). • Graph the solutions of a linear inequality in two variables (A1.AREI.12*).

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Subsequent Knowledge Related to this Unit

• A2.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, ~~quadratic, simple rational, and exponential relationships~~. Interpret the solutions and determine whether they are reasonable.
  ○ Will also be addressed in Algebra 2 Unit 5: Radical and Simple Rational Functions and Equations and Algebra 2 Unit 6: Exponential Functions and Equations.
• A2.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
  ○ Will also be addressed in Algebra 2 Unit 4: Quadratic Functions and Equations) and Algebra 2 Unit 6: Exponential Functions and Equations.
• A2.ACE.3 Use systems of equations and inequalities to represent constraints arising in real-world situations. Solve such systems using graphical and analytical methods, including linear programing. Interpret the solution within the context of the situation. (Limit to linear programming.)
  ○ Will also be addressed in Algebra 2 Unit 4: Quadratic Functions and Equations.
• A2.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
  ○ Will also be addressed in Algebra 2 Unit 4: Quadratic Functions and Equations, Algebra 2 Unit 5: Rational and Simple Rational Functions and Equations, and Algebra 2 Unit 6: Exponential Functions and Equations.

Relationship Among Standards in this Unit
All standards in this unit address linear equations/inequalities and systems of linear equations/inequalities. The focus is on creating and solving these systems with an emphasis on linear programming and real-world applications. It is important to note that this unit also includes compound inequalities and absolute value inequalities.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 2.*
Linear Equations (A2.ACE.1*; A2.ACE.2*) a. Activities for Creating an Equation Given a Situation Illustrative Mathematics: Planes and Wheat Illustrative Mathematics: Paying the Rent Illustrative Mathematics: Buying a Car Illustrative Mathematics: Clea on an Escalator
Compound/Absolute Value Inequalities (A2.ACE.1*) a. CK – 12: Overview of Solving Compound Inequalities b. Great Valley School District: Absolute Value Inequality Word Problems for Assessment c. Monterey Institute: Explanation of Compound Inequalities as a Union or Intersection of Inequalities d. Purple Math: Overview Of Solving Absolute Value Inequalities
Systems of Equations and Inequalities and Linear Programming (A2.ACE.3) a. Algebra-Class: Systems of Inequalities Practice Problems b. Henrico County Public Schools: Linear Programming PowerPoint from Henrico/Virginia c. Illuminations: Using TI-83/84 to Develop Understanding of Linear Programming: Dirt Bike Dilemma
Literal Equations and Formulas (A2.ACE.4*) a. Illustrative Mathematics: Equations and Formulas b. *When teaching solving literal equations, educators should make connections to relevant science, business, and CATE applications (STEM). Collaboration with teachers within such disciplines is encouraged.*

Resources
Algebra 2 Course Content Resources from Other States a. Henrico County Public Schools: Algebra 2 Resources b. EngageNY: Algebra 2 Resources c. Georgia Department of Education: 9 – 12 Resources d. Regents Prep: Algebra 2 and Trig

Activity Resources a. Illustrative Mathematics: Activities For All Levels b. Robert Kaplinksy: Problem-Based Learning Activities
Graphing Calculator Resources a. Desmos: Online Graphing Calculator with Many Pre-Made Activities b. Texas Instruments: Texas Instruments Algebra 2 Graphing Calculator Activities c. Wabbit: Online TI-84 Silver Edition Graphing Calculator Emulator
Interactive Resources a. Emergent Math: Emergent Math b. Explore Learning: Gizmo Online Simulations c. Interactive Quizzes: Interactive Quizzes for High School Assessments
Practice Tests and Assessment Resources a. California Department of Education: California Algebra Released Test Questions b. Practice Tests from Virginia for All Levels of Math - http://education.jlab.org/solquiz/ c. Problem-Attic: Sample Problems d. XL Math for Algebra 2 - XL Math for Algebra 2
Video Resources a. HippoCampus: Videos for Algebra 2 b. LearnersTV: Videos for Algebra 2 c. Virtual Nerd: Videos for Algebra 2

Sample Formative Assessment Tasks/Questions
Linear Equations, Inequalities, and Systems of Equations and Inequalities (A2.ACE.1*; A2.ACE.2*; A2.ACE.3; A2.ACE.4*) a. A local snack company makes circular cakes. The average circumference of the snack cakes is supposed to be 18.85 inches. When reviewing a recent batch of the snack cakes, the quality control manager thinks the cakes are not measuring the correct circumferences. To pass inspection, the circumferences of the cakes need to be within 0.5 inches of the average. i. Write an inequality to represent the situation. ($\lvert x-18.85\rvert<0.5$ ii. Solve the inequality to find the appropriate range of circumferences for the snack cakes. ($-0.5<x-18.85<0.5$ so $18.35\ inches<x<19.35\ inches$) b. Give students geometric, science, or business formulas and have them solve the equation for each of the different variables in the formula. For example: Solve the formula $C=\frac{5}{9}(F-32)$ that relates degrees Fahrenheit to degrees Celsius for F. Then then convert -5 degrees Celsius to Fahrenheit. c. Which equation states that the temperature, t, in a room is less than 30 from 680? i. $\lvert 3-t\rvert<68$ ii. $\lvert 3+t\rvert<68$ iii. $\lvert 68-t\rvert<3$ (correct answer) iv. $\lvert 68+t\rvert<3$ d. CK – 12: Multiple Choice Problems for Absolute Value Inequalities e. Illuminations: Dirt Bike Dilemma (*Addresses the development and assesses the conceptual understanding of linear programming; can be used for a resource for teaching the lesson or a formative assessment.*) f. Illustrative Mathematics: Activities For Writing A System Of Inequalities With Constraints g. Illustrative Mathematics: How Much Folate?
a. Inside Mathematics: Performance Assessment Tasks

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Algebra 2 Unit 3 Title
Polynomials

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
● A2.AAPR.1* Add, subtract, and multiply polynomials and understand that polynomials are closed under these operations. ○ Beyond quadratic polynomial expressions is the expectation for Algebra 2. ○ Emphasize that the polynomials are closed under these 3 operations which means when adding (subtracting or multiplying) two polynomials the result is a polynomial ○ Emphasize the distributive property and not mnemonic devices. ○ In Algebra 2, the polynomials should have rational and integer coefficients. ● A2.AAPR.3 Graph polynomials identifying zeros when suitable factorizations are available and indicating end behavior. Write a polynomial function of least degree corresponding to a given graph. (Limit to polynomials with degrees 3 or less.) ○ In Algebra 2, the polynomials should be 3rd degree and lower. ○ Support student understanding of the terms: roots, zeros, x-intercepts, and solutions of equations; essentially they all mean the same thing from a different perspective. ● A2.ASE.1* Interpret the meanings of coefficients, factors, terms, and expressions based on their real-world contexts. Interpret complicated expressions as being composed of simpler expressions. ○ In Algebra 1, students work with linear, exponential, and quadratic expressions. ○ In Algebra 2, students extend these concepts to general polynomials (Unit 3) and rational expressions (Unit 5). ● A2.ASE.2* Analyze the structure of binomials, trinomials, and other polynomials in order to rewrite equivalent expressions. ○ Linear, quadratic, and exponential expressions are the focus in Algebra 1, and integer exponents are extended to rational exponents in Algebra 2 (only those with square or cube roots). ○ In Algebra 2, the expectation is to extend to rational expressions (Unit 5).

New Academic Vocabulary for This Unit
• Closed

South Carolina College- and Career-Ready Standards for Mathematics
High School Support Document
Algebra 2 Unit 3: Polynomials

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Prior Knowledge Required for this Unit
The conceptual understanding of adding, subtracting, multiplying and dividing integers is developed beginning in Grade 7. Students should make connections between the arithmetic of integers and the arithmetic of polynomials. Understanding and applying the laws of exponents are essential for success with this concept. Students have been introduced to key features (intercepts and end behavior) of graphs in previous courses. Below are the standards covered in Algebra 1 in regards to polynomials: ● Add, subtract and multiply polynomials (A1.AAPR.1*) ● Interpret the meanings of parts of expressions based on real-world contexts (A1.ASE.1*) ● Analyze the structure of polynomials to rewrite equivalent expressions (A1.ASE.2*) ● Explain the properties of operations of rational and irrational numbers (A1.NRNS.3) ● Interpret the key features of functions and sketch the graph of a function from a verbal description (A1.FIF.4*) ● Graph functions and indicate key features (A1.FIF.7*)

Subsequent Knowledge Related to this Unit
The standards in this unit are not addressed in subsequent units.

Relationship Among Standards in this Unit
The standards in this unit address polynomial arithmetic and graphing polynomials. The unit investigations and activities should include polynomials of degree 3 or lower. The analysis of the polynomials to identify zeros and the algebraic form should assist with graphing the polynomials.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 3.*
Graphing Polynomials (A2.AAPR.3) The students should investigate the graphs of several polynomials and derive the following conclusions. (The teacher should not tell the students.) ● An *n*th degree polynomial has at most *n* roots and at most $n - 1$ "changes of direction" (i.e., from increasing to decreasing or vice versa). ● An even-degree polynomial has the same end-behavior in both the positive and negative directions: both heading to positive infinity, or both heading to negative infinity, depending upon the sign of the leading coefficient. ● An odd-degree polynomial has opposite end-behavior in the positive versus the negative directions, depending upon the sign of the leading coefficient. ● An odd-degree polynomial function must have at least one real root. ● Khan Academy: Using Zeros to Graph Polynomials ● Geogebra: Polynomial Parameters ● Desmos: Calculator
Polynomial Operations (A2.AAPR.1) ● Geogebra: Area Model ● Better Lesson: Adding and Subtracting Polynomials ● EngageNY: Polynomial Operations
Interpreting the Polynomial Expression and Its Parts (A2.ASE.1*, A2.ASE.2) ● Have students create their own expressions that meet specific criteria and verbalize how the expressions can be rewritten in different forms ● Help students establish a visual understanding of algebraic expressions and the meaning of terms, factors, and coefficients using algebra tiles ● Have students graph different algebraic expressions on the same coordinate plane using technology to verify the expressions are equivalent or they can review the values in an input output table ● Help students understand and interpret the meaning of coefficients, factors and terms by asking students to write a story context to support the algebraic expression ● Virtual Nerd: What is Quadratic Form of a Polynomial Equation?

Resources
Algebra 2 Course Content Resources from Other States a. Georgia Algebra 2 Resources: Georgia Algebra 2 Resources b. New York Algebra 2 Resources: New York Algebra 2 Resources c. Regents Resources from Oswego, New York: Regents Resources from Oswego, New York d. Virginia Algebra 2 Resources: Virginia Algebra 2 Resources
Activity Resources a. Illustrative Mathematics: Activities for all Levels b. Robert Kaplinsky: Problem-Based Learning Activities
Interactive Resources a. Emergent Math: Emergent Math b. Explore Learning: Gizmo Online Simulations c. Interactive Quizzes: Interactive Quizzes for High School Assessments
Practice Tests and Assessment Resources a. California Algebra Released Test Questions: California Algebra Released Test Questions b. Jefferson Lab: Practice Tests from Virginia for All Levels of Mathematics c. Problem Attic: Sample Problems d. XL Math for Algebra 2: XL Math for Algebra 2
Video Resources **a.** HippoCampus: Videos for Algebra & Geometry b. Learners TV: Videos for Algebra 2 **c.** Virtual Nerd: Videos for Algebra 2
Graphing Calculator Resources a. Desmos: Online Graphing Calculator with Many Pre-Made Activities b. Texas Instruments: Algebra 2 Graphing Calculator Activities c. Wabbit: Online TI-84 Silver Edition Graphing Calculator Emulator

Sample Formative Assessment Tasks/Questions
Polynomial Operations (A2.AAPR.1) a. Braingenie: Add, Subtract, and Multiply Polynomials b. Mathematics Assessment Project – Mathematics Assessment Resource Service: Generating Polynomials from Patterns c. Regents Prep Center: Regents Prep Center
Graphing Polynomials (A2.AAPR.3) a. IXL: Match Polynomials and Graphs b. Mathematics Assessment Project – Mathematics Assessment Resource Service: Representing Polynomials Graphically
Interpreting the Polynomial Expression and Its Parts (A2.ASE.1*/ASE.2) a. Georgia Department of Education: Exploration Task (page 54)

Algebra 2 Unit 4 Title
Quadratic Functions, Equations, & Inequalities

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

For all standards in this unit, keep in mind that in prior grades, quadratic expressions, equations, and inequalities have been an integral part of the curriculum. In Algebra 2, teachers should aid students in accessing this prior knowledge and extending it to applications. Students need to explore in greater detail and depth the use of quadratic equations and inequalities in modeling and investigating real world situations. Special attention should be placed on the domain and range and when and if the domain and range need to be restricted to fit the situation. These situations should include analysis of algebraic, tabular, and graphical representations and their solutions. A new concept to Algebra 2 is the introduction of complex solutions to quadratic functions in the form $a + bi$.

- A2.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving ~~linear~~, quadratic, ~~simple rational, and exponential~~ relationships. Interpret the solutions and determine whether they are reasonable.
  - Emphasis on modeling and solving real-world problems. Solving algebraically and graphically was taught in Algebra 1.
- A2.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
  - Limit to quadratic equations in this unit.
  - Emphasize and discuss the importance of appropriate labels and scales on axes of graphs.
  - Attention should be given to the importance of whether the boundary line for inequalities is included or not included in the solution set.
- A2.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
  - Limit to quadratic equations in this unit.
  - Remind students that when taking the square root of a variable squared, there will be a positive and negative solution.
- A2.AREI.4*b Solve mathematical and real-world problems involving quadratic equations in one variable.
  b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b. (Note: A2.AREI.4b is not a Graduation Standard.)
  - Compare and contrast the different approaches (listed above) to solving the same quadratic problem.
  - Provide students with the experience of analyzing the given problem to choose an appropriate method to solve.
  - Emphasis should be placed on helping students to understand and solve complex solutions, as this is the first time students are exposed to this concept. Have students observe that if a quadratic equation has complex solutions, the solutions always appear in conjugate pairs.

- A2.AREI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Understand that such systems may have zero, one, two, or infinitely many solutions. (Limit to linear equations and quadratic functions.)
  - Encourage conceptual and visual understanding of the solution/s to a system by beginning with solving graphically a system that consists of a linear and quadratic equation. Then solve the same system algebraically, showing it can be done both ways. This can also lead to a discussion as to why there are times when graphing is not as efficient as solving algebraically due to fractional or decimal answers.
  - Proceed by moving to a system that consists of two quadratic equations, solving first graphically and then algebraically.
- A2.AREI.11* Solve an equation of the form $f(x) = g(x)$ graphically by identifying the x-coordinate(s) of the point(s) of intersection of the graphs of $y = f(x)$ and $y = g(x)$.
  - Explain that when we set two quadratic equations (or one linear equation and one quadratic equation) equal and solve, it is essentially the same thing as solving a system of equations. This standard deals primarily with graphing but can easily be extended to an algebraic method.
- A2.ASE.3b* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (Note: A2.ASE.3b and 3c are not Graduation Standards.) b. Determine the maximum or minimum value of a quadratic function by completing the square.
  - Develop understanding that by completing the square, the coordinates (h, k) in the general form $y = a(x-h)^2 + k$ represents the vertex of the parabola; that h is the horizontal shift and y is the vertical shift.
  - Understand the connection between the vertex and the minimum/maximum point. Students should understand that when $a > 0$, the vertex is a minimum point and when $a < 0$, the vertex is a maximum point. Further emphasize the meaning of a minimum/maximum in real-world contextual situations.
- A2.FBF.1b* Write a function that describes a relationship between two quantities. (Note: IA.FBF.1a is not a Graduation Standard.) b. Combine functions using the operations addition, subtraction, multiplication, and division to build new functions that describe the relationship between two quantities in mathematical and real-world situations.
- A2.FBF.3* Describe the effect of the transformations $kf(x)$, $f(x) + k$, $f(x + k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.
  - Limit to quadratic equations in this unit.
  - Distinguish between what causes a vertical shifts versus a horizontal shift. Vertical shifts have been discussed in Algebra 1.
  - Extend the idea of transformations of a quadratic equation to include what causes the parabola to be narrower or wider.
  - Recognition of even and odd functions should be included in this unit.

- A2.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior ~~and periodicity~~.
    - Limit to quadratic functions in this unit.
    - Begin by identifying the key features of a given graph, then transitioning to using a set of characteristics to sketch the graph of a quadratic function.
- A2.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
    - Include the restrictions to the domain and range that are created by modeling real-world situations.
- A2.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.
    - Emphasis should be placed on interpreting what the average rate of change means in the given situation.
    - Extensions can be made to the discussion of the average rate of change between two points on the function that form a secant line and what it means as the distance between two points on the graph decreases.
- A2.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function.
    - Analyze, compare, and contrast quadratic functions in the form $y = ax^2 + bx + c$, $y = a(x-h)^2 + k$, and the factored form of a quadratic function to develop understanding of making the best selection of a particular form for different contextual situations.
    - Understand that $y = ax^2 + bx + c$ can often be factored to easily find the zeroes of the function, while the form $y = a(x-h)^2 + k$ allows for easy identification of the vertex of the function.
- A2.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal.
    - Compare two quadratic functions represented in different ways. For example, given a graph of one quadratic function and the algebraic expression for another, compare the maximums or minimums of the two quadratic functions.
- A2.NCNS.1* Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
    - Introduction of the complex number i begins in Algebra 2. The use of complex numbers is essential to mathematics and science.
- A2.NCNS.7* Solve quadratic equations in one variable that have complex solutions.
    - Knowledge of complex numbers makes every quadratic equation solvable. Prior to this, students have not had the ability to solve quadratic equations with a negative discriminant.
    - Emphasize understanding of what creates a complex solution and that complex solutions come in conjugate pairs.

New Academic Vocabulary for This Unit
• Complex Number • Complex Roots • Complex Solution • End Behavior • Even/Odd Functions • Imaginary Number (i)

Prior Knowledge Required for this Unit
Working with quadratic functions, equations, and inequalities is an integral part of the curriculum in Algebra 1. Standards that are in Algebra 1 and then *extended* in Algebra 2 are as follows: ● Create and solve quadratic equations and inequalities in one variable (A2.ACE.1*). ● Create and graph quadratic equations in two or more variables (A1.ACE.2*). ● Solve mathematical and real-world problems involving quadratic equations in one variable (A1.AREI.4*). ● Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring (A1.AREI.4b*). ● Solve an equation in the form of $f(x) = g(x)$ graphically (A2.AREI.11*). ● Describe the effect of transformations and combinations of transformations on a quadratic equation (A1.FBF.3*). ● Interpret key features including intercepts, increasing/decreasing intervals, etc. (A2.FIF.4*). ● Relating the domain and range of a function to its graph (A2.FIF.5*). ● Determining average rate of change of a function over a specified interval (A2. FIF.6). ● Translate between different but equivalent forms of an equation to reveal and explain different properties of the function(A.FIF.8). ● Compare properties of two functions in different representations such as algebraic, graphical, tabular, and verbal (A.FIF.9*). In Algebra 2, these skills are not to be retaught, but extended (by further development of prior knowledge) to more complex situations and modeling of real-world applications. This will also include solving a simple system consisting of a linear equation and a quadratic equation. Students will now explore in more depth the use and application of quadratic equations and inequalities to model real-world problems, including restricting domains and ranges to fit the problem's context. Algebra 1 curriculum includes solving quadratics by inspection, taking the square roots, completing the square, the quadratic formula, and factoring, but all answers are limited to non-complex roots. This is extended in Algebra 2 to include complex roots. Completing the square introduced in Algebra 1 is to be extended in Algebra 2 by using the form $y = a(x - h)^2 + k$ to determine the maximum or minimum value of a quadratic function.

Subsequent Knowledge Related to this Unit

- A2.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable.
    - Will also be addressed in Algebra 2 Unit 2 (Linear Equations & Inequalities), Algebra 2 Unit 5 (Radical & Simple Rational Functions & Equations), and Algebra 2 Unit 6 (Exponential Functions & Equations)
- A2.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
    - Will also be addressed in Algebra 2 Unit 2 (Linear Equations & Inequalities) and Algebra 2 Unit 6 (Exponential Functions & Equations)
- A2.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
    - Will also be addressed in Algebra 2 Unit 2 (Linear Equations & Inequalities), Algebra 2 Unit 5 (Radical & Simple Rational Functions & Equations), and Algebra 2 Unit 6 (Exponential Functions & Equations)
- A2.AREI.11* Solve an equation of the form $f(x) = g(x)$ graphically by identifying the x-coordinate(s) of the point(s) of intersection of the graphs of $y = f(x)$ and $y = g(x)$.
    - Will also be addressed in Algebra 2 Unit 5 (Radical & Simple Rational Functions & Equations) and Algebra 2 Unit 6 (Exponential Functions & Equations)
- A2.FBF.1b* Write a function that describes a relationship between two quantities. (Note: IA.FBF.1a is not a Graduation Standard.)
  b. Combine functions using the operations addition, subtraction, multiplication, and division to build new functions that describe the relationship between two quantities in mathematical and real-world situations.
    - Will also be addressed in Algebra 2 Unit 1 (Functions), Algebra 2 Unit 5 (Radical & Simple Rational Functions & Equations), and Algebra 2 Unit 6 (Exponential Functions & Equations)
- A2.FBF.3* Describe the effect of the transformations (x), $(x) + k$, $f(x + k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.
    - Will also be addressed in Algebra 2 Unit 1 (Functions), Algebra 2 Unit 5 (Radical & Simple Rational Functions & Equations), and Algebra 2 Unit 6 (Exponential Functions & Equations)
- A2.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior ~~and periodicity~~.
    - Will also be addressed in Algebra 2 Unit 5 (Radical & Simple Rational Functions & Equations) and Algebra 2 Unit 6 (Exponential Functions & Equations)

- A2.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
    - Will also be addressed in Algebra 2 Unit 5 (Radical & Simple Rational Functions & Equations) and Algebra 2 Unit 6 (Exponential Functions & Equations)
- A2.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified interval. Interpret the meaning of the average rate of change in a given context.
    - Will also be addressed in Algebra 2 Unit 5 (Radical & Simple Rational Functions & Equations) and Algebra 2 Unit 6 (Exponential Functions & Equations)
- A2.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function.
    - Will also be addressed in Algebra 2 Unit 5 (Radical & Simple Rational Functions & Equations) and Algebra 2 Unit 6 (Exponential Functions & Equations)

Relationship Among Standards in this Unit
All standards in this unit address quadratic equation and inequalities, both real and complex solutions. The overarching relationship among the standards in this unit is: • All quadratic equations/inequalities have a solution as we now include complex solutions. • The method for solving quadratic equations/inequalities varies based on which method is most appropriate. • Making connections between the graphical, algebraic (including standard and vertex form), and tabular form of a quadratic equation/inequality are essential. • Learning to use and interpret key features of the graph of a quadratic equation/inequalities allows students to explore and analyze the meaning of these key features. • Applications through modeling of real world situations should take into consideration any restriction/s to the domain or range.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 4.*
Quadratic Equation (Real Solutions) Investigations/Activities/Lessons a. Better Lesson: Connecting Real World Phenomena And The Average Rate Of Change Of A Quadratic Function (A2.FIF.6*) b. Better Lesson: Analyzing Quadratic Equations And Their Graphs In Varied Forms (A2.ASE.3*, A2.FIF.8*, A2.FIF.9*) c. Better Lesson: Using A Real World Context To Analyze A Quadratic Function To Maximize Profit (A2.ASE.3*, A2.AREI.4*, A2.FIF.5/6/7/8/9*, A2.FBF.1*) d. Better Lesson: Reviewing Quadratic Functions (A2.AREI.4*) e. Better Lesson: Modeling Quadratics - Projectile Motion, Revenue Maximization, Area, And Number Puzzles (A2.ACE.1*) f. Better Lesson: Graphing Using Various Algebraic Forms Of Quadratic Equations And Analyzation Of Key Features Of The Graph (A2.ASE.3*, A2.FIF.4/7*) g. Better Lesson: Translate Between Verbal, Algebraic, Numeric, And Graphical Representations Of Quadratic Equations (A2.AREI.4*, A2.ASE.3*)
Quadratic Inequalities Investigations/Activities/Lessons a. Better Lesson: Solving A System Of Quadratic And Linear Equations Graphically (A2.AREI.7*)
Complex Numbers Investigations/Activities/Lessons a. Better Lesson: Introducing Complex Numbers
Quadratic Equations (Complex Solutions) Investigations/Activities/Lessons a. Better Lesson: Solve Quadratics That Have Complex Solutions, Graph The Solutions, And Recognize Complex Conjugates (A2.NCNS.7) b. Better Lesson: Complex Solutions To Quadratic Functions (A2.NCNS.7*) c. Better Lesson: Understanding the Fundamental Theorem of Algebra (*understand that the degree of the polynomials tells us the number of solutions to expect if we include both real and imaginary solutions*) (A2.NCNS.7*) d. Better Lesson: Imaginary Answers To Real World Problems (A2.NCNS.7*) e. Better Lesson: Compare And Contrast The Effectiveness Of Multiple Methods Of Solving Quadratics Which Include Complex Solutions (A2.NCNS.7*, A2.FIF.8*)

Resources
Algebra 2 Course Content Resources from Other States a. New York Algebra 2 Resources: New York Algebra 2 Resources b. Georgia Algebra 2 Resources: Georgia Algebra 2 Resources c. Regents Resources from Oswego, New York: Regents Resources from Oswego, New York d. Virginia Algebra 2 Resources: Virginia Algebra 2 Resources
Activity Resources a. Illustrative Mathematics: Activities for all Levels b. Robert Kaplinsky: Problem-Based Learning Activities
Interactive Resources a. Emergent Math: Emergent Math b. Explore Learning: Gizmo Online Simulations c. Interactive Quizzes: Interactive Quizzes for High School Assessments
Practice Tests and Assessment Resources a. California Algebra Released Test Questions: California Algebra Released Test Questions b. Jefferson Lab: Practice Tests from Virginia for All Levels of Mathematics c. Problem Attic: Sample Problems d. XL Math for Algebra 2: XL Math for Algebra 2
Video Resources a. HippoCampus: Videos for Algebra & Geometry b. Learners TV: Videos for Algebra 2 c. Virtual Nerd: Videos for Algebra 2
Graphing Calculator Resources a. Desmos: Online Graphing Calculator with Many Pre-Made Activities b. Texas Instruments: Algebra 2 Graphing Calculator Activities c. Wabbit: Online TI-84 Silver Edition Graphing Calculator Emulator

Sample Formative Assessment Tasks/Questions
a. Complex Numbers Formative Assessment: Complex Number Maze

Algebra 2 Unit 5 Title
Radical and Rational Functions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- A2.ACE.1* Create and solve equations and inequalities in one variable that model real-world problems involving ~~linear, quadratic~~, simple rational, ~~and exponential~~ relationships. Interpret the solutions and determine whether they are reasonable.
    - Create, solve, and evaluate rational equations with monomial and polynomial denominators.
    - Interpret the solutions, noting if any are extraneous.
- A2.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
    - Use equations and formulas from a variety of subjects and disciplines.
- A2.AREI.2* Solve simple rational and radical equations in one variable and understand how extraneous solutions may arise.
    - Understand that not all solutions generated algebraically are actually solutions to the original equations, therefore extraneous solutions must be explored.
- A2.AREI.11* Solve an equation of the form f(x) = g(x) graphically by identifying the x- coordinate(s) of the point(s) of intersection of the graphs of y = f(x) and y = g(x).
    - Extend this understanding as the standard was taught in Algebra 1 and previous units.
    - Explain that when we set two equations equal and solve, it is essentially the same thing as solving a system of equations. This standard deals primarily with graphing but can easily be extended to an algebraic method.
    - Limit to rational and radical functions in this unit.
- A2.FBF.1a* *(Note: IA.FBF.1a is not a Graduation Standard.)* Write a function that models a relationship between two quantities using both explicit expressions and ~~a recursive process~~ and by combining standard forms using addition, subtraction, multiplication and division to build new functions.
    - Understand that rational functions are the result of the division of two polynomial functions.
- A2.FBF.1b* Combine functions using the operations addition, subtraction, multiplication, and division to build new functions that describe the relationship between two quantities in mathematical and real-world situations.
    - Understand that many rational functions are the result of the combination of other functions. For example, f(x) = 1, g(x) = x, and h(x) = x+2. m(x) = f(x)/[g(x)*h(x)] = 1/[x(x+2)] = 1/x^2+2x
- A2.FBF.3* Describe the effect of the transformations k*f*(x), f(x) + k, f(x + k), and combinations of such transformations on the graph of y = f(x) for any real number *k*. Find the value of *k* given the graphs and write the equation of a transformed parent function given its graph.
    - Understand that many rational functions are the result of a transformation of f(x) = 1/x. For example, g(x) = f(x+a) = 1/(x+a).
    - Understand that many radical functions are the result of a transformation of f(x) = $\sqrt{x}$. For example, g(x) = f(x-2) = $\sqrt{x-2}$

- A2.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity.
  - A key feature of rational functions is the possibility of asymptotes and points of discontinuity.
- A2.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - Emphasize that the domain of a radical function is restricted due to the requirement that the radicand must be greater than or equal to zero.
- A2.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases.
  - Use the key features to graph the rational and radical functions.
  - Emphasize asymptotes for rational functions and restricted domains for radical functions.
- A2.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function.
  - Simplifying (by factoring or other strategies) the rational equations will help explain the discontinuity, asymptotes, and other features of the graph of the function.
  - Include rationalizing any radical monomials by using the exponent properties or binomials that include radicals by multiplying by the conjugate.

New Academic Vocabulary for This Unit
• Asymptotes (vertical and horizontal) • Conjugate • Extraneous solution • Point of discontinuity (removable and non-removable) • Rational expression

Prior Knowledge Required for this Unit
● Notice the arithmetic operations of rational expressions is governed by the same rules as the arithmetic operations of rational numbers (7.NS.3) ● Recall and make use of their knowledge of polynomial functions as well as compositions of functions to investigate the characteristics of rational functions (AAPR.1 and AAPR.3)

Subsequent Knowledge Related to this Unit
● A2.ACE.1 Unit 5 addresses solving radical and rational equations. ○ Will also be addressed in Algebra 2 Unit 6 (Exponential Functions & Equations) ● A2.ACE.4 Unit 5 addresses solving radical and rational literal equations and formulas. ○ Will also be addressed in Algebra 2 Unit 6 (Exponential Functions & Equations) ● A2.AREI.11 Unit 5 addresses solving the radical and/or rational equations graphically. ○ Will also be addressed in Algebra 2 Unit 6 (Exponential Functions & Equations) ● A2.FBF.1 Unit 5 writes a function that models a relationship between two quantities. ○ Will also be addressed in Algebra 2 Unit 6 (Exponential Functions & Equations) ● A2.FBF.3* Unit 5 describes the effect of the transformations on the graphs of radical and rational functions and writes the equation of a transformed parent function given its graph. ○ Will also be addressed in Algebra 2 Unit 6 (Exponential Functions & Equations) ● A2.FIF.4 Unit 5 interprets the key features of radical and rational functions. ○ Will also be addressed in Algebra 2 Unit 6 (Exponential Functions & Equations) ● A2.FIF.5 Unit 5 relates the domain and range of radical and rational functions to the relationships described. ○ Will also be addressed in Algebra 2 Unit 6 (Exponential Functions & Equations) ● A2.FIF.8 Unit 5 translates between equivalent forms of rational functions. ○ Will also be addressed in Algebra 2 Unit 6 (Exponential Functions & Equations)

Relationship Among Standards in this Unit
The standards in this unit include functions not specifically covered in previous math courses. Students are expected to write, graph, apply, and interpret these functions. The focus in Unit 5 is on the key features (asymptotes and points of discontinuity) of rational and radical functions. The goal is for students to develop, understand, and make connections between a variety of function forms: equations, graphs, verbal descriptions, and tables. Emphasis is also placed on the transformation of these functions in the coordinate plane. The terms extraneous solutions, asymptotes, and points of discontinuity are introduced for the first time.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 5.*
In previous grades, students should have noted rational numbers extend the arithmetic of integers by allowing division by all numbers except zero. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. Sign charts are recommended when evaluating rational functions to show the change in directions of the graphs of rational functions. For both rational and radical functions, it is essential for students to see and analyze many graphs of both types of functions. During the analysis of the graphs of the functions, the students should look for patterns and notice what features remain the same and what changes to the graphs occur. After observing several graphs, students should understand that the inverses of the power functions $y = x^n$ (with domains restricted as needed) form parent functions $y = \sqrt[n]{x}$ for families of radical functions. ● Illustrative Mathematics: Graphing Rational Functions (A2.FIF.7*) ● West Contra Costa Unified School District: Key Features and Vocabulary for Rational Functions (A2.FIF.4*) ● West Contra Costa Unified School District: Solving Radical Equations (A2.AREI.2*) ● Brain Genie: Rational Exponents and Radical Functions (A2.AREI.2*) ● Brain Genie: Solve Rational Equations (A2.AREI.2*) ● Illustrative Mathematics: Who Wins the Race (A2.AREI.2*, A2.ACE.1*, A2.AREI.11*) ● Geogebra: Graphs of Radical Functions (A2.FBF.3*) ● Geogebra: Characteristics of Radical Functions(A2.FBF.3*) ● Brain Genie: Graph Simple Rational (A2.FBF.3*)

Resources
Algebra 2 Course Content Resources from Other States a. Henrico County Public Schools: Algebra 2 Resources b. EngageNY: Algebra 2 Resources c. Georgia Department of Education: 9 – 12 Resources d. Regents Prep: Algebra 2 and Trig
Activity Resources a. Illustrative Mathematics: Activities For All Levels b. Robert Kaplinksy: Problem-Based Learning Activities
Graphing Calculator Resources a. Desmos: Online Graphing Calculator with Many Pre-Made Activities b. Texas Instruments: Texas Instruments Algebra 2 Graphing Calculator Activities c. Wabbit: Online TI-84 Silver Edition Graphing Calculator Emulator
Interactive Resources a. Emergent Math: Emergent Math b. Explore Learning: Gizmo Online Simulations c. Interactive Quizzes: Interactive Quizzes for High School Assessments
Practice Tests and Assessment Resources a. California Department of Education: California Algebra Released Test Questions b. Practice Tests from Virginia for All Levels of Math - http://education.jlab.org/solquiz/ c. Problem-Attic: Sample Problems d. XL Math for Algebra 2 - XL Math for Algebra 2
Video Resources a. HippoCampus: Videos for Algebra 2 b. LearnersTV: Videos for Algebra 2 c. Virtual Nerd: Videos for Algebra 2

Sample Formative Assessment Tasks/Questions
Solving Equations (A2.ACE.1) a. Problem-Attic: Sample Problems b. Math Bits Notebook: Radical Equations (#1 and 10) c. Math Bits Notebook: Radical Equations (#3, 11, 12)
Rational Function that Models Two Quantities (A2.FBF.1, A2.FIF.5) a. Georgia State Department of Education: Unit 4 (tasks begin on page 63)
Functions that Model Two Quantities a. Mathematics Assessment Project: Representing Functions of Everyday Situations

Algebra 2 Unit 6 Title
Exponential Functions and Equations

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*

- A2.ACE.1* Create and solve equations and ~~inequalities~~ in one variable that model real-world problems involving ~~linear, quadratic, simple rational, and~~ exponential relationships. Interpret the solutions and determine whether they are reasonable.
  - Limit to exponential equations for this unit.
  - Extend beyond integer exponents (from Algebra 1).
- A2.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales.
  - Emphasis should be on the appropriate use of labels, units, and scales when graphing.
  - Limit to exponential equations for this unit.
- A2.ACE.4* Solve literal equations and formulas for a specified variable including equations and formulas that arise in a variety of disciplines.
  - Solve using logarithms as an extension for Honors Algebra 2
- A2.ASE.3* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (Note: A2.ASE.3c are not Graduation Standards.) c. Use the properties of exponents to transform expressions for exponential functions.
  - An example of the use of an exponent property would be 1.15^t can be written as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$.
- A2.AREI.11* Solve an equation of the form f(x) = g(x) graphically by identifying the x- coordinate(s) of the point(s) of intersection of the graphs of y = f(x) and y = g(x).
  - Extend this standard to include higher-order polynomials, rational, radical, absolute value and exponential functions.
  - Understand that graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically.
  - Include the use of a graphing calculator for finding points of intersection.
- A2.FBF.1* Write a function that describes a relationship between two quantities. (Note: IA.FBF.1a is not a Graduation Standard.)
  a. Write a function that models a relationship between two quantities using both explicit expressions and a recursive process and by combining standard forms using addition, subtraction, multiplication and division to build new functions.
  - Make the connection to geometric sequences taught in Algebra 1 and unit 1 of Algebra 2.
  - Emphasize that there are times when one form (explicit and recursive) to describe the function is preferred over the other.
- A2.FBF.1* Write a function that describes a relationship between two quantities.
  b. Combine functions using the operations addition, subtraction, multiplication, and division to build new functions that describe the relationship between two quantities in mathematical and real-world situations.

- More complex exponential functions should be developed by using the operations addition, subtraction, and multiplication (such as $f(x) = 100 + 6^t$)
- Emphasis should be on how the operation affects the function and the relationship it is modeling

- A2.FBF.2* Write ~~arithmetic and~~ geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
  - Intermediate Algebra students will have been exposed to this standard whereas this is the first time these concepts will be introduced to students coming from Algebra 1.
  - Connect geometric sequences to exponential functions.
  - Exponential functions are the explicit form of recursively-defined geometric sequences.
  - The recursive formula for a geometric sequence uses multiplication and the explicit formula uses exponentiation.
  - Understand the difference between an explicit and recursive formula. An explicit formula is a function rule that relates each term of a sequence to the term number/position. An explicit formula allows the students to find any element of a sequence without knowing the element before it. Recursive formula is a function rule that relates each term of a sequence to the prior term.
- A2.FBF.3* Describe the effect of the transformations $kf(x)$, $f(x) + k$, $f(x + k)$, and combinations of such transformations on the graph of $y = f(x)$ for any real number k. Find the value of k given the graphs and write the equation of a transformed parent function given its graph.
  - Exploration of the effects of a constant, k, should include comparing the shape and position and analyzing the differences between the graphs of the original equation and transformed equation.
  - Limit to exponential functions.
- A2.FIF.3* Define functions recursively and recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
  - Explain that a recursive formula tells how a sequence starts and how to use the previous value(s) to generate the next element of the sequence.
- A2.FIF.4* Interpret key features of a function that models the relationship between two quantities when given in graphical or tabular form. Sketch the graph of a function from a verbal description showing key features. Key features include intercepts; intervals where the function is increasing, decreasing, constant, positive, or negative; relative maximums and minimums; ~~symmetries~~; end behavior and ~~periodicity~~.
  - Make a connection to F.BF.1b and F.BF.3 when teaching.
- A2.FIF.5* Relate the domain and range of a function to its graph and, where applicable, to the quantitative relationship it describes.
  - Emphasize the domain includes all real numbers as students may think otherwise when reviewing the visual representation.
  - Explore graphical representations to analyze the variability in the range.
- A2.FIF.6* Given a function in graphical, symbolic, or tabular form, determine the average rate of change of the function over a specified

interval. Interpret the meaning of the average rate of change in a given context.
  - Because exponential functions increase and decrease so rapidly, it may be necessary to look at smaller intervals when determining average rate of change.
- A2.FIF.7* Graph functions from their symbolic representations. Indicate key features including intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior and periodicity. Graph simple cases by hand and use technology for complicated cases.
  - Identify such features as the asymptote, the intercepts, end behaviors of the functions.
- A2.FIF.8* Translate between different but equivalent forms of a function equation to reveal and explain different properties of the function. (Note: A2.FIF.8b is not a Graduation Standard.) b. Interpret expressions for exponential functions by using the properties of exponents.
  - Make a connection to ASE.3.
- A2.FIF.9* Compare properties of two functions given in different representations such as algebraic, graphical, tabular, or verbal.
  - Describe and explain the characteristics (i.e. increasing/decreasing, end behaviors, asymptotes, etc) of two exponential functions when presented in different forms
- A2.FLQE.1* Distinguish between situations that can be modeled with linear functions or exponential functions by recognizing situations in which one quantity changes at a constant rate per unit interval as opposed to those in which a quantity changes by a constant percent rate per unit interval. (Note: A2.FLQE.1b is not a Graduation Standard.) b. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
  - Understand that exponential functions change by equal factors over time. If this factor is greater than 1, it indicates exponential growth. If the factor is between 0 and 1, it indicates exponential decay.
- A2.FLQE.2* Create symbolic representations of ~~linear and~~ exponential functions, including ~~arithmetic and~~ geometric sequences, given graphs, verbal descriptions, and tables.
  - Explore translating from the graph, verbal description, or table to an algebraic model.
- A2.FLQE.5* Interpret the parameters in a linear or exponential function in terms of the context.
  - Use real-world contexts to help students understand how the parameters of exponential functions depend on the context. Students can be given different parameters of a function to manipulate and compare the results to draw conclusions about the effects of the changes.

New Academic Vocabulary for This Unit
• Explicit formula/rule • Percent rate of change • Recursive formula/rule

Prior Knowledge Required for this Unit
In earlier grades, students have developed conceptual knowledge and been expected to: ● Apply exponent rules (8.EEI.1) ● Create and solve equations, but limited to integer exponents (A1.ACE.1) ● Graphing equations using appropriate labels, units, and scales (A1.ACE.2) ● Identify x-coordinates and points of intersection of graphs (A1.AREI.11) ● Identifying key features of a function and sketching the graph from a verbal description. (A1.FIF.4) ● Relate the domain and range of a function to its graph.(A1.FIF.5) ● Determine the average rate of change of a function over a specified interval. (A1.FIF.6) ● Effects of transformation with exponential with integer exponents (A1.FBF.3) ● Create symbolic representations of linear functions (A1.FLQE.2) ● Interpret the parameters in a linear function in terms of context (A1.FLQE.5)

Subsequent Knowledge Related to this Unit
The following standards are also included in Pre-Calculus: • AREI.11 Identifying x-coordinates and points of intersection of graphs. • FBF.1b Combining functions using operations. • FBF.3 Describing transformations of a graph. • FIF.4 Identifying key features of a function and sketching the graph from a verbal description. • FIF.5 Relating the domain and range of a function to its graph. • FIF.6 Determining the average rate of change of a function over a specified interval.

Relationship Among Standards in this Unit
All standards in this unit address exponential functions and equations. The overarching relationship amongst the standards in this unit is: • Making connections between the graphical, algebraic, and tabular form of exponential functions and equations are essential. • Learning to use and interpret key features of the graph of exponential functions allows students to explore and analyze the meaning of these key features. • Applications through modeling of real world situations should take into consideration any restriction/s to the domain or range.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 6.*
Transforming Exponential Graphs a. Explore Learning: Exponential Functions-Activity A b. Better Lesson: "Stretching Exponential Functions and Your Mind"
Exponential Growth and Decay a. College Preparatory Mathematics: Exponential Growth and Decay b. Alabama Learning Exchange: Exponential Growth and Decay c. Georgia Standards of Excellence Framework: Zombie Apocalypse Simulation d. Georgia Standards of Excellence Framework: Bacteria in the Swimming Pool
Key Features and Characteristics of Exponential Functions a. Georgia Standards of Excellence Framework: Graphs of Exponential Functions
Geometric Sequences - Explicit and Recursive Formulas a. Algebra Lab: Geometric Sequences b. Riverside Unified School District: Writing Sequences Explicitly and Recursively c. Teacher Web: Overview of Explicit and Recursive Formulas d. Georgia Standards of Excellence Framework: Community Service, Sequences and Functions

Resources
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Activity Resources a. Illustrative Mathematics: Activities For All Levels b. Robert Kaplinksy: Problem-Based Learning Activities
Graphing Calculator Resources a. Desmos: Online Graphing Calculator with Many Pre-Made Activities b. Texas Instruments: Texas Instruments Algebra 2 Graphing Calculator Activities c. Wabbit: Online TI-84 Silver Edition Graphing Calculator Emulator
Interactive Resources a. Emergent Math: Emergent Math b. Explore Learning: Gizmo Online Simulations c. Interactive Quizzes: Interactive Quizzes for High School Assessments
Practice Tests and Assessment Resources a. California Department of Education: California Algebra Released Test Questions b. Practice Tests from Virginia for All Levels of Math - http://education.jlab.org/solquiz/ c. Problem-Attic: Sample Problems d. XL Math for Algebra 2 - XL Math for Algebra 2
Video Resources a. HippoCampus: Videos for Algebra 2 b. LearnersTV: Videos for Algebra 2 c. Virtual Nerd: Videos for Algebra 2

Sample Formative Assessment Tasks/Questions
• Engage New York: Algebra 1 Module 3 Lesson 22 • Engage New York: Algebra 1 Module 3 Lesson 23 • Engage New York: Algebra 1 Module 3 Lesson 3 • Engage New York: Algebra 1 Module 3 Lesson 5 • Illustrative Mathematics: Exponential Functions • Illustrative Mathematics: A Saturating Exponential

South Carolina College- and Career-Ready Standards for Mathematics
High School Support Document
Geometry Coversheet

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Points, Lines, Planes, Angles, and Proofs	Triangles	Quadrilaterals	Similarity	Right Triangles and Trigonometry	Area and Volume	Circles	Statistics
Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
G.GCO.1* G.GCO.8a* G.GCO.8b* G.GCO.8d* G.GCO.11* G.GGPE.4* G.GGPE.5* G.GGPE.6 G.GGPE.7* G.GM.1* G.GM.2	G.GCI.3 G.GCO.2* G.GCO.3* G.GCO.4* G.GCO.5* G.GCO.6* G.GCO.7* G.GCO.8c* G.GCO.9a* G.GCO.9b* G.GCO.9d* G.GCO.11* G.GM.1* G.GM.2 G.GSRT.5*	G.GCO.6* G.GCO.8* G.GCO.9* G.GCO.10a* G.GCO.10b* G.GCO.10c* G.GCO.10d* G.GCO.10e* G.GCO.11* G.GGPE.4* G.GM.1* G.GM.2 G.GSRT.5*	G.GCO.2* G.GCO.5* G.GCO.9c* G.GCO.11* G.GM.1* G.GM.2 G.GSRT.1 G.GSRT.2* G.GSRT.3* G.GSRT.4a* G.GSRT.4b* G.GSRT.5*	G.GM.1* G.GM.2 G.GSRT.4c* G.GSRT.6* G.GSRT.7 G.GSRT.8*	G.GCI.5* G.GCO.1* G.GCO.11* G.GGPE.7* G.GGMD.1* G.GGMD.2 G.GGMD.3* G.GGMD.4* G.GM.1* G.GM.2	G.GCI.1 G.GCI.2* G.GCI.3 G.GCI.4 G.GCO.1* G.GGPE.1* G.GM.1* G.GM.2 G.GSRT.5*	G.SPID.1* G.SPID.2* G.SPID.3*

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Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

1. Make sense of problems and persevere in solving them. a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

Geometry Unit 1 Title
Points, Lines, Planes, Angles, and Proofs

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- G.GCO.1* Define angle, perpendicular line, parallel line, line segment, ray, circle, and skew in terms of the undefined notions of point, line, and plane. Use geometric figures to represent and describe real-world objects.
- G.GCO.8a* Prove, and apply in mathematical and real-world contexts, theorems about lines and angles, including the following: vertical angles are congruent.
- G.GCO.8b* Prove, and apply in mathematical and real-world contexts, theorems about lines and angles, including the following: when a transversal crosses parallel lines, alternate interior angles are congruent, alternate exterior angles are congruent, and consecutive interior angles are supplementary.
    - Establish the Corresponding Angles Postulate first and use this postulate to prove the other theorems.
- G.GCO.8d* Prove, and apply in mathematical and real-world contexts, theorems about lines and angles, including the following: perpendicular lines form four right angles.
- G.GCO.11* Construct geometric figures using a variety of tools, including a compass, a straightedge, dynamic geometry software, and paper folding, and use these constructions to make conjectures about geometric relationships.
- G.GGPE.4* Use coordinates to prove simple geometric theorems algebraically
    - Relate point, line, and plane to Coordinate Geometry.
- G.GGPE.5* Analyze slopes of lines to determine whether lines are parallel, perpendicular, or neither. Write the equation of a line passing through a given point that is parallel or perpendicular to a given line. Solve geometric and real-world problems involving lines and slope.
- G.GGPE.6 Given two points, find the point on the line segment between the two points that divides the segment into a given ratio.
    - To divide a segment into lengths that have a ratio of $\frac{a}{b}$, use the formula $P = (\frac{ax_1+bx_2}{a+b}, \frac{ay_1+by_2}{a+b})$ and relate this formula to the midpoint formula.
- G.GGPE.7* Use the distance and midpoint formulas to determine distance and midpoint in a coordinate plane, as well as areas of triangles and rectangles, when given coordinates.
    - Students should be able to explain how the distance formula relates to the Pythagorean Theorem.
- G.GM.1* Use geometric shapes, their measures, and their properties to describe real-world objects.
    - This standard is used throughout the course. Include shapes, measures, and properties applicable to this unit.
- G.GM.2 Use geometry concepts and methods to model real-world situations and solve problems using a model.
    - This standard is used throughout the course. Include concepts and methods applicable to this unit.

New Academic Vocabulary for This Unit
• Alternate exterior angles • Alternate interior angles • Conjecture • Consecutive interior angles • Construction • Postulate • Proof • Skew • Theorem • Transversal • Vertical angles

Prior Knowledge Required for this Unit
In earlier grades/courses, students have developed conceptual knowledge and have had the opportunity to learn how to: • Students should know basic geometric terminology from elementary and middle school, such as point, line, plane, ray, segment, angle, supplementary angles, complementary angles, parallel lines, and perpendicular lines. • Students should know how to plot points on a coordinate plane (5.G.1). • Students should know how to write linear equations given two points or given a point and the slope (8.F.4c). • Students should know how to measure accurately using a ruler and a protractor (4.MDA.5). • Students should have general application knowledge of parallel lines and transversal (8.GM.5c).

Subsequent Knowledge Related to this Unit
• Foundational definitions will be extended and applied in subsequent units; for example, angle bisector and perpendicular bisectors will be utilized in incenter and circumcenter of triangles (Geometry Unit 2: Triangles). • Students will use the relationships involving lines and angles that are established in this unit when they explore relationships and prove theorems that involve triangles, quadrilaterals, and other polygons in future units. • Constructions and coordinate geometry are introduced here and are meant to be applied throughout the course in order for students to make critical connections among geometric relationships synthetically (without coordinates) and analytically (with coordinates). • Students will construct logical arguments and formal proofs of geometric relationships throughout the course as they develop their deductive reasoning skills and understanding of more sophisticated theorems based on the simpler axioms introduced in Geometry Unit 1.

Relationship Among Standards in this Unit
This unit includes all of the standards that involve the most basic geometric shapes. The standards focus on analyzing geometric relationships both with and without coordinates that will carry through the rest of the course.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 1.*
Logic, Reasoning, and Proof (G.GCO.8*; G.GM.1*; G.GM.2) a. Euclidean Foundation Khan Academy: Euclidean Geometry Beginnings Math Open Reference: Euclid b. Venn Diagram Reasoning NCTM Illuminations: Venn Diagrams and Logic Virtual Nerd: Venn Diagrams c. Reasoning and Proof Dictionary.Reference.Com: Syllogism Khan Academy: Deductive Reasoning Khan Academy: Proof by Contradiction (see problems 4 and 6) Math Goodies: Conditional Statement and Truth Tables Math Goodies: Extension: Conditional Statement MathBitsNotebook: Indirect Proof (Proof by Contradiction) and More Proof By Contradiction MathBitsNotebook: Types of Direct Proofs Virtual Nerd: Conditional Statement and Converse, Inverse, and Contrapositive Virtual Nerd: Inductive Reasoning Virtual Nerd: Law of Detachment
Undefined Terms and Foundational Geometry Properties (G.GCO.1*; G.GCO.8*; G.GM.1*; G.GM.2) a. Point, Line, Plane (Undefined Terms), and Collinear/Coplanar Cliff Notes Math: Point, Line, Plane, and Collinear And Coplanar Grade A Math Help: Undefined Terms and Key Concepts Khan Academy: Drawing with 3D Plane Diagrams Math Open Reference: Introduction to Plane Geometry Math Open Reference: Point, Line, Collinear, and Coplanar applets Math Open Reference: Point, Line, Plane, Collinear, and Coplanar MathBitsNotebook: Point, Line, Plane, and Collinear And Coplanar Virtual Nerd: Point and Plane

b. Definitions: Segments, Midpoints, Rays, Angles, Angle Bisector, and Perpendicular Bisector
Cliffs Notes Math: Midpoints and Rays (Including Ruler Postulate and Segment Addition) and Angles (Including Adjacent and Angle Addition)
Grade A Math Help: Segments, Rays, and Angles
Khan Academy: Lines, Segments, Rays and Angles
Math Open Reference: Segment, Midpoint, Segment Bisector, Intersecting Lines, Ray, Opposite Rays, Angles, Angle Interior, and Angle Bisector
MathBitsNotebook: Explanations of Definition Concept
MathBitsNotebook: Segment, Midpoint, Intersecting Lines, Segment Bisector, Ray, Opposite Rays, Angle , Angle Interior, and Angle Bisector applets
Virtual Nerd: Segment, Ray, Angle, and Perpendicular Bisector

c. Symbols: Points, Lines, Segments, Rays, and Angles
Khan Academy: Basic Language and Symbols
MathBitsnotebook: Key Symbols

d. Foundational Postulates
Cliff Notes Math: Postulates
EngageNY: Review of Geometry Assumptions
MathBitsNotebook: Postulates and Auxiliary Lines

e. Measuring Segments And Angles
Grade A Math Help: Measuring Angles
Khan Academy: Measuring Angles
MathBitsNotebook: Angle Measures and Classifications
MathBitsNotebook: Notation of Measurements and Congruence
MathBitsNotebook: Notes Include Segment Length and Ruler Postulate
Math Open Reference: Congruence, Congruent Segments, and Congruent Angles
Math Open Reference: Congruent Segments and Congruent Angles applets
Virtual Nerd: What Does Congruence Mean?
Virtual Nerd: What Does Degree Represent? and Acute, Right, Obtuse And Right Angles

Segment Relationships (G.GCO.8*; G.GM.1*; G.GM.2)

a. Segment Addition Postulate
   Cliffs Notes Math: Notes in Ruler and Segment Addition Postulates
   Khan Academy: Segment Addition Postulate Example
   MathBitsNotebook: Notes Include Segment Addition Postulate and Segment Bisector
b. Segment Bisector
   Khan Academy: Midpoint Example
   Math Open Reference: Midpoint, Segment Bisector, and Perpendicular Bisector
   Math Open Reference: Midpoint, Segment Bisector, and Perpendicular Bisector applets
   MathBitsNotebook: Practice With Segment Lengths (Including Segment Addition And Midpoint)

Angle Relationships (G.GCO.8*; G.GCO.8a*; G.GCO.8d*; G.GM.1*; G.GM.2)

a. Right Angle and Perpendicular Lines
   Cliffs Notes Math: Intersecting, Parallel, and Perpendicular Lines
   Math Open Reference: Perpendicular Lines and Right Angles
   Math Open Reference: Perpendicular Lines and Right Angles Applet
   Mathbitsnotebook: Perpendicular Lines and Related Theorems
   Mathbitsnotebook: Practice with Right Angles
   Virtual Nerd: Parallel and Perpendicular Lines Application
   Virtual Nerd: Perpendicular Lines Have Four Right Angles – Explanation
b. Complementary and Supplementary Angles
   Cliffs Notes Math: Notes Include Complementary and Supplementary Angles
   Math Open Reference: Complementary and Supplementary Angles
   Math Open Reference: Complementary and Supplementary Angles applets
   MathBitsNotebook: Notes Include Complementary and Supplementary Angles
   Virtual Nerd: Complementary Angles and Supplementary Angles
c. Adjacent Angles and Angle Addition Postulate
   Cliffs Notes Math: Notes Include Angle Addition and Angle Bisector
   Khan Academy: Angle Addition Postulate Example
   Math Open Reference: Adjacent Angles
   Virtual Nerd: Angle Addition Postulate Example

d. Angle Bisector
   Math Open Reference: Angle Bisector applet
d. Linear Pairs with Linear Pair Postulate and Vertical Angles With Vertical Angle Theorem
   Cliffs Notes Math - Notes Include Vertical Angles
   Khan Academy: Linear Pair And Vertical Angles
   Math Open Reference: Linear Pair and Vertical Angles
   Math Open Reference: Linear Pair and Vertical Angles applets
   MathBitsNotebook - Pair Types Of Angles (Including Linear Pair) and Vertical Angles
   MathBitsNotebook: Practice With Angle Measures
   Virtual Nerd: Vertical Angles
e. Proofs with Segments and Angles (G.GCO.8*)
   EngageNY: Angle Proof Applications
   MathBitsNotebook: Proofs With Segments and Angles

Parallel Lines and Transversals (G.GCO.8*; G.GCO.8a*; G.GM.1*; G.GM.2)

a. Parallel Concepts
   MathBitsNotebook: Parallel Postulate (Euclid's 5th Postulate) and Parallel, Perpendicular and Transversal Lines
   Virtual Nerd: Parallel Lines and Skew Lines
b. Corresponding Angles Postulate
   Grade A Math Help: Parallel Lines and Transversal Notes
   Math Open Reference: Corresponding Angles (Two Parallel Lines and a Transversal)
   Math Open Reference: Corresponding Angles (Two Parallel Lines and a Transversal) applet
   Virtual Nerd: Corresponding Angle Postulate and Converse of Corresponding Angle Postulate
c. Parallel Lines and Transversal Theorems
   EngageNY: Angle Proof With Parallel Lines and Transversal Applications
   EngageNY: Review Of Corresponding Angles With Parallel Lines and Transversal
   Khan Academy: Parallel Lines and Transversal Explanations and Unique Parallel and Perpendicular Lines Example
   MathBitsNotebook: Angle Pair With Two Parallel Lines and Transversal Identifying Angle Pairs and Notes
   Virtual Nerd: Finding Angle Measure Within Parallel Lines and Transversal Example

Coordinate Plane Geometry (G.GGPE.4*; G.GGPE.5*; G.GGPE.6; G.GGPE.7*)

a. Point-Line-Plane Coordinate Geometry

Cliffs Notes Math: General Coordinate Information
Math Open Reference: Coordinate Plane
Math Open Reference: Coordinate Plane applet
Math Open Reference: Introduction Coordinate Plane
Math Open Reference: Points On Coordinate Plane
Math Open Reference: Points On Coordinate Plane applet
Virtual Nerd: Endpoints Of A Segment On Coordinate Plane

b. Midpoint, Slope and Distance Formulas
Cliffs Notes Math: Midpoint With Equal Distance Shown and Distance Formula
EngageNY: Using Distance Formula to Find Perimeter And Area (extension in Unit 1; see again in Unit 6)
Khan Academy: Midpoint and Distance formulas examples
Math Open Reference: Midpoint and Distance Formula
Math Open Reference: Midpoint and Distance formulas applets
Virtual Nerd: Midpoint Explanation, Midpoint Example, Distance Formula Explanation, and Distance Formula Example

c. Divide a Segment into Lengths that have a Ratio of $\frac{a}{b}$
Khan Academy: Ratio of Distances on a Segment, and Find Point with Given Ratio Lengths

d. Parallel and Perpendicular Lines
Cliffs Notes Math: Slope and Slope of Line review; Parallel and Perpendicular Segments
EngageNY: Parallel And Perpendicular Lines Varied Applications
Khan Academy: Introduction To Parallel And Perpendicular Lines, and Verifying Two Lines Are Parallel, Perpendicular, Or Neither
Math Open Reference: Point-Slope Form applet
Math Open Reference: Point-Slope Form review
Virtual Nerd: Point-Slope Form review, and Writing Equations Of Lines Parallel or Perpendicular

Constructions (G.GCO.11*; G.GM.1*; G.GM.2)
MathBitsNotebook: Basic Construction Information
Math Open Reference: Euclid And Constructions

a. Segment Congruence
EngageNY: Equilateral Triangle Construction
Math Open Reference: Constructing A Segment Into Congruent Parts extension
Math Open Reference: Constructing Congruent Segments and extension: Construct Equilateral Triangle

Math Open Reference: Constructing Congruent Segments applet
Math Open Reference: Constructing Congruent Triangles (SSS Congruence)
Math Open Reference: Constructing Congruent Triangles applet
Math Open Reference: Construction A Segment Into Congruent Parts applet
MathBitsNotebook: Constructing Congruent Segments (Plus Congruent Angles) and Constructing Equilateral Triangle

b. Segment Bisector and Perpendicular Lines
EngageNY: Segment Bisector Construction
Math Open Reference: Construct Segment Bisector; Constructing A Line Perpendicular To Another Line (Point On) and (Point Off) applet
Math Open Reference: Constructing a 90-Degree Angle applet
Math Open Reference: Constructing a 90-Degree Angle Extension
Math Open Reference: Constructing Segment Bisector; Constructing A Line Perpendicular To Another Line (Point On) and (Point Off)
MathBitsNotebook: Constructing Segment Bisector (Plus Angle Bisector) and Constructing Perpendicular Lines
Virtual Nerd: Perpendicular Bisector

c. Angle Congruence and Line Parallel to Another (Corresponding Angle-Transversal Method)
MathBitsNotebook: Constructing Congruent Angles (Plus Previous Congruent Segments) And Construction Similar Triangles
Math Open Reference: Constructing Congruent Angles and Constructing A Line Parallel To Another Line (Congruent Angle Method)
Math Open Reference: Constructing Congruent Angles and Constructing A Line A Parallel To Another Line (Corresponding Angle-Transversal Method) applet
Math Open Reference: Angle Addition (Using Congruent Angles) extension
Math Open Reference: Angle Addition (Using Congruent Angles) applet extension

d. Angle Bisector
MathBitsNotebook: Constructing Angle Bisector (Plus Previous Segment Bisector)
Math Open Reference: Constructing An Angle Bisector
Math Open Reference: Constructing An Angle Bisector applet
EngageNY: Angle Constructions
Math Open Reference: Constructing A 30-Degree Angle and Constructing A 45-Degree Angle extensions
Math Open Reference: Constructing A 30-Degree Angle and Constructing A 45-Degree Angle extensions

Resources
• Cliff Notes Math: Geometry • Emergent Math: Emergent Math • EngageNY: Geometry • Grade A Math Help: Geometry • Grade A Math Help: Geometry Resources • Illuminations: Grades 9 - 12 Resources • Khan Academy: Geometry • Math Education Page: Geometry Book of Labs • Math Education Page: Sum of the Angles in a Triangle • Math Goodies: Math Goodies • Math Open Reference: Geometry Resources • MathBitsNotebook: Geometry Online Study Resources • Patty Paper – *Patty Paper Geometry* by Michael Serra; resource of activities and discovery lessons utilizing *(patty paper can be purchased, or possibly donated from your local butcher or grocer's meat department)* • Virtual Nerd: Geometry Skills Videos

Sample Formative Assessment Tasks/Questions
Undefined Terms and other foundational properties (G.GCO.1*; G.GCO.8*; G.GM.1*; G.GM.2) a. Khan Academy: Point, Line, Plane, and Other Terms
Axiom System Review a. EngageNY: Review of Assumptions
a. Inside Mathematics: Performance Assessment Tasks

Geometry Unit 2 Title
Triangles

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- G.GCI.3 Construct the inscribed and circumscribed circles of a triangle using a variety of tools, including a compass, a straightedge, and dynamic geometry software, and prove properties of angles for a quadrilateral inscribed in a circle.
- G.GCO.2* Represent translations, reflections, rotations, and dilations of objects in the plane by using paper folding, sketches, coordinates, function notation, and dynamic geometry software, and use various representations to help understand the effects of simple transformations and their compositions.
    - Omit dilations which will be used in Unit 4 Similarity.
- G.GCO.3* Describe rotations and reflections that carry a regular polygon onto itself and identify types of symmetry of polygons, including line, point, rotational, and self-congruence, and use symmetry to analyze mathematical situations.
- G.GCO.4* Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
    - Now that formal axiom process has been established in Unit 1, these basic transformation can be re-investigated under necessary descriptions. For example, in a reflection, a line segment that joins a point to its image is perpendicular to the line of reflection and the line of reflection will pass through the midpoint of the segment joining the point to its image. OR the line of reflection is the perpendicular-bisector of the segment connect each pre-image to image points.
- G.GCO.5* Predict and describe the results of transformations on a given figure using geometric terminology from the definitions of the transformations, and describe a sequence of transformations that maps a figure onto its image.
- G.GCO.6* Demonstrate that triangles and quadrilaterals are congruent by identifying a combination of translations, rotations, and reflections G.GCI.3 - Construct the inscribed and circumscribed circles of a triangle using a variety of tools, including a compass, a straightedge, and dynamic geometry software, and prove properties of angles for a quadrilateral inscribed in a circle. in various representations that move one figure onto the other.
- G.GCO.7* Prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions.
    - Include the Side-Side-Side congruence condition.
- G.GSRT.5* Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
    - Congruence criteria only in this unit. Similarity criteria will be used in Unit 4 Similarity.
- G.GCO.8* Prove, and apply in mathematical and real-world contexts, theorems about lines and angles.

- This overarching standard is used to verify many essential properties, such as Angle-Bisector Theorem - any point that in the interior of an angle that is equidistant from the sides of the angle must be on the angle's bisector.

- G.GCO.8c* Prove, and apply in mathematical and real-world contexts, theorems about lines and angles, including the following: any point on a perpendicular bisector of a line segment is equidistant from the endpoints of the segment.
- G.GCO.9 Prove, and apply in mathematical and real-world contexts, theorems about the relationships within and among triangles.
  - This overarching standard is applied throughout the Geometry course as the axiom system continuous to validate itself with prior knowledge. For example, verifying that an exterior angle of a triangle is equal to the sum of its non-adjacent (remote) interior angles.
- G.GCO.9a* Prove, and apply in mathematical and real-world contexts, theorems about the relationships within and among triangles, including the following: measures of interior angles of a triangle sum to 180°.
- G.GCO.9b* Prove, and apply in mathematical and real-world contexts, theorems about the relationships within and among triangles, including the following: base angles of isosceles triangles are congruent.
- G.GCO.9c* Prove, and apply in mathematical and real-world contexts, theorems about the relationships within and among triangles, including the following: the segment joining midpoints of two side of a triangle is parallel to the third side and half the length.
  - Prove and use the Midsegment Theorem of a triangle
- G.GCO.9d* Prove, and apply in mathematical and real-world contexts, theorems about the relationships within and among triangles, including the following: the medians of a triangle meet at a point.
  - Include the perpendicular bisectors of a triangle meet at a point.
- G.GCO.11* Construct geometric figures using a variety of tools, including a compass, a straightedge, dynamic geometry software, and paper folding, and use these constructions to make conjectures about geometric relationships.
  - Omit proving properties of angles for a quadrilateral inscribed in a circle, as this will be explored in Unit 7 Circles.
- G.GM.1* Use geometric shapes, their measures, and their properties to describe real-world objects.
  - This standard is used throughout the course. Include shapes, measures, and properties applicable to this unit.
- G.GM.2 Use geometry concepts and methods to model real-world situations and solve problems using a model.
  - This standard is used throughout the course. Include concepts and methods applicable to this unit.
- G.GSRT.5* Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometry figures.
  - Much of the Geometry course depends on triangle congruence to verify later properties, such as seen in G.GCO.9b* with the Isosceles Triangle Base Angle Theorem, and also in application to verify other congruences such as applied with corresponding parts of congruent triangles are congruent.

New Academic Vocabulary for This Unit
• Bisect, Bisector • Circumscribe • Concurrent • Congruent • Inscribe • Median of a Triangle

Prior Knowledge Required for this Unit
In earlier grades/courses/units, students have developed conceptual knowledge and have had the opportunity to learn how to: • Students should be able to classify triangles (7.GM.2). • Students should know and be able to apply relationships of alternate interior angles between parallel lines from previous unit, and build upon their Grade 8 experience with triangle sum and angle relationships (8.GM.5a; 8.GM.5b; G.GCO.8b*). • Students should know that translations, rotations, and reflections are rigid transformations that preserve length and angle measures, and understand how it relates to congruency (8.GM.1b, c, and d).

Subsequent Knowledge Related to this Unit
• Students will use properties of triangles and congruence of triangles to prove other geometric relationships later in the course; for example, the diagonals of rectangle ABCD can be proven congruent by showing that $\Delta ACD \cong \Delta BDC$. • Transformations are applicable to algebraic concepts of function families and their graphs that students encounter in Algebra 2 and Pre-Calculus courses.

Relationship Among Standards in this Unit
In this unit, students will explore the common characteristics of all triangles, discover ways to prove that two triangles are congruent, and then use these congruence relationships to prove properties of isosceles and equilateral triangles.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 2.*
Triangle Angle Sum and Exterior Angle Theorems (G.GCO.9; G.GCO.9a*) Cliffs Notes Math: Exterior Angle Theorem EngageNY: Interior And Exterior Angles Of A Triangle Extension: Geometry Book of Labs - Section 1 - Complete Lab 1.4 To Review Triangle Sum As The Base Of Polygon Angle Sum Geometry Book of Labs: Section 1; Complete Lab 1.5 Angles in a Triangle to Explore Classification and Interior Angles Geometry Book of Labs: Section 1; Complete Lab 1.6 Exterior Angle Theorem Khan Academy: Triangle Angle Sum Theorem Verification with Examples 1, 2, 3, and Challenging 4 Math Education Page: Triangle Sum Theorem applet MathBitsNotebook: Exterior Angle Theorem with Verification-Proof MathBitsNotebook: Triangle Angle Sum Theorem with Many Varied Verification-Proofs Virtual Nerd: Triangle Angle Sum Theorem and Extension with Equilateral Triangle Angles Are 60-Degrees Each

Congruency and Triangle Congruence (G.GCO.7*)

a. Congruence Concept
   Cliffs Notes Math: Triangle Congruence Summary Of Notes
   Math Open Reference - Congruence Concept, SSS Postulate, SAS Postulate, ASA Postulate, AAS Theorem, and HL Theorem
   Math Open Reference: Congruence Concept, SSS Postulate, SAS Postulate, ASA Postulate, AAS Theorem, and HL Theorem applets
   Math Open Reference: Why These Concepts Do Not Work For Congruence? SSA (ASS) attempt and AAA attempt
   Math Open Reference: Why These Concepts Do Not Work For Congruence? SSA (ASS) attempt and AAA attempt applet
   MathBitsNotebook: Concept of Congruence, and Triangle Congruence Properties
b. Congruence Theorems, Applications, and Proof
   NOTE: Review EngageNY rigid motions (attached in Transformations and Congruence section) for possible SAS, ASA, SSS, AAS, and HL introduction.
   EngageNY: Triangle Congruence Proofs-1 and Triangle Congruence Proofs-2
   Khan Academy: Discussion Video on More On SSA Failure with Establishment Of Special Exception Of Hypotenuse-Leg
   Khan Academy: Discussion Videos on SSS Postulate and SAS And ASA Postulates (Includes AAA And SSA Congruence Failures)
   Khan Academy: Manipulating AAA Triangles And Demonstrating Failed Congruency applet
   Khan Academy: Outline of a Specific Two-Column Proof (Note: "Reasons" Within Reason Column Are Not Typical Geometry Language)
   MathBitsNotebook: Triangle Congruence Proofs Structuring (Includes Rigid Motions), and Analyzing Strategies For Congruence Proofs
   MathBitsNotebook: Two Column Proof Problems, Proof Steps, and Applying CPCTC To Produce Another Pair Of Congruency
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Evaluating Conditions for Congruency

Isosceles Triangle Base Angles Theorem (G.GCO.9b*; G.GSRT.5*)

   EngageNY: Rigid Motion Of Isosceles Triangle Base Angles Theorem
   MathBitsNotebook: If Two Sides Of A Triangle Are Congruent, Then The Angle Opposite (Base Angles) Are Congruent

Transformation and Congruence (G.GCO.2*; G.GCO.4*; G.GCO.5*; G.GCO.6*)

a. Translations, Reflections, Rotations, and Combinations
   Mathematics Assessment Project – Mathematics Assessment Resource Service: Transformations review
   Transformations
b. Congruency Within Transformations
   Math Education Page: Rigid Motions; Translations, Reflections, and Rotations applets
   Math Education Page: Translation
   Math Open Reference: Modeled Demonstration Of Preserved Congruence With All Combinations Of Rigid Motions applet
   MathBitsNotebook: Rigid Motions (Transformations) And Congruence and Triangle Congruence Proofs Structuring Including Rigid Motions
c. Congruence and Similarity
   EngageNY: Rigid Motion For SAS Postulate (*can be used to introduce SAS postulate*)
   EngageNY: Rigid Motion For ASA And SSS Postulates (*can be used to introduce ASA and SSS postulates*)
   EngageNY: Rigid Motion For AAS And HL Theorems (*can be used to introduce AAS and HL Theorems*)

Points of Concurrency and Triangle Midsegment (G.GCI.3; G.GCO.8*; G.GCO.8c*; G.GCO.9*; G.GCO.9c*; G.GCO.9d*; G.GSRT.5*)

a. Angle Bisector and Perpendicular Bisector Theorems
   Khan Academy: Video Discussion on A Point On An Angle Bisector Is Equidistant From The Sides And Proof Or Theorem
   Math Open Reference: Equidistant Points (Models Perpendicular Bisector Theorem)
   Math Open Reference: Equidistant Points applet
   Virtual Nerd: Perpendicular Bisector Theorem application
b. Incenter, Circumcenter, Orthocenter, and Centroid
   Cliffs Notes Math: Altitude, Median, And Angle Bisectors Within A Triangle
   Khan Academy: Lesson Videos of Incenter and Circumcenter
   Math Open Reference: Angle Bisectors - Incenter, Incenter Circle , Perpendicular Bisectors - Circumcenter, and Circumcenter Circle applets
   Math Open Reference: Angle Bisectors - Incenter, Incenter Circle, Perpendicular Bisectors - Circumcenter, and Circumcenter Circle
   Math Open Reference: Incenter On Coordinate Plane; applet: Incenter On Coordinate Plane
   Math Open Reference: Orthocenter and Centroid
   Math Open Reference: Orthocenter and Centroid applets
   MathBitsNotebook: Median-Centroid, Altitude-Orthocenter, Angle Bisector-Incenter, And Perpendicular Bisector-Circumcenter Notes
   Virtual Nerd: Incenter, Circumcenter, and Median-Centroid
c. Median-Centroid Applications
   Virtual Nerd: Application of Median-Centroid
d. Midsegment and Midsegment of a Triangle Theorem
   EngageNY: Midsegments Of A Triangle And Extensions
   Math Open Reference: Midsegment Is Half The Length Of The Parallel Side
   Math Open Reference: Midsegment Is Half The Length Of The Parallel Side applet
   MathBitsNotebook: Midsegment Theorem And Different Proofs
   Virtual Nerd: Triangle Midsegment Theorem Explained On Coordinate Plane

Constructions of Incenter and Circumcenter (G.GCI.3) EngageNY: Construct a Square and Nine-Point Circle EngageNY: Constructions of Points of Concurrencies and Related Theorems Math Open Reference: Constructions of Incenter and Circumcenter Math Open Reference: Constructions of Incenter and Circumcenter applets Math Open Reference: Constructions of Orthocenter and Centroid Math Open Reference: Constructions of Orthocenter and Centroid applets
Constructions of with Triangle Congruence (G.GCO.11*) Geometry Book of Labs: Section 6; Complete Lab 6.1 Constructing Non-Congruent Triangles Math Open Reference: Constructing Congruent Triangles using SSS Math Open Reference: Constructing Congruent Triangles using SSS applet

Resources
• Cliff Notes Math: Geometry • Emergent Math: Emergent Math • EngageNY: Geometry • Grade A Math Help: Geometry • Grade A Math Help: Geometry Resources • Illuminations: Grades 9 - 12 Resources • Khan Academy: Geometry • Math Education Page: Geometry Book of Labs • Math Education Page: Sum of the Angles in a Triangle • Math Goodies: Math Goodies • Math Open Reference: Geometry Resources • MathBitsNotebook: Geometry Online Study Resources • Mathematics Assessment Project – Mathematics Assessment Resource Service: Homepage • Virtual Nerd: Geometry Skills Videos

Sample Formative Assessment Tasks/Questions
Congruency and Triangle Congruence (G.GCO.7*) a. MathBitsNotebook: Varied Congruence Questions and Full Two-Column Proofs
Constructions of with Triangle Congruence (G.GCO.11*) a. MathBitsNotebook: Rigid Motion Triangle Congruence b. Math Education Page: Determining Type of Transformations Examples 1, 2, 3, and 4; applets
Proofs using segments and angles within a triangle - not congruence proofs (G.GCO.8*, G.GCO.9*, G.GCO.9c*) a. MathBitsNotebook: Proofs Using Segments And Angles Within A Triangle (Not Congruence Proofs)
Points of Concurrency and Triangle Midsegment (G.GCI.3; G.GCO.9*) a. MathBitsNotebook: Segments within a Triangle and Triangle Midsegment practice
a. Inside Mathematics: Performance Assessment Tasks

Geometry Unit 3 Title
Quadrilaterals

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- G.GCO.6* Demonstrate that triangles and quadrilaterals are congruent by identifying a combination of translations, rotations, and reflections in various representations that move one figure onto the other.
  - Demonstrate congruency of transformed quadrilaterals under rigid motions, as so with previous triangles unit.
- G.GCO.8* Prove, and apply in mathematical and real-world contexts, theorems about lines and angles,
- G.GCO.9* Prove, and apply in mathematical and real-world contexts, theorems about the relationships within and among triangles
  - Included these overarching standards of G.GCO.8 and G.GCO.9 for definition, theorem verification, and application with quadrilaterals containing special features not covered in G.GC0.10, such as Isosceles Trapezoid and Kites.
  - Include analysis of Isosceles Trapezoid and Kite which extends previous learning from congruent triangles, particularly how corresponding parts of congruent triangles are congruent.
  - Prove, model and apply Polygon Interior Angle and Exterior Angle Sum Theorems and make connections to Quadrilaterals such that the sum of the interior angles is 360 degrees and sum of the exterior angles is 360 degrees.
- G.GCO.10* Prove, and apply in mathematical and real-world contexts, theorems about parallelograms
  - Include this overarching standard to address parallelogram properties of opposite sides are parallel and consecutive angles are supplementary.
  - Include this overarching standard to prove a quadrilateral is a parallelogram by showing both pairs of opposite sides are parallel, and one pair of opposite sides is both parallel and congruent.
  - Include this overarching standard to define rectangle, rhombus, and squares from parallelograms, and prove that from quadrilaterals rectangles can be proven as a quadrilateral with four right angles, rhombus can be proven as a quadrilateral with four congruent sides, and squares can be proven as quadrilaterals with four right angles and four congruent sides.
  - Include this overarching standard to address that the parallelogram properties also exist in rectangle, rhombus, and squares.
  - Include this overarching standard to address that each diagonal in a rhombus bisects a pair of opposite angles.
- G.GCO.10a* Prove, and apply in mathematical and real-world contexts, theorems about parallelograms, including the following: opposite sides of a parallelogram are congruent.
  - Include using this property in application that opposite sides are congruent and proving a quadrilateral is a parallelogram by showing both pairs of opposite sides are congruent.

- G.GCO.10b* Prove, and apply in mathematical and real-world contexts, theorems about parallelograms, including the following: opposite angles of a parallelogram are congruent.
  - Include using this property in application that opposite angles are congruent and proving a quadrilateral is a parallelogram by showing both pairs of opposite angles are congruent.
- G.GCO.10c* - Prove, and apply in mathematical and real-world contexts, theorems about parallelograms, including the following: diagonals of a parallelogram bisect each other.
  - Include using this property in application that diagonals bisect each other and proving a quadrilateral is a parallelogram by diagonals bisect each other.
- G.GCO.10d* - Prove, and apply in mathematical and real-world contexts, theorems about parallelograms, including the following: rectangles are parallelograms with congruent diagonals.
  - Include using this property in application that diagonals in a rectangle are congruent and proving a parallelogram is a rectangle by showing diagonals are congruent.
- G.GCO.10e* - Prove, and apply in mathematical and real-world contexts, theorems about parallelograms, including the following: a parallelogram is a rhombus if and only if the diagonals are perpendicular.
  - Include using this property in application that diagonals are perpendicular to each other and proving a parallelogram is a rhombus by showing diagonals are perpendicular.
  - Connect that quadrilaterals with perpendicular diagonals can also be kites (a special quadrilateral)
- G.GCO.11* - Construct geometric figures using a variety of tools, including a compass, a straightedge, dynamic geometry software, and paper folding, and use these constructions to make conjectures about geometric relationships.
  - Include foundational constructions to construct squares and parallelograms.
- G.GGPE.4* - Use coordinates to prove simple geometric theorems algebraically.
  - Prove quadrilaterals are parallelograms, specific types of parallelograms, or special quadrilaterals in conjunction with G.GCO.8, G.GCO.9, and G.GCO.10 through given coordinates matching these type of figures by applying distance, slope and midpoint formula accordingly.
  - Verify through coordinate formula of distance, slope and midpoint generalized quadrilateral theorems, such as to demonstrate a quadrilateral with the given coordinates of (0, 0), (a, b), (c, d), and (c-a, d-b) is a parallelogram as indicated in G.GCO.10, 10a, 10b, 10c, 10d, and 10e.
  - Proof concept can be modeled in multiple formats. Coordinate Proofs include both deductive and inductive reasoning along with written explanations connecting coordinate formula calculations to the specific geometry being proven.

- G.GM.1* - Use geometric shapes, their measures, and their properties to describe real-world objects.
    - Include shapes, measures, and properties applicable to this unit.
- G.GM.2 - Use geometry concepts and methods to model real-world situations and solve problems using a model.
    - Include concepts and methods applicable to this unit.
- G.GSRT.5* - Use congruence ~~and similarity~~ criteria for triangles to solve problems and to prove relationships in geometric figures.
    - Include analysis of Isosceles Trapezoid and Kite extends previous learning from congruent triangles, particularly how corresponding parts of congruent triangles are congruent.
    - Congruence criteria only in this unit. Similarity criteria will be used in Unit 4 Similarity.

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New Academic Vocabulary for This Unit
• Parallelogram – *exposed to concept under area (6.GM.1) and construction (7.GM.2c) but without formal definition or analysis* • Regular Polygon • Diagonal of a polygon

Prior Knowledge Required for this Unit
In earlier grades/courses/units, students have developed conceptual knowledge and have had the opportunity to learn how to: • Students should be able to apply the Triangle Angle Sum Theorem for interior angles and possess knowledge of the relationships of exterior angles (Unit 2). • Students have been exposed to the concepts of parallelograms, rectangle, rhombus, trapezoid, isosceles trapezoid, and kite through area (6.GM.1) and generalized construction (7.GM.2c), but likely without formal definition. • Students should be able to draw with protractor and ruler parallelograms and special quadrilaterals under specified criteria (7.GM.2c) • Students should "understand that shapes in different categories (e.g., rhombus, rectangle, square, and other 4-sided shapes) may share attributes (e.g., 4-sided figures) and the shared attributes can define a larger category (e.g., quadrilateral). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories." (3.G.1) • Students should have a firm understanding of triangle congruency and that corresponding parts of congruent triangles are congruent and how these apply to create new geometry theorems (Unit 2). • Student should have functional application skills of midpoint, slope, and distance formulas (Unit 1). • Students should be able to construct congruent segments, segment bisectors, congruent angles, angle bisector, a line parallel through a given point to another line, and a line perpendicular to another line through a given point on or off that line (Unit 1). • Students should be able to demonstrate rigid motion congruency of segments, angles, and triangles (Units 1 and 2). • Students should be able to recognize quadrilaterals by observing shapes and their attributes (various K-6 geometry standards). • Students should be able to precisely describe transformations in terms of angles, perpendicular and parallel lines to describe a specific mapping from one figure to another (G.GCO.4, Unit 2).

Subsequent Knowledge Related to this Unit
• Students will be able to derive a formula by showing the product of the diagonal lengths produces area of a rhombus (Unit 6). • Student will recognize various quadrilaterals and apply in deriving corresponding area formula for circles and within three dimensional shapes surface area and volume formulas (Unit 6).

Relationship Among Standards in this Unit
Within this unit, students will extend their knowledge of congruent triangles and that corresponding parts of congruent triangles are congruent to each other and then produce various quadrilateral theorems. Students must possess firm understanding of how within the quadrilateral family properties of a parent figure also apply to more specified figures, such as general quadrilateral and parallelogram properties also are true within specified rectangle, rhombus and squares. Application of previous knowledge of coordinate geometry and concept of proof are applied to produce coordinate proofs (G.GPE.4*). Extension of rigid motion transformational geometry from triangles demonstrates quadrilateral congruencies; students should be able to describe in terms of angles, midpoints, perpendicular lines, etc. how one triangle maps onto another to produce triangle congruencies within a quadrilateral (G.GCO.10*) and then ultimately congruency between quadrilaterals (G.GCO.6*).

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 3.*
Polygon Sum for Interior and Exterior Angles (G.GCO.9*) Cliff Notes Math: Polygons Angles Math Open Reference: Triangles within a Polygon Math Open Reference: Triangles within a Polygon applet Math Open Reference: Interior Angle Sum within a Polygon Math Open Reference: Interior Angle Sum within a Polygon applet Math Open Reference: Exterior Angle Sum within a Polygon Math Open Reference: Exterior Angle Sum within a Polygon applet Virtual Nerd: What is a Regular Polygon Virtual Nerd: Sum of Interior Angles of a Polygon Virtual Nerd: Finding Measure of Exterior angle of a Polygon
Quadrilateral Concepts (G.GCO.9*, G.GGO.10*) Math Open Reference: Quadrilateral Concepts Math Is Fun: Quadrilateral Concepts Math is Fun: Quadrilaterals Interactive applet MathBitsNotebook Quadrilateral Principles Regents Prep: Quadrilateral Family Virtual Nerd: Quadrilaterals Virtual Nerd: Classifying Quadrilaterals Virtual Nerd: Finding angle measure within a Quadrilateral Virtual Nerd: Finding angle measure within a Quadrilateral with Variables School Yourself: Quadrilaterals Khan Academy: Quadrilateral Discussion Khan Academy: Quadrilateral Analysis Mathematics Assessment Project – Mathematics Assessment Resource Service: Describing and Defining Quadrilaterals

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Parallelograms and its properties (G.GCO.10*, G.GCO.10a*, G.GCO.10b*, G.GCO.10c*, G.GCO.10d*)

Cliffs Notes Math: Properties of Parallelograms
MathBitsNotebook: Parallelogram Properties
Regents Prep: Parallelogram Properties
Math Open Reference: Parallelogram Definition and Notes
Math Open Reference: Parallelogram Definition applet
Math Open Reference: Parallelogram Inscribed within a Quadrilateral
Virtual Nerd: Parallelogram
Virtual Nerd: Parallelogram Application – Solving Using Opposite Sides Congruent
School Yourself: Parallelograms Angles
School Yourself: Parallelograms Sides
School Yourself: Parallelograms Diagonals
Khan Academy: Proving Opposite Sides in a Parallelogram are Congruent
Khan Academy: Proving Opposite Angles in a Parallelogram are Congruent
Khan Academy: Proving Diagonals in a Parallelogram Bisect Each Other

Proving Parallelograms (G.GCO.10*, G.GCO.10a*, G.GCO.10b*, G.GCO.10c*, G.GCO.10d*)

Cliffs Notes Math: Proofs of Parallelograms
Regents Prep: Proof of Parallelograms
Math Open Reference: Proofs of Parallelograms

Special Parallelograms (G.GCO.10*, G.GCO.10e*) Cliffs Notes Math: Properties of Special Parallelograms MathBitsNotebook: Special Parallelogram Properties and Proofs Regents Prep: Special Parallelograms Math Open Reference: Definition of Rectangle Math Open Reference: Diagonals of a Rectangle Math Open Reference: Diagonals of a Rectangle applet Math Open Reference: Definition of Rhombus Math Open Reference: Diagonals of a Rhombus Math Open Reference: Diagonals of a Rhombus applet Math Open Reference: Definition of Square Math Open Reference: Diagonals of a Square Math Open Reference: Diagonals of a Square applet Virtual Nerd: What is a Rectangle Virtual Nerd: Diagonals of a Rectangle application Virtual Nerd: What is a Rhombus Virtual Nerd: Diagonals of a Rhombus application Virtual Nerd: What is a Square School Yourself: Rectangles Diagonals School Yourself: Rhombus Diagonals
Trapezoids, Isosceles Trapezoid, and Kites – special quadrilaterals (G.GCO.8*, G.GCO.9*, G.GCO.10e*, and G.GSRT.5*) Cliffs Notes Math: Properties of Trapezoids MathBitsNotebook: Properties of Trapezoids and Kites Regents Prep: Theorems with Trapezoids Math Open Reference: Definition of Trapezoid Math Open Reference: Median (or Midsegment) of a Trapezoid Math Open Reference: Median of a Trapezoid applet Virtual Nerd: What is a Trapezoid

Quadrilaterals and Coordinate Geometry Proofs (G.GGPE.4*) Math Open Reference: Coordinate Geometry with Rectangle Math Open Reference: Coordinate Geometry with Rectangle applet Virtual Nerd: Example of Coordinate Reasoning West Contra Costa Unified School District – The Parallelogram Law
Quadrilaterals and Transformations (G.GCO.6*) MathBitsNotebook: Conceptual Notes of Viewing Symmetry with Quadrilaterals

Quadrilaterals Constructions (G.GCO.11*)

Math Open Reference: Inscribed Square
Math Open Reference: Inscribed Square applet
Math Open Reference: Constructing Inscribed Square applet
Virtual Nerd: Constructing a Square

Resources
• Cliff Notes Math: Geometry • Discovering Geometry: Discovering Geometry • Emergent Math: Emergent Math • EngageNY: Geometry • Georgia Department of Education, Standards and Teacher Support: Geometry • Grade A Math Help: Geometry • Grade A Math Help: Geometry Resources • Illuminations: Grades 9 - 12 Resources • Khan Academy: Geometry • Math Education Page: Geometry Book of Labs • Math Education Page: Sum of the Angles in a Triangle • Math Goodies: Math Goodies • Math Open Reference: Geometry Resources • MathBitsNotebook: Geometry Online Study Resources • Mathematics Assessment Project – Mathematics Assessment Resource Service: Homepage • School Yourself: Geometry • Virtual Nerd: Geometry Skills Videos • West Contra Costa Unified School District: Geometry

Sample Formative Assessment Tasks/Questions
Quadrilaterals (G.GCO.8*, G.GCO.9*, G.GCO.10*, G.GCO.10a*, G.GCO.10b*, G.GCO.10c*, G.GCO.10d*, G.GCO.10e*, G.GM.1* G.GM.2, G.GSTR.5*) a. Discovering Geometry: Polygons and Quadrilaterals b. Grade A Mathematics: Triangles and Quadrilaterals c. Inside Mathematics: Performance Assessment Tasks - Quadrilaterals d. Khan Academy: Quadrilateral Angles Assessment e. Khan Academy: Quadrilateral Types Assessment f. Math Education Page: Geometry Book of Labs – Triangles and Quadrilaterals g. MathBitsNotebook: Application Problems with Quadrilaterals h. MathBitsNotebook: Proofs with Quadrilaterals i. MathBitsNotebook: Quadrilateral Flow Chart j. MathBitsNotebook: Quadrilateral Practice with Missing Measures k. MathBitsNotebook: Quadrilateral Practice with Using Algebra l. Regents Prep: Proofs with Quadrilaterals m. Regents Prep: Quadrilaterals and Parallels n. Regents Prep: Quadrilaterals Mixed Practice
Quadrilaterals and Coordinate Geometry Proofs (G.GGPE.4*) a. MathBitsNotebook: Coordinate Proofs b. Georgia Department of Education: Geometry Unit 5

Geometry Unit 4 Title
Similarity

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*

• G.GCO.2* Represent ~~translations, reflections, rotations, and~~ dilations of objects in the plane by using paper folding, sketches, coordinates, function notation, and dynamic geometry software, and use various representations to help understand the effects of simple transformations and their compositions.
• G.GCO.5* Predict and describe the results of transformations on a given figure using geometric terminology from the definitions of the transformations, and describe a sequence of transformations that maps a figure onto its image.
• G.GCO.9c* Prove, and apply in mathematical and real-world contexts, theorems about the relationships within and among triangles, including the following: the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.
  ○ Prove and use the Midsegment Theorem of a triangle
• G.GCO.11* Construct geometric figures using a variety of tools, including a compass, a straightedge, dynamic geometry software, and paper folding, and use these constructions to make conjectures about geometric relationships.
• G.GM.1* Use geometric shapes, their measures, and their properties to describe real-world objects.
  ○ Include shapes, measures, and properties applicable to this unit.
• G.GM.2 Use geometry concepts and methods to model real-world situations and solve problems using a model.
  ○ Include concepts and methods applicable to this unit.
• G.GSRT.1 Understand a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. Verify experimentally the properties of dilations given by a center and a scale factor. Understand the dilation of a line segment is longer or shorter in the ratio given by the scale factor.
• G.GSRT.2* Use the definition of similarity to decide if figures are similar and justify decision. Demonstrate that two figures are similar by identifying a combination of translations, rotations, reflections, and dilations in various representations that move one figure onto the other.
• G.GSRT.3* Prove that two triangles are similar using the Angle-Angle criterion and apply the proportionality of corresponding sides to solve problems and justify results.
  ○ SSS and SAS should also be used to prove two triangles are similar.
• G.GSRT.4a* Prove, and apply in mathematical and real-world contexts, theorems involving similarity about triangles, including the following: A line drawn parallel to one side of a triangle divides the other two sides into parts of equal proportion.
  ○ Prove and use the Side-Splitter Theorem

- G.GSRT.4b* Prove, and apply in mathematical and real-world contexts, theorems involving similarity about triangles, including the following: If a line divides two sides of a triangle proportionally, then it is parallel to the third side.
    - Prove and use the Converse of the Side-Splitter Theorem.
- G.GSRT.5* Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

New Academic Vocabulary for This Unit
None

Prior Knowledge Required for this Unit
• Students should know how relate scale drawings to dilations of geometric figures (8.GM.3). • Students should know how to dilate geometric figures using scale factors that are positive rational numbers (8.GM.4). • Students should know how to recognize that two-dimensional figures are only similar if a series of transformations can be performed to map the pre-image to the image (8.GM.4). • Students should know how to describe the series of transformations that justifies this similarity given two similar figures (8.GM.4). • Students should know how to use proportional reasoning to find the missing side lengths of two similar figures (8.GM.4).

Subsequent Knowledge Related to this Unit
• Properties of similar right triangles will be used to define trigonometric ratios and to determine sine, cosine, and tangent of an acute angle in a right triangle (Unit 5: G.GSRT.6). • In addition, similarity will be used again when working with chords and secants in circles (Unit 7: G.GCI.2).

Relationship Among Standards in this Unit
In this unit, students will explore the properties of similarity and will use criteria such as Angle-Angle to prove triangles are similar and to solve problems involving congruent and similar triangles. In order to solve such problems, students will explore theorems including the Midsegment Theorem and the Side-Splitter Theorem (Triangle Proportionality Theorem) and its converse.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 4.*
Similar Figures (G.GSRT.1) a. VirtualNerd: Similar Figures b. VirtualNerd: What is a Dilation? c. MathBitsNotebook: Dilations and Lines (Segments) d. Geometry Book of Labs: Scaling on a Geoboard, Section 10.1 e. Geometry Book of Labs: Similar Rectangles, Section 10.2 f. BetterLesson: Perspective Drawings
Similarity and Transformations (G.GSRT.2*) a. VirtualNerd: Identifying Similar Transformations b. Kahn Academy: How To Determine Whether Two Shapes Are Similar c. EngageNY: Similarity Transformations d. IXL Learning Inc: Similarity Transformations Practice
Proving Triangle Similarity (G.GSRT.3*) a. MathBitsNotebook: Proving Similar Triangles b. BetterLesson: Introduction to Similar Triangles Proofs c. Kahn Academy: Similarity Postulates d. Math Nspired: Angles and Similarity e. Math Open Reference: Triangle Similarity Test (SSS) f. Math Open Reference: Triangle Similarity Test (SAS)
The Side-Splitting Theorem (G.GSRT.4a*; G.GSRT.4b*) a. MathBitsNotebook: Side Splitting Theorem b. MathBitsNotebook: Side Splitting Theorem Practice c. IXL Learning Inc: Triangle Proportionality Practice d. BetterLesson: Proving Theorems Involving Similar Triangles

Using Congruence and Similarity Criteria for Triangles to Solve Problems (G.GSRT.5*)

a. Kahn Academy: Solving Problems with Similar and Congruent Triangles
b. Illustrative Mathematics: Bank Shot
c. Illustrative Mathematics: Finding Triangle Coordinates
d. Math Nspired: Applications of Similar Figures

Resources
• BetterLesson: Math • EngageNY: Geometry • Geogebra: Geometry • Illustrative Mathematics: High School Geometry • IXL Learning Inc: Geometry • Kahn Academy: Geometry • Math Education Page: Geometry Book of Labs • Math Nspired: Geometry • Math Open Reference: Geometry Resources • MathBitsNotebook: Geometry Online Study Resources • VirtualNerd: Geometry Tutorials

Sample Formative Assessment Tasks/Questions
Similar Figures (G.GSRT.1) a. Better Lesson: Assessment for Sweet Similar Shapes b. Georgia Department of Education: Dilations in the Coordinate Plane c. Georgia Department of Education: Similar Triangles d. MathBitsNotebook: Similarity
Similarity and Transformations (G.GSRT.2*) a. Georgia Department of Education: Shadow Math b. MathBitsNotebook: Dilations and Lines
Proving Triangle Similarity (G.GSRT.3*) a. Georgia Department of Education: Proving Similar Triangles
The Side-Splitting Theorem (G.GSRT.4a*; G.GSRT.4b*) a. MathBits Notebook: Side Splitter Theorem Practice
Using Congruence and Similarity Criteria for Triangles to Solve Problems (G.GSRT.5*) a. Illustrative Mathematics: Unit Squares and Triangles b. Illustrative Mathematics: Points from Directions

Geometry Unit 5 Title
Right Triangles and Trigonometry

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*

• G.GSRT.4c* - Prove, and apply in mathematical and real-world contexts, theorems involving similarity about triangles, including the following: The square of the hypotenuse of a right triangle is equal to the sum of squares of the other two sides.
  ○ Prove and use the Pythagorean Theorem
  ○ Include special right triangles, 45-45-90 and 30-60-90
• G.GSRT.6* - Understand how the properties of similar right triangles allow the trigonometric ratios to be defined and determine the sine, cosine, and tangent of an acute angle in a right triangle.
• G.GSRT.7 - Explain and use the relationship between the sine and cosine of complementary angles.
• G.GSRT.8* - Solve right triangles in applied problems using trigonometric ratios and the Pythagorean Theorem.
  ○ Include angle of elevation and angle of depression problems.
• G.GM.1* - Use geometric shapes, their measures, and their properties to describe real-world objects.
  ○ Include shapes, measures, and properties applicable to this unit.
• G.GM.2 - Use geometry concepts and methods to model real-world situations and solve problems using a model.
  ○ Include concepts and methods applicable to this unit.

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New Academic Vocabulary for This Unit
• Cosine • Sine • Tangent • Trigonometry

Prior Knowledge Required for this Unit
In earlier grades, students have developed conceptual knowledge and have had the opportunity to learn how to: • Transform and apply the Pythagorean Theorem particularly as it relates to rational and irrational squares (8.EEI.2a, b, and d). • Similar Triangles (Geometry, Unit 4, G.GSRT.2*, G.GSRT.3*, G.GSRT.5*)

Subsequent Knowledge Related to this Unit
Student will use their basic knowledge of the trigonometric ratios to explore trigonometric functions in Pre-Calculus. Students will extend their knowledge of the trigonometric ratios as they relate them to the unit circle and will define the other trigonometric ratios in terms of sine and cosine. In addition, students will explore the Law of Sines and the Law of Cosines (PC.FT. 4, PC.FT.5, and PC.FT.6).

Relationship Among Standards in this Unit
In this unit, students will be introduced to basic right triangle trigonometry for the first time. Students will use their knowledge of similar triangles and the Pythagorean Theorem to define the trigonometric ratios of sine, cosine, and tangent (G.GSRT.4c*, G.GSRT.6*). They will also use their knowledge of the complementary angles in a right triangle to explore the relationship between the sine and cosine of complementary angles (G.GSRT.7). Using their knowledge of the basic trig ratios and the Pythagorean Theorem, students will solve real – world problems involving right triangle trig (G.GSRT.8*, G.GM.1*, G.GM.2).

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 5.*
Right Triangles, Similarity, and Trig Ratios (G. GSRT.4c*, G.GSRT.6*) a. MathBitsNotebook: Pythagorean Theorem b. West Contra: Pythagorean Theorem and Its Converse c. Kahn Academy: Triangle Similarity and the Trig Ratios d. IXL Learning Inc: Trigonometric Ratio in Similar Right Triangles Practice e. Kahn Academy: Introduction to the Trig Ratios f. Kahn Academy: Finding Trig Ratios in Right Triangles g. VirtualNerd: 30-60-90 Special Right Triangle h. VirtualNerd: 45-45-90 Special Right Triangle i. VirtualNerd: Finding Missing Side in 30-60-90 j. VirtualNerd: Finding Missing Side in 45-45-90 k. West Contra Costa Unified School District: Intro to Trig Ratios l. Regents Prep: Solving for a Side m. Math Is Fun: Finding a Side n. Mathematics Vision Project: Similarity and Right Triangles pages 24 - 31
Trigonometric Ratios and Complementary Angles (G.GSRT.7) a. MathBitsNotebook: Sine and Cosine of Complementary Angles b. Kahn Academy: Showing the Relationship between the Sine and Cosine of Complements c. EngageNY: Sine and Cosine of Complementary Angles
Solving Real-world Problems involving Right Triangle Trigonometry and the Pythagorean Theorem (G.GSRT.8*, G.GM.1*, G.GM.2) a. MathBitsNotebook: Trigonometry Word Problems b. Kahn Academy: Angles of Elevation and Depression c. VirtualNerd: How to Solve a Problem using an Angle of Elevation d. TeachEngineering: Trig River

Resources
• EngageNY: Geometry • Illustrative Mathematics: High School Geometry • IXL Learning Inc: Geometry • Kahn Academy: Geometry • MathBitsNotebook: Geometry - Trigonometry • MathematicsVisionProject: Right Triangle Trig • MathisFun: Trigonometry • RegentsPrep: Trigonometry • TeachEngineering: Trigonometry • VirtualNerd: Geometry Right Triangle Trig • West Contra Costa Unified School District: Geometry • Yukon Education: Trigonometry

Sample Formative Assessment Tasks/Questions
Right Triangles, Similarity, and Trig Ratios (G. GSRT.4c*, G.GSRT.6*) a. Georgia Department of Education: Discovering Trigonometric Ratio Relationships b. Georgia Department of Education: Finding that Side or Angle c. Illustrative Mathematics: Defining Trig Ratios d. MathBitsNotebook: Trigonometry Numerical Practice
Trigonometric Ratios and Complementary Angles (G.GSRT.7) a. Illustrative Mathematics: Sine and Cosine of Complementary Angles b. MathBitsNotebook: Sine and Cosine of Complementary Angles Practice
Solving Real-world Problems involving Right Triangle Trigonometry and the Pythagorean Theorem (G.GSRT.8*, G.GM.1*, G.GM.2) a. Georgia Department of Education: Hypsometer Activity b. Georgia Department of Education: Formative Assessment: Right Triangles in Your Environment c. Illustrative Mathematics: Setting Up Sprinklers d. MathBitsNotebook: Trig Word Problem Practice e. Yukon Education: Applications of Trig Ratios f. Yukon Education: Designing a Solution

Geometry Unit 6 Title
Area and Volume

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*

• G.GCI.5* Derive the formulas for the length of an arc and the area of a sector in a circle and apply these formulas to solve mathematical and real-world problems.
  ○ Develop understanding of circle circumference and area formulas and then apply a fractional part to generate corresponding arc length and sector area formulas.
  ○ Model both explicit formula calculation and proportional reasoning of total circumference or area compared to 360 degrees for student understanding and application in solving for unknown parts or whole.
  ○ Apply arc length or sector area to calculate whole circumference or area in applicable problems.
  ○ Extend to arc-segment area calculation from sector area such that the included triangle area is subtracted from the total sector area to find remaining arc-segment area.
• G.GCO.1* Define angle, perpendicular line, parallel line, line segment, ray, circle, and skew in terms of the undefined notions of point, line, and plane. Use geometric figures to represent and describe real-world objects.
  ○ Connect coplanar concepts and terminology into modeled two and three dimensional figures and real world applications.
• G.GCO.11* Construct geometric figures using a variety of tools, including a compass, a straightedge, dynamic geometry software, and paper folding, and use these constructions to make conjectures about geometric relationships.
  ○ Perform constructions such as a regular hexagon or creating a platonic (regular polyhedron) to enrich understanding of geometry figures.
• G.GGPE.7* Use the distance and midpoint formulas to determine distance and midpoint in a coordinate plane, as well as areas of triangles and rectangles, when given coordinates.
  ○ Apply distance formula to calculate area of triangles, rectangles, and other combination of 2-dimensional figures.
• G.GGMD.1* - Explain the derivations of the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. Apply these formulas to solve mathematical and real-world problems.
  ○ Understand that π is defined as circumference divided by diameter; hence, circumference of a circle derives from this definition.
  ○ Connect square unit foundational rectangular, triangle, parallelogram area concepts to model and derive area of circle.
  ○ Extend cubic unit foundational prism into volume of cylinder formula.
  ○ Connect cubic unit foundational prism and cylinder and related to 1/3 relationship to corresponding Base Area and height of pyramid and cone.
  ○ Apply volume formulas in modeled and real life applications, including varied dimensions and comparisons to other like and unlike solids.

- G.GGMD.2 - Explain the derivation of the formulas for the volume of a sphere and other solid figures using Cavalieri's principle.
- G.GGMD.3* - Apply surface area and volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems and justify results. Include problems that involve algebraic expressions, composite figures, geometric probability, and real-world applications.
    - Apply volume and surface area in modeled and real life applications
- G.GGMD.4* - Describe the shapes of two-dimensional cross-sections of three-dimensional objects and use those cross-sections to solve mathematical and real-world problems.
    - Relate cross-sections to foundational area concepts
    - Relate cross-sections area with surface area and volume.
- G.GM.1* - Use geometric shapes, their measures, and their properties to describe real-world objects.
    - Include area of faces of solids and relate to surface area.
    - Relate composite area, circle-sector area, surface area and volume to real-world applications.
- G.GM.2 - Use geometry concepts and methods to model real-world situations and solve problems using a model.
    - Include area of faces of solids and relate to surface area.
    - Relate composite area, circle-sector area, surface area and volume to real-world applications.

Return to High School Overview, High School Table of Contents, or Geometry Coversheet

New Academic Vocabulary for This Unit
• Arc • Arc length • Cavalieri's Principle • Geometric Probability • Sector Area • Segment (Sector-Segment) Area

Prior Knowledge Required for this Unit
• Students should be able to identify and apply appropriate area formulas for rectangles, triangles, and other quadrilaterals, and understand the composite combination of rectangular and triangular area in generating common polygon formulas (Grade 3 to 5, 6.GM.1, and 7.GM.6). • Students should understand the proportional relationship between the constant π and radius-diameter and circumference, and be able to use both circumference and area formulas for circle in application and real-life situations (7.GM.4). • Students should be able to recognize scale model to actual measurement and how area relates to those situations. (7.GM.1). • Student possess appropriate unit measurement understanding and are able to develop and use circumference and area formulas for circles, general polygons and solids (7.GM.4 and 7.GM.6). • Students should be able to unfold three dimensional solids into two dimensional nets (6.GM.4 and 7.GM.6). • Students should recall Unit 2 Triangles and Unit 3 Quadrilaterals specifics such as altitude (height) or diagonals and recognize key features within polygons, including regular polygons (Units 2 and 3). • Students should recall Unit 5 Right Triangles to find corresponding measures utilized in area, surface area, and volume formulas (G.GCI.5* and G.GGMD.3*). • Student should have foundational construction skills and experience in applications, such as inscribing and circumscribing circles and creating a square (Units 1 – 3 and G.GCO.11*).

Subsequent Knowledge Related to this Unit
• Students will apply concept of constructions and arc within the unit on circles (Unit 7). • Students again analyze the development of arc length formulas in Pre-Calculus (PC.GCI.5*)

Relationship Among Standards in this Unit
The relationship of standards within this unit begins first with an understanding that for many students they have not worked with Area and Volume in application or discovery since Grade 7 and before that in grades 3 through 5, as indicated by South Carolina mathematics standards (7.GM.1, 7.GM.4, 7.GM.6 and Grades 3 to 5). Continued scaffolding of dimensional measurement concepts of length, area, and volume (as first seen in Grades 3 to 5 and then 7.GM.6) are once again built upon to develop in-depth understanding of appropriate unit measurement and the development of circumference and area formulas for circles, general polygons and solids (7.GM.4; 7.GM.6) and extended into circles, cylinders, cones, and spheres and apply the Cavalieri's principle to those figures and other solids (G.GGMD.1*; G.GGMD.2). The applications of geometric probability and constructions both enrich the previous geometric formula applications in modeled and real life applications, and create new avenues of study. Finally, the concepts of arc length, sector area, and arc-section area formulas are developed and area calculations of sector and arc-section can be calculated (G.GCI.5*).

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 6.*
Foundational Review of Area and Square Units (Grade 7 standards G.GCO.1*, G.GM.1*, G.GM.2, G.GGMD.4*) a. Khan Academy: Rectangular & Parallelogram Area video b. Khan Academy: Triangle Area applet c. Khan Academy: Area of Composite Figure-1 video d. Khan Academy: Area and Perimeter of Composite Figure-2 video e. NCTM Illuminations: Review of Foundational Area Concepts
Coordinate Grid Area (G.GCO.1*, G.GGPE.7*) a. Coordinate Grid and Perimeter & Area MathBitsNotebook Coordinate Perimeter & Area Open Math Reference: Coordinate Triangle Area-Boxing by “rectangular boxing” Open Math Reference: Coordinate Triangle Area-Boxing by “rectangular boxing” applet Open Math Reference: Coordinate Triangle Area Open Math Reference: Coordinate Triangle Area applet Open Math Reference: Coordinate Rectangle Area Open Math Reference: Coordinate Rectangle Area applet Khan Academy: Coordinate Area of Quadrilaterals video b. Explore: Relating Coordinate Grid and Area Concept NRICH: http://nrich.maths.org/2293 c. Explore: Coordinate Area for Polygons Open Math Reference: Polygon Area Open Math Reference: Polygon Area applet

Circle Circumference & Area including Arcs & Sectors (G.GCO.1*, G.GCI.5*, G.GGMD.1*, G.GGMD.4*, G.GM.1*, G.GM.2)

a. Review of Circle Circumference and its relationship to π definition
   π discussion at http://www.angio.net/pi/piquery
   Virtual Nerd: π and Circumference discussion
   Khan Academy: π, diameter, and circumference relationship
b. Arc Length, Sector Area and Annulus
   MathBitsNotebook: Arc Length
   MathBitsNotebook: Sector Area
   NY Regents: Sector Area and Sector-Segment Area notes
   Open Math Reference: Sector Area
   Open Math Reference: Sector Area applet
   Open Math Reference: Sector-Segment Area
   Open Math Reference: Sector-Segment Area applet
   School Yourself: Earth Circumference application
c. Enrichment: Annulus
   Open Math Reference Annulus Area
   Open Math Reference Annulus Area applet

Understanding and Applying Volume of Prisms and Cylinders (G.GCO.1*, G.GGMD.1*, G.GGMD.2, G.GGMD.3*, G.GGMD.4*, G.GM.1*, G.GM.2)

a. Foundations of Volume
   Virtual Nerd: Nets and Nets & Solids videos
   Virtual Nerd: What is Volume? Video
   MathBitsNotebook: Polyhedra
   Open Math Reference: Cubic Units
b. Volume Calculation
   MathBitsNotebook: Prisms and Cylinders V=Hh notes
   Virtual Nerd: Volume Formulas for Prism and Cylinder videos
   Virtual Nerd: Rectangular Prism, Triangular Prism, and Cylinder volume videos
   Open Math Reference: Cylinder Volume
   Open Math Reference: Cylinder Volume applet
   Open Math Reference: Cylinder Partial Volume

Khan Academy: Prism Volume video Khan Academy: Cylinder Volume & Surface Area video
Understanding and Applying Volume of Pyramids and Cones (G.GCO.1*, G.GGMD.1*, G.GGMD.2, G.GGMD.3*, G.GGMD.4*, G.GM.1*, G.GM.2) a. Volume Calculation MathBitsNotebook: Prisms and Cones V=1/3Bh notes Virtual Nerd: Volume Formulas for Pyramid and Cone videos Virtual Nerd: Rectangular Pyramid, Triangular Pyramid, and Cone volume videos Open Math Reference: Pyramid Volume Open Math Reference: Pyramid Volume applet Khan Academy: Cone Volume School Yourself: Cone Volume School Yourself: Pyramid Volume Explanation (connects Cavalieri's Principle) b. Composite Figures Volume Virtual Nerd: Finding Volume of a Composite Figure

Understanding and Applying Surface Area (G.GCO.1*, G.GGMD.2, G.GGMD.4*, G.GM.1*, G.GM.2)

a. Foundations of Surface Area
   Virtual Nerd: Nets and Nets & Solids videos
   MathBitsNotebook: Polyhedra
   Open Math Reference: Surface Area
   Khan Academy: Polyhedra nets
b. Volume Calculation
   MathBitsNotebook: Prisms , Cylinder, Pyramids, and Cones see surface area notes
   Virtual Nerd: Rectangular Prism, Cylinder, Pyramid and Cones lateral and surface area videos
   Virtual Nerd: Slant Height of Pyramids and Cones video
   Open Math Reference: Cylinder Surface Area Formula Foundation
   Open Math Reference: Cylinder Surface Area Formula Foundation applet
   Open Math Reference: Pyramid Area Formula Foundation
   Open Math Reference: Pyramid Area Formula Foundation applet
   Open Math Reference: Slant Height Comparisons
   Open Math Reference: Slant Height Comparisons applet
   Khan Academy: Cylinder Volume & Surface Area video
   School Yourself: Prism Surface Area
   School Yourself: Pyramid Surface Area
   School Yourself: Cylinder Surface Area
   School Yourself: Cone Surface Area
   matematicasVisuales: Prisms visual nets, applets, and formula notes
   matematicasVisuales: Cylinder visual nets, applets, and formula notes

Spheres and Cavalieri's Principle (G.GCO.1*, G.GGMD.4*, G.GM.1*, G.GM.2) a. Sphere Concept and Cavalieri's Principle MathBitsNotebook: Sphere Surface Area and Volume notes Virtual Nerd: Surface Area of Sphere concept Virtual Nerd: Surface Area of Sphere calculation Virtual Nerd: Volume of Sphere concept Virtual Nerd: Volume of Sphere calculation Open Math Reference: Surface Area and Volume of Sphere Open Math Reference: Surface Area and Volume of Sphere apples School Yourself: Sphere Volume and Surface Area b. Cavalieri's Principle MathBitsNotebook: Cavalieri's Principle notes School Yourself: Cavalieri's Principle Prisms, Cylinders, Pyramids, and Cones School Yourself: Cavalieri's Principle Spheres matematicasVisuales: Sphere Volume Formula with Cavalier's Principle notes and applets c. Extensions MathBitsNotebook: Rotational Area and Volume notes matematicasVisuales: Barrel Volume applets
Constructions (G.GCO.11*) • Possible: MathBitsNotebook: Constructing a Regular Hexagon • Possible: Math Is Fun: Manipulating Platonic Solids

Resources
• Cliff Notes Math: Geometry • Emergent Math: Emergent Math • EngageNY: Geometry • Georgia Department of Education, Standards and Teacher Support: Geometry • Grade A Math Help: Geometry • Grade A Math Help: Geometry Resources • Illuminations: Grades 9 - 12 Resources • Khan Academy: Geometry • matematicasVisuales: Geometry • Math Education Page: Geometry Book of Labs • Math Education Page: Sum of the Angles in a Triangle • Math Goodies: Math Goodies • Math Open Reference: Geometry Resources • MathBitsNotebook: Geometry Online Study Resources • Mathematics Assessment Project – Mathematics Assessment Resource Service: Homepage • West Contra Costa Unified School District: Geometry • Virtual Nerd: Geometry Skills Videos

Sample Formative Assessment Tasks/Questions
Coordinate Grid Area (G.GGPE.7*) a. MathbitsNotebook: Practice with Coordinate Area and Perimeter b. Khan Academy: Area of Coordinate Triangle
Circle Circumference & Area including Arcs & Sectors (G.GCI.5*, G.GGMD.1*) a. Georgia Department of Education: Geometry Unit 4
Applying Surface Area and Volume (G.GCO.1*, G.GGMD.1*, G.GGMD.2, G.GGMD.3*, G.GGMD.4*, G.GM.1*, G.GM.2) a. EngageNY: Prisms & Cylinders and Pyramids & Cones b. MathBitsNotebook: Applied Problems and Modeling Volume Questions c. MathBitsNotebook: Cross-Sectional d. MathBitsNotebook: Prisms & Cylinders, Pyramids & Cones, Spheres, and Cavalieri's Principle

Geometry Unit 7 Title
Circles

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*

• G.GGPE.1* Understand that the standard equation of a circle is derived from the definition of a circle and the distance formula.
    ○ Identify center and radius as reading standard equation of a circle.
    ○ Write circle of equation in standard form as viewing a circle, and find such parts as specific criteria is given such as the center and a point on the circle.
    ○ Sketch a graph of a circle when given or finding a center and radius of a circle.
    ○ Possible extension could build upon "completing the square" algebra skills to set-up the equation of a circle written in standard form.
• G.GCI.1 Prove that all circles are similar.
    ○ Relate Triangle Similarity tests to extend to circles to verify that all circles are similarity.
    ○ Relate dilation of circle to demonstrate that all circle are similar.
• G.GCI.2* Identify and describe relationships among inscribed angles, radii, and chords; among inscribed angles, central angles, and circumscribed angles; and between radii and tangents to circles. Use those relationships to solve mathematical and real-world problems.
    ○ Relate radius to tangent as same point is perpendicular, and extending into right triangles.
    ○ Relate congruent chords as being equidistance from center and congruent chords forming congruent arcs.
    ○ Discover that the diameter drawn perpendicular to any chord is its perpendicular bisector, and any chord's perpendicular bisector is a diameter of a circle.
    ○ Define and relate intercept arc with central and inscribed angles.
    ○ Find corresponding arc and angle measures of central and inscribe angles, angles formed by tangent and chords, and two intersecting chords.
    ○ Find corresponding arc and angle measures of angles formed by two secants, two tangents, and by a tangent and secant.
    ○ Apply triangle similarity to develop a means for calculating lengths of segments formed by intersecting chords.
    ○ Find corresponding secant and tangent segment lengths as two of which are drawn to a circle from an exterior point.
• G.GCI.3 Construct the inscribed and circumscribed circles of a triangle using a variety of tools, including a compass, a straightedge, and dynamic geometry software, and prove properties of angles for a quadrilateral inscribed in a circle.
    ○ Extend upon constructions developed in Unit 3 and related to new vocabulary and properties associated within this unit.
    ○ Connect measure of inscribed angles to verify that opposite angles in an inscribed quadrilateral are supplementary.
• G.GCI.4 Construct a tangent line to a circle through a point on the circle, and construct a tangent line from a point outside a given circle to the circle; justify the process used for each construction.
• G. GCO.1* Define angle, perpendicular line, parallel line, line segment, ray, circle, and skew in terms of the undefined notions of point, line,

and plane. Use geometric figures to represent and describe real-world objects.

- G.GM.1* Use geometric shapes, their measures, and their properties to describe real-world objects.
  - Connect circle chord-diameter relationship to identify location of equidistant "center" from three points.
  - Recognize various real-world examples as they apply to concepts associated with circles.
- G.GM.2 Use geometry concepts and methods to model real-world situations and solve problems using a model.
  - Connect circle chord-diameter relationship to find the location which is equidistant from three points.
- G.GSRT.5* Use congruence and similarity criteria for triangles to solve problems and prove relationships in geometry figures.
  - Relate triangle similarity to verify segments formed by intersecting chords.
  - Apply HL Triangle Congruence to verify various relationships within a circle, such as two tangents drawn from an exterior point are congruent.
  - Solve for various measurements within real-world examples as they apply to concepts associated with circles.

Return to High School Overview, High School Table of Contents, or Geometry Coversheet

New Academic Vocabulary for This Unit
• Inscribed angle • Chord of a circle • Central angle • Tangent of a circle • Secant of a circle

Prior Knowledge Required for this Unit
• Students should be able to Investigate the concept of circles (7.GM.4). • Students should related circumscribe and inscribed to concepts found within this unit (G.GGCI.3 and Unit 2). • Students should extend conceptual understanding of arc as gained in Unit 6 (G.GCI.5* and Unit 6). • Students understand angle interior and exterior angle relationships within triangles and quadrilaterals (Unit 2 and Unit 3). • Students should be able to verify triangle similarity and apply generate corresponding segment lengths (Unit 4). • Students understand language of “Tangent” from Right Triangle Trigonometry (Unit 5). • Students should be able to apply Right Triangle geometry to find missing angle and segment lengths (Unit 5). • Students understand the area and arc length as how they relate to circles (Unit 6). • Students should apply methods of solving quadratic equations as gained in Algebra One (AREI.4*)

Subsequent Knowledge Related to this Unit
• Students will apply understanding gain from the foundation of the circle equation in advanced conics (PC.GGPE.2 and PC.GGPE.3). • Students again analyze the development of arc length formulas in Pre-Calculus (GCI.5*). • Students will develop concept of radian and unit circle and build upon knowledge gained in this unit (PC.FT.1 and PC.FT.4).

Geometry Unit 7: Circles

Return to High School Overview, High School Table of Contents, or Geometry Coversheet

Relationship Among Standards in this Unit
Students should recognize the relationship of radii, tangent, chords between each other (G.GCI.2*) and be able to find corresponding angle, arc, and identified lengths relating to varied parts associated within circles (G.GCI.2*) including the ability to solve for natural quadratics which form from distances from an exterior point as tangents and secants are drawn to a circle (AREI.4*). As students conduct constructions (G.GCI.3; G.GCI.4) they will relate the geometry properties established with the circle (G.GCI.2*). Topics such as Inscribed and Circumscribed Triangles (Unit 2) and Regular Hexagon and Square (Unit 6) constructions are reexamined to extend meaningful understandings to this Circle Unit. Finally, the standard form of a circle extends conceptual understandings of circle back into the coordinate plane and algebraic reasoning (G.GGPE.1*). Throughout the unit, intentional real world connections deepen understanding and offer application with meaning.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 7.*
Standard Form Equation of a Circle (G.GGPE.1*) a. MathBitsNotebook: Standard Form Circle Equation – notes b. MathBitsNotebook: General Form of Equation c. RegentsNY: Circle Equation – notes d. Open Math Reference: Basic Circle e. Open Math Reference: Basic Circle applet f. Open Math Reference: Standard Form Circle Equation g. Open Math Reference: Standard Form Circle Equation applet h. Virtual Nerd: Standard Form Circle Equation video i. Khan Academy: Standard Form Circle Equation video j. Khan Academy: Writing from General Form to Standard Form and Graph video k. West Contra Costa Unified School District: Circles Unit lesson notes l. Extension: Open Math Reference: Relating to Unit Circle
Tangent-Radius Diameter and Chord and Congruent Tangents (G.GCI.2*, G.GCO.1*, G.GM.1, G.GM.2, G.GSRT.5*) a. MathBitsNotebook: Tangent and angle measure theorems b. RegentsNY: Chord and Arc Theorems and Tangent-Radius and Two-Tangents Theorems c. Open Math Reference: Tangent-Radius d. Open Math Reference: Tangent-Radius applet e. Virtual Nerd: Equidistant Chords video f. Virtual Nerd: Determining Tangency video g. Khan Academy: Radius (diameter) Perpendicular-Bisector to Chord Theorems Proofs video-1 and video-2 h. Khan Academy: Tangent-Radius application video i. SchoolYourself: Congruent Tangents j. Illustrative Mathematics: Radius Drawn to Point of Tangency is Perpendicular lesson

Central and Inscribed Angles, and Arc and Angle Measure Relationships (G.GCI.2*, G.GCO.1*, G.GM.1, G.GM.2, G.GSRT.5*)

a. Central and Inscribed Angles
   MathBitsNotebook: Chords and Arcs
   Open Math Reference: Minor and Major Arcs, Central Angles, Inscribed Angles, Inscribed Right Triangle
   Open Math Reference: Minor and Major Arcs, Central Angles, Inscribed Angles, Inscribed Right Triangle
   RegentsNY: Arcs in Circles
   Virtual Nerd: Inscribed Angle and Intercept Arc Relationship
   Khan Academy: Minor Arc, Central Angle and Arc, and more Central Angle and Arc, Inscribed Angles, and Inscribed Angle Proof
   SchoolYourself: Minor and Major Arcs, Central and Inscribed Angles, Inscribed Right Triangle, Inscribed Quadrilateral, and Parallel Chords
b. Arc and Angle Measure Relationships
   MathBitsNotebook: Tangents, Secants, and Chords angle measure theorems
   Open Math Reference: Angles formed by Two Secants
   Open Math Reference: Angles formed by Two Secants
   RegentsNY: Arcs, Chords, and Tangent Angles
   Virtual Nerd: Arc and Angle Relationship with Intersecting Chords-1 and Arc and Angle Relationship with Intersecting Chords-2
   Virtual Nerd: Inscribed Quadrilateral Theorem
   Khan Academy: Two Tangents from Exterior Point Theorem Proof , Application 1, and Application 2
   SchoolYourself: Two Tangents from Exterior Point

Finding segment lengths of intersecting chords, secants, and tangents (G.GCI.2*, G.GCO.1*, G.GM.1, G.GM.2, G.GSRT.5*)

a. MathBitsNotebook: Chords, Secants, and Tangent Segment Lengths
b. RegentsNY: Chords, Secants, and Tangents Segment Lengths
c. Open Math Reference: Intersecting Chords and Intersecting Secants
d. Open Math Reference: Intersecting Chords and Intersecting Secants
e. SchoolYourself: Intersecting Chords and Intersecting Secants
f. Extension: SchoolYourself: Tangent Lengths and Area Relationships

Circle Similarity (G.GCI.1, G.GSRT.5*)

a. MathBitsNotebook: Proving Similar Circles
b. Khan Academy: Circle Similarity

Constructions (G.GC1.2*, G.GCI.3, G.GCI.4)

a. MathBitsNotebook: Regular Hexagon and Square, Tangent to Circle, and Incenter and Circumcenter
b. Open Math Reference: Regular Hexagon and Square (from Unit 6), and Incenter and Circumcenter
c. Open Math Reference: Tangent to Circle Exterior Point, Tangent to Point On Circle, Circle Through Three Points, and Finding Center of Circle
d. Open Math Reference: Tangent to Circle Exterior Point, Tangent to Point On Circle, Circle Through Three Points, and Finding Center of Circle
e. Math Is Fun: Inscribed and Circumscribed Triangles
f. Math Is Fun: Tangent to Circle Exterior Point, Circle Through Three Points, and Finding Center of Circle
g. Math Is Fun – Extension: Regular Pentagon

Resources
• Cliff Notes Math: Geometry • Emergent Math: Emergent Math • EngageNY: Geometry • Georgia Department of Education, Standards and Teacher Support: Geometry • Grade A Math Help: Geometry • Grade A Math Help: Geometry Resources • Illuminations: Grades 9 - 12 Resources • Khan Academy: Geometry • matematicasVisuales: Geometry • Math Education Page: Geometry Book of Labs • Math Education Page: Sum of the Angles in a Triangle • Math Goodies: Math Goodies • Math Open Reference: Geometry Resources • MathBitsNotebook: Geometry Online Study Resources • Mathematics Assessment Project – Mathematics Assessment Resource Service: Homepage • West Contra Costa Unified School District: Geometry • Virtual Nerd: Geometry Skills Videos

Sample Formative Assessment Tasks/Questions
Standard Form Equation of a Circle (G.GGPE.1*) a. Khan Academy: Circle Equation b. MathBitsNotebook: Circle Equation c. RegentsNY: Circle Equation
Find angle measures and arc degrees within circles (G.GCI.2*, G.GCO.1*, G.GM.1, G.GM.2, G.GSRT.5*) a. MathBitsNotebook: Central and Inscribed Angles and Arcs b. MathBitsNotebook: Tangent and Chord Angles and Arcs c. MathBitsNotebook: Tangent and Secant Angles and Arcs d. RegentsNY: Angles in Circles e. RegentsNY: Angles-Tangents-Chords f. RegentsNY: Tangent and Secant Angles and Arcs
Finding segment lengths of intersecting chords, secants, and tangents (G.GCI.2*, G.GCO.1*, G.GM.1, G.GM.2, G.GSRT.5*) a. EngageNY: Circles Unit b. MathBitsNotebook: Chords, Secants, and Tangent Segment Lengths c. MathBitsNotebook: Tangent Segment Lengths d. RegentsNY: Chords Lengths (with some arc-angle) e. RegentsNY: Chords, Secants, and Tangent Segment Lengths f. RegentsNY: Two Tangents Lengths

Geometry Unit 8 Title
Statistics

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
• G.SPID.1* Select and create an appropriate display, including dot plots, histograms, and box plots, for data that includes only real numbers. • G.SPID.2* Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets that include all real numbers. • G.SPID.3* Summarize and represent data from a single data set. Interpret differences in shape, center, and spread in the context of the data set, accounting for possible effects of extreme data points (outliers).

New Academic Vocabulary for This Unit
• Skew – with respect to the shape of data

Prior Knowledge Required for this Unit

In earlier grades, students have developed conceptual knowledge and have had the opportunity to learn how to:
 	
• Select and create an appropriate display for numerical data, including dot plots, histograms, and box plots. (6.DS.4)
• Describe numerical data sets in relation to their real-world context. (6.DS.5)
    ○ State the sample size.
    ○ Describe the qualitative aspects of the data (e.g., how it was measured, units of measurement).
    ○ Give measures of center (median, mean).
    ○ Find measures of variability (interquartile range, mean absolute deviation) using a number line.
    ○ Describe the overall pattern (shape) of the distribution.
    ○ Justify the choices for measure of center and measure of variability based on the shape of the distribution.
    ○ Describe the impact that inserting or deleting a data point has on the measures of center (median, mean) for a data set.

Subsequent Knowledge Related to this Unit
The standards covered in this unit build the foundation for the standards that will be covered more formally in Probability and Statistics (PS.SPID.1, PS.SPID.2, and PS.SPID.3).

Relationship Among Standards in this Unit
In this unit, students will bring some prior knowledge with them that they will build upon. They will be selecting and creating an appropriate display for data (G.SPID.1*). In addition, they will use appropriate statistics to compare center and spread of two or more different data sets (G.SPID.2*). Students will summarize a set of data by interpreting differences in shape, center, and spread in the context of the data while accounting for the effects of outliers (G.SPID.3*).

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 8.*
Displaying Data (G.SPID.1*) a. VirtualNerd: How to Make a Histogram b. Kahn Academy: How to Interpret a Histogram c. Kahn Academy: Constructing a Box and Whisker Plot d. Kahn Academy: Interpreting a Box and Whisker Plot e. MathNspired: Introduction to Box Plots (for those with TI-Nspire Technology) f. MathBitsNotebook: Representing Data Graphically g. MathBitsNotebook: Histograms h. MathBitsNotebook: Dot Plots i. MathBitsNotebook: Box Plots
InterpretingData (G.SPID.2*, G.SPID.3*) a. MathNspired: Center and Spread (for those with TI-Nspire Technology) b. Kahn Academy: Comparing Means of Distributions c. Kahn Academy: Comparing Means and Medians of Different Distributions d. Kahn Academy: Thinking about Shapes of Distributions e. Kahn Academy: Analyzing Clusters, Gaps, Peaks, and Outliers f. MathBitsNotebook: Shapes of Distributions g. MathBitsNotebook: Measures of Center h. MathBitsNotebook: Measures of Spread **i.** MathBitsNotebook: Comparing Statistical Graphs

Resources
• Georgia Department of Education: Algebra 1 • Illustrative Mathematics: Statistics and Probability • Kahn Academy: Descriptive Statistics • MathBitsNotebook: Data Distributions • MathNspired: Statistics • VirtualNerd: Probability and Data Analysis

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Sample Formative Assessment Tasks/Questions
Displaying and Interpreting Data (G.SPID.1*; G.SPID.2*; G.SPID.3*)
a. Kahn Academy: Interpreting and Comparing Data Distributions
b. Illustrative Mathematics: Speed Trap
c. Illustrative Mathematics: Haircut Costs
d. Kahn Academy: Clusters, Gaps, Peaks, and Outliers Practice
e. Georgia Department of Education: Math Class
f. Georgia Department of Education: The Basketball Star
g. MathBitsNotebook: Practice with Box Plots
h. MathBitsNotebook: Practice Interpreting Graphs

South Carolina College- and Career-Ready Standards for Mathematics
High School Support Document
Probability and Statistics Coversheet

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Categorical Data (Venn Diagrams and Contingency Tables)	Quantitative Data (Graphing and Exploring Univariate Data)	Normal Distribution	Bivariate Data and Scatterplots	Sampling and Simulation Design	Basic Probability Concepts and Applications	Sampling Distributions and Expected Value	Extension Topics
Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
PS.SPCR.1 PS.SPCR.4 PS.SPID.5*	PS.SPID.1* PS.SPID.2* PS.SPID.3*	PS.SPID.4	PS.SPID.6* PS.SPID.7* PS.SPID.8* PS.SPID.9 PS.SPID.10	PS.SPMJ.3 PS.SPMJ.4 PS.SPMJ.5 PS.SPMJ.6	PS.SPCR.2 PS.SPCR.3 PS.SPCR.5 PS.SPCR.6 PS.SPCR.7 PS.SPCR.8 PS.SPMJ.2*	PS.SPMD.1 PS.SPMD.2 PS.SPMD.3 PS.SPMD.4* PS.SPMD.5* PS.SPMD.6* PS.SPMJ.1*	PS.SPMJ.1*

South Carolina College- and Career-Ready Standards for Mathematics
High School Support Document
Probability and Statistics Coversheet

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Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

1. Make sense of problems and persevere in solving them. a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

Probability and Statistics Unit 1 Title
Categorical Data (Venn Diagrams and Contingency Tables)

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
● PS.SPCR.1 Describe events as subsets of a sample space and a. Use Venn diagrams to represent intersections, unions, and complements. ○ Represent intersections, unions and complements using venn diagrams. b. Relate intersections, unions, and complements to the words and, or, and not. ○ Translate between verbal representation and symbolic form (e.g. $A \cap B$, A and B, A intersected with B; $A \cup B$, A or B, A union B; $A \cap B'$, $A \cap B^C$, A and not B, A intersect B complement) c. Represent sample spaces for compound events using Venn diagrams. ○ Shade two (and three for extension) part venn diagrams for subsets given in verbal and symbolic form of intersections, unions and complements. ○ Translate between venn diagrams, symbolic form and set notation. ● PS.SPCR.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. ○ Construct two-way frequency (contingency) tables. ○ Interpret two-way frequency (contingency) tables. ○ Use the two-way table as a sample space to decide if events are independent. ○ Use the two-way table as a sample space to approximate conditional probabilities/proportions. ● PS.SPID.5 Analyze bivariate categorical data using two-way tables and identify possible associations between the two categories using marginal, joint, and conditional frequencies. ○ e.g., If using variables gender (male/female) and grade (11/12), a marginal frequency could be the total number of females, or the total number of students in grade 11. These values can be found on the perimeter of the two-way table. ○ e.g., If using the variables gender (male/female) and grade (11/12), a joint frequency could be the number of females in grade 12. ○ e.g., If using the variables gender (male/female) and grade (11/12), a conditional frequency could be the number of students in grade 11, given that they are male.

New Academic Vocabulary for This Unit
• Complement • Compound Events • Conditional Frequency • Conditional Probability • Contingency Table • Dependent • Independent • Intersection • Joint Frequency • Marginal Frequency • Sample Space • Two-way Frequency Table • Union • Venn Diagram

Probability and Statistics Unit 1: Categorical Data (Venn Diagrams and Contingency Tables)

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Prior Knowledge Required for this Unit
None

Subsequent Knowledge Related to this Unit
• Recognize and explain conditional and independent events (PS.SPCR.5) • Calculate probability for combined events, independent and dependent events (PS.SPCR.2, PS.SPCR.7) • Calculate conditional probability (PS.SPCR.3, PS.SPCR.6) • Apply counting principles and use to calculate probability (PS.SPCR.8)

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Relationship Among Standards in this Unit
The standards in Probability and Statistics Unit 1 are organized to introduce students to the study of statistics and organizing categorical data. Students begin this unit by organizing data in venn diagrams and two-way (contingency) tables. Students describe subsets of the sample space in multiple forms, including verbal, symbolic, set notation, as well as by shading appropriate regions on diagrams. This unit prepares students to study quantitative data in the next unit and their eventual study of probability.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 1.*
Describing and Organizing Events and Subsets (PS.SPCR.1) a. AMSTAT: First Day Stats Activity (Organizing Data) b. Interactivate: Introduction to Sets & Venn Diagrams c. Better Lesson: Leading Group Discussion on Describing Data d. Illustrative Mathematics: Describing Data e. Better Lesson: Intersection and Union f. Better Lesson: Subsets g. Better Lesson: Complements h. Better Lesson: Venn Diagram i. Geogebra: Shading Venn Diagram
Using Two-Way Tables to Analyze Data (PS.SPCR.4, PS.SPID.5) a. Better Lesson: Creating Two-Way Tables b. Better Lesson: Interpreting Two-Way Tables c. Stat Crunch: Contingency Table Activity d. Causeweb.org: Categorical Data Activity e. AMSTAT: A Sweet Task (Create and Interpret Frequency Tables) f. AMSTAT: Classifying Fortune Cookie Fortunes (Analyzing Categorical Data) g. AMSTAT: Using Data About the Donner Party to Illustrate Descriptive Statistics

Resources

- Study.com: Marginal, Joint, & Conditional Video/Lesson
- NY Regents Prep: Sets and Venn Diagrams
- Kahn Academy: Intersection and Union of Sets
- SSDAN: Contingency Table Powerpoint
- Stattrek: Two-way Tables

Sample Formative Assessment Tasks/Questions
a. Illustrative Mathematics: Two-Way Tables & Relative Frequency b. Illustrative Mathematics: Two-Way Tables & Relative Frequency

Probability and Statistics Unit 2 Title
Quantitative Data (Graphing and Exploring Univariate Data)

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- PS.SPID.1* Select and create an appropriate display, including dot plots, histograms, and box plots, for data that includes only real numbers.
  - Create graphical representations of data sets using dot plots, histograms and box plots.
- PS.SPID.2* Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets that include all real numbers.
  - Determine 5-number summary (minimum, Q_1, median, Q_3, maximum).
  - Calculate range and interquartile range (IQR).
  - Calculate mean and standard deviation.
  - Determine the best representation of center (mean or median) based on symmetry, outliers.
  - Determine the best representation for spread (standard deviation or IQR) based on measures of center used.
  - Identify mode.
- PS.SPID.3* Summarize and represent data from a single data set. Interpret differences in shape, center, and spread in the context of the data set, accounting for possible effects of extreme data points (outliers).
  - Determine the relationship between mean and median based on the shape of the data.
  - Calculate fences to determine outlier cutoffs for box plots.
  - Modify box plots to represent outliers.
  - Identify corresponding representations of data (box plots, histograms, measures of center and spread).
  - Identify modality of a histogram.

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New Academic Vocabulary for This Unit
• Boxplot • Dot Plot • Modality • Outlier

Prior Knowledge Required for this Unit
None

Subsequent Knowledge Related to this Unit
• Normal distribution(PS.SPID.4) • Analyzing bivariate data (PS.SPID.5) • Expected value (PS.SPMD.2)

Return to High School Overview, High School Table of Contents, or Probability and Statistics Coversheet.

Relationship Among Standards in this Unit
The standards in Probability and Statistics Unit 2 are organized to introduce students to working with quantitative data. Students begin this unit by organizing data in dot plots, histograms, and boxplots. Students use modality, symmetry and outliers to describe the plots. Students calculate and use appropriate measures of center (mean and median) and measures of spread (IQR, range, standard deviation) based on symmetry and outliers. Students learn how to calculate fences in order to identify outliers.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 2.*
Represent data using dot plots, histograms and boxplots and describe the shape of the data set (PS.SPID.1*) a. NCTM: Advanced Data Grapher b. NCTM: Histogram Tool c. NCTM: Investigating Mean and Median with Boxplots d. Illustrative Mathematics: Represent and Compare Data using Boxplots e. Wiley: Matching Histograms and Boxplots - Online Activity f. Better Lesson: Analyze Boxplots g. Better Lesson: Connecting Boxplots with Histograms h. CPalms.org: Formative Assessment, Representing Data i. CPalms.org: Haircut Costs, Representing Data with Boxplots j. Actuarial Foundation: Module 1 - Full Lesson k. AMSTAT: Using SeeIt to Compare Dotplots and Boxplots
Analyze measures of central tendency (PS.SPID.2*) a. Regents Prep: Fruit Loops Activity b. Better Lesson: Measures of central tendency
Analyze measures of spread and interpret outliers (PS.SPID.2*, PS.SPID.3*) a. Illustrative Mathematics: Analyzing Boxplots and Standard Deviation b. Illustrative Mathematics: Measuring Variability c. Better Lesson: The Stroop Effect d. Better Lesson: Identifying Outliers and Describing Their Effect e. Texas Instruments: Analyze Shape, Measures of Center and Spread f. Share My Lesson: Matching Card Game (Measures of Shape, Center and Spread) g. NASA: Dusty Dilemma Guide

Resources
• Prentice Hall: Analyzing Measures of Central Tendency • Regents Prep: Analyzing Data Activity • Khan Academy: Descriptive Statistics • FloridaSchoolLeaders.org: Descriptive Statistics • Master Math Mentor: Descriptive Statistics Unit • Stattrek: Data Patterns • Stattrek: Dotplots • Stattrek: Histograms • Stattrek: Boxplots • Stattrek: Comparing Plots

Sample Formative Assessment Tasks/Questions
a. Illustrative Mathematics: Understanding Standard Deviation b. Illustrative Mathematics: Representing Data and Analyzing Measures of Center c. Better Lesson: Writing Assignment (Using Data to Construct an Argument) d. CPalms.org: Univariate and Bivariate Data Assessments e. NASA: Team Extreme Experiment

Probability and Statistics Unit 3 Title
Normal Distribution

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
● PS.SPID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ○ Given the mean, standard deviation and a specific data value determine the corresponding z-score on the standard normal distribution curve. ○ Given the mean, standard deviation and z-score, determine the corresponding data value. ○ Determine the area or percentage of data values between two values on a normal distribution curve using a graphing calculator (normalcdf), spreadsheet or standard normal distribution table. ○ When given the area or percentage of data values on the normal distribution curve, determine the corresponding cut-off(s) using a graphing calculator (invNorm), spreadsheet, or standard normal distribution table.

New Academic Vocabulary for This Unit
• Normal Distribution • Standard Normal Distribution Curve • Z-score

Prior Knowledge Required for this Unit
• Mean (PS.SPID.2) • Standard deviation (PS.SPID.2) • Describing shape of data (unimodal, symmetric) (PS.SPID.3)

Subsequent Knowledge Related to this Unit
• Calculating Probability of Compound Events (PS.SPCR.2) • Probability Distributions (PS.SPMJ.1)

Relationship Among Standards in this Unit
The standards in Probability and Statistics Unit 3 are organized to introduce students to the normal distribution. At the beginning of this unit students identify characteristics of the normal distribution, specifically the standard normal distribution curve. Students will convert between data that is normally distributed and the standard normal distribution curve by finding z-scores. Finally, students will use technology, spreadsheets, and tables to determine the proportion of data values that lie within an interval. The same tools are used to determine the cut-off data value for given percentiles.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 3.*
Determine what it means for data to be normally distributed. Discuss characteristics of the normal distribution and determine whether data sets are normally distributed (PS.SPID.4). a. Better Lesson: Introduction to the Normal Distribution b. Shodor.org: Interactive Normal Distribution c. AMSTAT: What Does the Normal Distribution Sound Like (Collect and Analyze Data) d. Actuarial Foundation: Module 2
Convert between data sets that are normally distributed and the corresponding z-scores and area on the standard normal distribution. Teachers may wish to introduce the empirical rule to estimate area prior to using tables and technology (PS.SPID.4). a. Better Lesson: Exploring Standard Deviation b. Khan Academy: The Normal Distribution
Use the standard normal distribution to determine the proportion of data values within a given interval and data values that enclose a given proportion or area (PS.SPID.4). a. Math = Love Blog: Normal Distribution Cards b. Better Lesson: Z-Scores and the Normal Distribution c. Illustrative Mathematics: Analyzing SAT Scores d. Regents Prep: Analyzing Sleep Using the Normal Distribution e. Virginia Department of Education: Analyzing and Using the Standard Normal Curve f. Maryland Department of Education: Introduce Normal Distribution and the Empirical Rule g. National Security Agency: M&M Activity

Resources
• Math Bits: The Normal Distribution in TI Calculator • Minitab: The Normal Distribution • Autograph-maths.com: Idea 3 (Standardizing the Normal Distribution) • Core Plus Mathematics: The Normal Distribution • College Preparatory Mathematics: The Normal Distribution Supplementary Materials • Master Math Mentor: Descriptive Statistics Unit • Stattrek: Normal Distribution • Stattrek: Standard Normal

Sample Formative Assessment Tasks/Questions
a. Illustrative Mathematics: Analyzing Test Scores b. Illustrative Mathematics: Do You Fit In This Car? c. Radford.edu: Collect and Analyze Data

Probability and Statistics Unit 4 Title
Bivariate Data and Scatterplots

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*

• PS.SPID.6* Using technology, create scatterplots and analyze those plots to compare the fit of linear, quadratic, or exponential models to a given data set. Select the appropriate model, fit a function to the data set, and use the function to solve problems in the context of the data.
    ○ Use technology to create scatterplots.
    ○ Analyze the shape of scatterplots to determine if a linear, quadratic, or exponential model is appropriate.
    ○ Use technology to create a regression equation.
    ○ Use the regression equation to make predictions (interpolate and extrapolate)
• PS.SPID.7* Find linear models using median fit and regression methods to make predictions. Interpret the slope and intercept of a linear model in the context of the data.
    ○ Use technology to find least squares regression line (LinReg (a+bx) or (ax+b) in TI graphing calculator).
    ○ Use technology to find median fit regression line (Med-Med in TI graphing calculator).
    ○ Interpret the slope in context, using appropriate units.
    ○ Interpret the y-intercept in context, using appropriate units.
• PS.SPID.8* Using technology, compute and interpret the correlation coefficient of a linear fit.
    ○ Calculate r, the correlation coefficient, using technology when finding the linear regression equation. Note that on a TI calculator students will need to turn their diagnostics on (2nd:0, scroll down to find DiagnosticOn, press enter twice).
    ○ Interpret the value of r to determine the strength (weak, moderate, strong) and direction (positive, negative) of the linear relationship.
• PS.SPID.9 Differentiate between correlation and causation when describing the relationship between two variables. Identify potential lurking variables which may explain an association between two variables.
    ○ Provide examples of pairs of quantitative variables that may be correlated, but that are clearly not cause and effect (e.g. ice cream sales and crime rates in June).
    ○ Identify lurking variables which may explain why two correlated variables do not imply causation (e.g. higher temperatures, less accountability for adolescents in the summer time).
• PS.SPID.10 Create residual plots and analyze those plots to compare the fit of linear, quadratic, and exponential models to a given data set. Select the appropriate model and use it for interpolation.
    ○ Create residual plot (with or without technology) to evaluate the difference between predicted and actual values.
    ○ Analyze residual plots to determine the most effective regression model (linear, quadratic or exponential).

New Academic Vocabulary for This Unit
• Causation • Correlation • Extrapolate • Interpolate • Lurking Variable • Regression • Residual plot

Prior Knowledge Required for this Unit
• Scatterplot (A1.SPID.6) • Linear Functions (A1.SPID.6, A1.SPID.7, A1.FLQE.1, A1.FIF.4) • Exponential Functions (A1.SPID.6, A1.FLQE.1, A1.FIF.4) • Quadratic Functions (A1.SPID.6, A1.FIF.4) • Solving Equations (A1.ACE.1, A1.SPID.6) • Evaluating Functions (A1.ACE.1, A1.SPID.6)

Subsequent Knowledge Related to this Unit
The standards in this unit are not addressed in subsequent units.

Relationship Among Standards in this Unit
The standards in Probability and Statistics Unit 4 are organized to introduce students to analyzing bivariate data. This unit begins using technology to create scatterplots and multiple linear regression models. The remainder of this unit focuses on interpreting the slope and y-intercept of the model in context of the data, using the model to make predictions, and interpreting the strength of the model using the correlation coefficient. Finally, students will create quadratic and exponential regression models for the data, using residual plots to identify the best model.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 4.*
Graph Data and Create Linear Model (PS.SPID.6*, PS.SPID.7*) a. NCTM: Exploring Linear Data b. Annenberg Learner: Video - Fitting Lines to Data c. NCTM: Barbie Bungee d. Annenberg Learner: Video - Scatterplots
Interpret Linear Model in Context of Data and Analyze the Strength of the Linear Model (PS.SPID.7* , PS.SPID.8*). a. NCTM: Interpreting Model, Evaluating Strength of Model b. Illustrative Mathematics: Five Tasks Analyzing Bivariate c. NCTM: Bathtub Water Levels d. Better Lesson: Analyzing Residuals Using Online Applet e. Better Lesson: Explore Correlation on Gapminder f. Better Lesson: Correlation vs Causation g. Better Lesson: Least Square Regression and Interpreting Linear Models h. AMSTAT: Analyze Text Message Data i. AMSTAT: The United States of Obesity j. AMSTAT: Analyzing NFL Quarterback Salaries k. Actuarial Foundation: Module 4
Analyze Alternate Regression Models (PS.SPID.10) a. NCTM: Shrinking Candles, Running Water, Folding Boxes b. Better Lesson: Modeling with Exponential Functions c. MrsKrummel.com: Non Linear Regression Techniques

Resources
• Glencoe: Finding Median Fit of Regression Line on Calculator • Engageny: New York Curriculum Module • Gapminder: Data Sets, Motivating Videos, Data Representation Software • Master Math Mentor: Regression Unit • Stattrek: Correlation • Stattrek: Linear Regression • Stattrek: Residuals

Sample Formative Assessment Tasks/Questions
a. Illustrative Mathematics: Interpreting Linear Model (Texting & Grades) b. Illustrative Mathematics: Interpreting Linear Model (Subaru) c. Illustrative Mathematics: Create, Interpret & Analyze Linear Model (Coffee & Crime) d. Illustrative Mathematics: Represent Bivariate Data & Interpret Linear Model (100m Dash) e. Illustrative Mathematics: Represent Bivariate Data & Interpret Relationship (Restaurant Bill) f. Better Lesson: Performance Task (The Big Mac Index) g. NASA: Spacewalk Training

Probability and Statistics Unit 5 Title
Sampling and Simulation Design

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- PS.SPMJ.3 Plan and conduct a survey to answer a statistical question. Recognize how the plan addresses sampling technique, randomization, measurement of experimental error and methods to reduce bias.
    - ○ Contrast and compare different sampling techniques (Simple Random, Cluster, Systematic, Stratified, Convenience).
        - How each sampling method is conducted?
        - What are the similarities and differences among the different sampling technique?
        - How does each sampling technique address randomization?
        - What opportunities exist for bias within each sampling technique?
        - How might experimental errors occur within each sampling technique?
    - ○ Plan and conduct a survey
        - Identify an appropriate sampling technique and discuss:
            - ○ Randomization.
            - ○ Potentials for bias.
            - ○ Potentials for experimental error and how these could be measured.
        - Conduct a survey
- PS.SPMJ.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
    - ○ Use data from a sample survey to estimate a population mean or proportion.
    - ○ Use simulation models for random sampling to develop a margin of error.
- PS.SPMJ.5 Distinguish between experiments and observational studies. Determine which of two or more possible experimental designs will best answer a given research question and justify the choice based on statistical significance.
    - ○ Define observational studies and when they are appropriate.
    - ○ Define experimental studies and when they are appropriate.
    - ○ Compare and contrast observational and experimental studies.
    - ○ Given a research question, determine which of the experimental designs may work best and be able to justify the choice based on statistical significance.
- PS.SPMJ.6 Evaluate claims and conclusions in published reports or articles based on data by analyzing study design and the collection, analysis, and display of the data.
    - ○ Examine the use of data in a published report/article utilizes data.

- What sampling technique was used?
- What types of bias or experimental error might exist in the study?
- How is the data displayed and what do you think the author(s) wants readers to believe based on the display?

New Academic Vocabulary for This Unit
• Bias • Blinding • Census • Cluster Sampling • Convenience Sampling • Double Blinding • Experimental Error • Experimental Study • Margin of Error • Observational Study • Population • Random • Randomization • Sample • Sampling Technique • Simple Random Sample • Simulation • Statistical Significance • Stratified Sampling • Survey • Systematic Sample

Prior Knowledge Required for this Unit
• Mean (PS.SPID.2) • Standard Deviation (PS.SPID.2)

Subsequent Knowledge Related to this Unit
● Analyze Decisions and Strategies (PS.SPMD.6)

Probability and Statistics Unit 5: Sampling and Simulation Design

Return to High School Overview, High School Table of Contents, or Probability and Statistics Coversheet.

Relationship Among Standards in this Unit
The standards in Probability and Statistics Unit 5 are organized to introduce students to constructing and conducting surveys, determining how to obtain a sample from a given population, and the differences between observational and experimental studies. This unit also integrates the use of means and proportions. After working through standards PS.SPMJ.3, PS.SPMJ.4, and PS.SPMJ.5, the students should be able to analyze data presented in published articles.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 5.*
Randomization, Bias and Experimental Error (PS.SPMJ.3) a. Better Lesson: Biased Versus Unbiased Samples
Sampling Techniques (PS.SPMJ.3) a. Better Lesson: Sampling Simplified b. Kahn Academy: Reasonable Samples c. AMSTAT: Sampling in Archaeology (Multiple Sampling Techniques) d. AMSTAT: Types of Average and Sampling
Developing Questions & Conducting a Survey (PS.SPMJ.3) a. AMSTAT: Chocolicious
Experiments versus Observational Studies (PS.SPMJ.5) a. Better Lesson: Statistical Studies
Estimate Population Mean, Proportion and find the Margin of Error (PS.SPMJ.4) a. Kahn Academy: Population Mean b. AMSTAT: How Wet is the Earth
Evaluate Published Reports (PS.SPMJ.6) a. AMSTAT: Consuming Cola b. Kahn Academy: Valid Claims

Resources
• Stattrek: Margin of Error • Stattrek: Simple Random Sampling • Stattrek: Stratified Sampling • Master Math Mentor: Experimental Design Unit

Sample Formative Assessment Tasks/Questions
a. Illustrative Mathematics: Musical Preferences b. Illustrative Mathematics: Strict Parents c. Illustrative Mathematics: High Blood Pressure d. Illustrative Mathematics: Body Temperature

Probability and Statistics Unit 6 Title
Basic Probability Concepts & Applications

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- PS.SPCR.2 Use the multiplication rule to calculate probabilities for independent and dependent events. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
  - $P(B \cap A)$ is read as the probability of B, given A.
  - Two events are independent if $P(A \cap B) = P(A) \cdot P(B)$.
  - If two events are dependent, $P(A \cap B) = P(A) \cdot P(B|A)$.
  - Find probability of a sequence of events occurring, such as the probability of drawing two specific cards from a deck of cards.
  - Highlight circumstances that could change two events from being independent to dependent (e.g. finding the probability of drawing 2 cards of the same suit from a standard deck of card when the cards are replaced after each draw and not replaced after each draw).
  - Could use tree diagrams to represent independent and dependent events and the sample space.
- PS.SPCR.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
  - If two events are dependent, $P(B \cap A) = P(B) \cdot P(A|B)$, thus it follows $P(A|B) = P(A \cap B)/P(B)$.
  - Two events A and B are independent if $P(B) = P(B|A)$, and $P(A) = P(A|B)$.
- PS.SPCR.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
  - Determine whether daily events are independent or dependent (e.g. likelihood of rain and likelihood of going to the beach; likelihood of rain and likelihood of a pop quiz).
- PS.SPCR.6 Calculate the conditional probability of an event A given event B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
  - $P(A|B)$=$P(A \cap B)/P(B)$
  - If not addressed in Unit 1, use two-way tables and venn diagrams to find conditional probability.
  - When using two-way tables and venn diagrams calculate the conditional probability with and without the formula. Highlight cases when the formula given above is necessary based on the given information.
- PS.SPCR.7 Apply the Addition Rule and the Multiplication Rule to determine probabilities, including conditional probabilities, and interpret the results in terms of the probability model.
  - Addition Rule if events A and B are mutually exclusive: $P(A \cup B) = P(A) + P(B)$
  - Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- Multiplication Rules: $P(A \cap B) = P(A) \cdot P(B)$ (independent events) or $P(A \cap B) = P(A) \cdot P(B|A)$ (dependent events).

- PS.SPCR.8 Use permutations and combinations to solve mathematical and real-world problems, including determining probabilities of compound events. Justify the results.
  - Combination (when order does not matter): ${}_nC_r = \frac{n!}{r!(n-r)!}$
  - Permutation (when the order matters): ${}_nP_r = \frac{n!}{(n-r)!}$
  - Use counting principles to determine the number of ways you could form specific subsets from a larger group.
  - Use counting principles to determine the probability of forming specific subsets (e.g. A club consists of 20 juniors and 30 seniors. Seven students are going to be randomly selected to form a committee. Find the probability that this committee consists of 4 juniors and 3 seniors.)
- PS.SPMJ.2*Distinguish between experimental and theoretical probabilities. Collect data on a chance event and use the relative frequency to estimate the theoretical probability of that event. Determine whether a given probability model is consistent with experimental results.
  - Experimental probability is the ratio of the number of times an event occurs to the total number of trials or times the activity is performed. This is the probability that results from observation of a sample.
  - Theoretical probability is the number of ways that the event can occur, divided by the total number of outcomes. This probability is the understood or believed probability for the population.
  - Collect data on the number of times an event occurs out after a series of trials. Calculate the resulting experimental probability and compare to the theoretical probability.

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New Academic Vocabulary for This Unit
• Addition Rule • Combination • Compound Event • Conditional Probability • Dependent • Experimental Probability • Factorial • Independent • Multiplication rule • Permutation • Theoretical Probability • Tree Diagram

Prior Knowledge Required for this Unit
• Intersection (PS.SPCR.1) • Probability of Simple and Compound Events (7.DSP.5 - 8) • Marginal Frequency (PS.SPID.5) • Joint Frequency (PS.SPID.5) • Conditional Frequency (PS.SPID.5) • Venn Diagrams (PS.SPCR.1) • Two-Way Tables/Contingency Tables (PS.SPCR.4)

Subsequent Knowledge Related to this Unit
• Probability Distributions (PS.SPMD.1) • Expected Value (PS.SPMD.2) • Binomial Distribution (PS.SPMJ.3) • Normal Distribution (PS.SPID.4)

Relationship Among Standards in this Unit
The standards in Probability and Statistics Unit 6 are organized to introduce students to finding both theoretical and experimental probability. This unit begins by introducing students to finding the probability of simple and combined events using the addition and multiplication rule. This prepares students for finding conditional probability and determining whether events are independent. Students may use tree-diagrams, two-way (contingency) tables, or venn diagrams as an organizational and visual tool. Finally, students will extend on these topics by using counting principles to determine the probability of forming specific subsets of a population. This prepares students for studying probability distributions in the subsequent unit.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 6.*
Distinguish Between Experimental and Theoretical Probabilities (PS.SPMJ.2) a. Khan Academy: Comparing Theoretical to Experimental Probabilities b. Algebra-Class:Theoretical Probability versus Experimental Probability c. Regents Prep: Theoretical vs. Empirical Probability d. Online Math Learning: Theoretical Probability and Experimental Probability e. NCTM: Analyze the Relationship Between Experimental and Theoretical Probabilities
Determining Probability (PS.SPCR.2, PS.SPCR.7) a. Regents Prep: Hitting the Target Activity b. Better Lesson: Monty Hall Problem c. Better Lesson: Mixed Probability Problem Set d. Illustrative Mathematics: Three Addition Rule Tasks e. Illustrative Mathematics: Multiplication Rule Task f. Mathwire: Tossing 1 Die (multiple activities) g. Bull Head Schools: Probability Foldable
Conditional Probability (PS.SPCR.3, PS.SPCR.5, PS.SPCR.6) a. Better Lesson: Introduction to Conditional Probability b. Better Lesson: Conditional Probability & Independence Activity c. Illustrative Mathematics: Four Conditional Probability Tasks d. NCTM: Explorations with Chance, Analyzing Fairness
Counting Principles (PS.SPCR.8) a. Better Lesson: Permutations Practice b. Better Lesson: Counting Principles c. Better Lesson: Using Counting Principles to Find Probability d. Illustrative Mathematics: Four Counting Principles Tasks

Resources
● Khan Academy: Making Predictions with Probability ● Khan Academy: Constructing Probability Models from Data ● Illustrative Mathematics: Activities, Performance Tasks ● Henrico County Public Schools, Virginia: Probability Resources ● Master Math Mentor: Probability Unit

Sample Formative Assessment Tasks/Questions
a. Better Lesson: Introduction to Probability Formative Assessment b. Better Lesson: Probability Performance Task (Analyzing the Lottery) c. Better Lesson: Mixed Probability Problem Set d. Illustrative Mathematics: Four Conditional Probability Tasks e. Illustrative Mathematics: Three Addition Rule Tasks f. Illustrative Mathematics: Multiplication Rule Task g. Illustrative Mathematics: Four Counting Principles Tasks

Probability and Statistics Unit 7 Title
Sampling Distributions & Expected Value

Content Standards with Clarifying Notes
Open bullets indicate clarifying notes.

- PS.SPMD.1 Develop the probability distribution for a random variable defined for a sample space in which a theoretical probability can be calculated and graph the distribution.
  - Construct a probability model/distribution based on theoretical probabilities. This could be in the form of any table, graph, or formula that gives each possible value and the probability of that value.
  - Graph the probability distribution (Random variable on x-axis. Corresponding probability on y-axis. Discrete random variables are typically shown as histograms.)
- PS.SPMD.2 Calculate the expected value of a random variable as the mean of its probability distribution. Find expected values by assigning probabilities to payoff values. Use expected values to evaluate and compare strategies in real-world scenarios.
  - Calculate the expected value of a random variable as the mean of its probability distribution.
    - $E(x) = \Sigma x P(x)$
  - Relate expected value formula to finding the mean of data arranged in a frequency table to strengthen student understanding.
  - Compare expected values for different games of chance to determine which games should be played and which games have high and low payoffs.
- PS.SPMD.3 Construct and compare theoretical and experimental probability distributions and use those distributions to find expected values.
  - Construct experimental probability distributions using either sample data or trials.
  - Compare the experimental probability distribution to the theoretical distribution.
  - Determine the expected value of both distributions.
- PS.SPMD.4* Use probability to evaluate outcomes of decisions by finding expected values and determine if decisions are fair.
  - Weigh possible outcomes of a decision by assigning probabilities to payoff values and finding the expected value.
  - Find the expected payoff for a game of chance (e.g. find the expected winnings from buying a lottery ticket).
  - Determine whether events are fair by analyzing the expected value.
- PS.SPMD.5* Use probability to evaluate outcomes of decisions. Use probabilities to make fair decisions.
  - Evaluate and compare strategies on the basis of expected value (e.g. compare high and low deductible auto insurance policies using various chances of having an accident).
  - Determine whether events or games are fair by calculating the expected value.
  - Determine conditions needed to make events or games considered fair.

- PS.SPMD.6* Analyze decisions and strategies using probability concepts.
  - Evaluate data to determine whether or not the best decision was made.
  - Consider all available strategies to determine and defend a strategy to recommend.
  - Critique the decision-making process before, during, and after an event's occurrence. This could include critiquing an article, medical testing, product testing, coaching decisions made in athletic events, etc.
- PS.SPMJ.1* Understand statistics and sampling distributions as a process for making inferences about population parameters based on a random sample from that population.
  - Recognize that the sample distribution is a subset of the population, such that it is sizeable enough to make inferences about ultimate parameters of the population.
  - Use sample statistics to make predictions for the population. This could include predicting the number of green M&M's produced by Mars Chocolate based on the number of green M&M's found in samples.
  - Confidence Intervals and Hypothesis Testing is saved for Unit 8 as an extension unit.

New Academic Vocabulary for This Unit
• Expected Value • Fair Game/Decision • Probability Distribution • Random Variable

Prior Knowledge Required for this Unit
• Understand what probability means (7.DSP.5 - 7.DSP.8 and Unit 6) • Understand what a sample space is (7.DSP.8, PS.SPCR.1) • Distinguish between theoretical and experimental probability (7.DSP.8, PS.SPMJ.2*) • Understand what it means for data to be normally distributed and how to relate this data to the standard normal distribution (PS.SPID.4)

Subsequent Knowledge Related to this Unit
● Unit 8 includes instruction on confidence intervals and hypothesis testing, which will further address PS.SPMJ.1.

Relationship Among Standards in this Unit
The standards in Probability and Statistics Unit 7 are organized to introduce students to sampling distributions and expected values. This unit begins by introducing students to probability distributions of theoretical probabilities for discrete random variables. The students then calculate expected values based on these distributions. After exploring theoretical probability distributions, students compare experimental probability to the corresponding theoretical probability. As the concluding topics for this course, students use probability to evaluate decisions and to use sample data while making inferences about the population.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 7.*
Probability Distributions (PS.SPMD.1) a. AMSTAT: Using Dice to Introduce Sampling Distribution b. Kahn Academy: Discrete Probability Distributions c. Kahn Academy: Probability Distribution Activity
Expected Value (PS.SPMD.2, PS.SPMD.3, PS.SPMD.4, PS.SPMD.5) a. Better Lesson: Probability Distribution & Expected Value b. NCTM: Explorations With Chance: Is It Fair
Use Sample Data to Make Inferences (PS.SPMJ.1) a. Illustrative Mathematics: Use Probability to Make Decisions b. AMSTAT: Odd or Even c. Actuarial Foundation: Module 3
Use Probability to Make Decisions (PS.SPMD.6) a. NCTM: Stick or Switch (Iteration of the Monty Hall problem) b. Better Lesson: Game Project - Playing Against All Odds

Resources
• Annenberg Learner Video - Probability Models • Stattrek: Probability Distributions

Sample Formative Assessment Tasks/Questions
a. AMSTAT: Odd or Even page 7 b. Illustrative Mathematics: Bagel Shop c. Illustrative Mathematics: Fred's Fun Factory

Probability and Statistics Unit 8 Title
Extension Topics

Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Possible Instructional Strategies/Lessons
New Academic Vocabulary for This Unit	Subsequent Knowledge Related to this Unit	Resources
	Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

Probability and Statistics Unit 8: Extension Topics

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Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
● SPMJ.1* Understand statistics and sampling distributions as a process for making inferences about population parameters based on a random sample from that population. (Extension from Unit 7) ○ Construct, describe and use confidence intervals to make inferences about population parameters. ○ Hypothesis testing • Formulate the null hypothesis. • Identify the test statistic to be used. • Compute the p-value. • Compare the p-value to the significance value.

New Academic Vocabulary for This Unit
• Confidence Interval • Hypothesis Testing • Null Hypothesis • P-value • Significance Value

Probability and Statistics Unit 8: Extension Topics

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Prior Knowledge Required for this Unit
● Cumulative Course Standards

Subsequent Knowledge Related to this Unit
The standards in this unit are not addressed in subsequent units.

Relationship Among Standards in this Unit
This unit is optional and serves as an extension to the standards covered throughout the course. Students can be given the opportunity to construct confidence intervals, develop and test hypothesis, work with p-values and explore chi-square testing.

Possible Instructional Strategies/Lessons *The order of the topics below illustrates a possible instructional order for Unit 8.*
Confidence Intervals and Hypothesis Testing a. MrsKrummel.com: Introducing Margin of Error b. Kahn Academy: Confidence Intervals c. Brownmath: M&M Confidence Interval Activity d. University of California: Chips Ahoy Hypothesis Testing e. NASA: A Human Factor of Space Flight f. AMSTAT: An AMazing Comparison (Create and Conduct a Hypothesis Test)
Chi-Squared Test for Categorical Data a. AMSTAT: The Case of the Careless Zoo Keeper b. Online Stat Book: Chi Square c. AMSTAT: The Psychic Staring Effect (Four Components of Statistical Problem Solving)

Resources
• Stattrek: Hypothesis Testing • Stattrek: How to Test Hypothesis • Stattrek: Chi-Square • Master Math Mentor: Inference Unit

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Sample Formative Assessment Tasks/Questions
a. AMSTAT: The Blob b. AMSTAT: Walk the Line c. MrsKrummel.com: Final Inference Project

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