Education Analytics INC.

Technical Report on the South Carolina Teacher Value-Added Model

Academic Year 2018-2019

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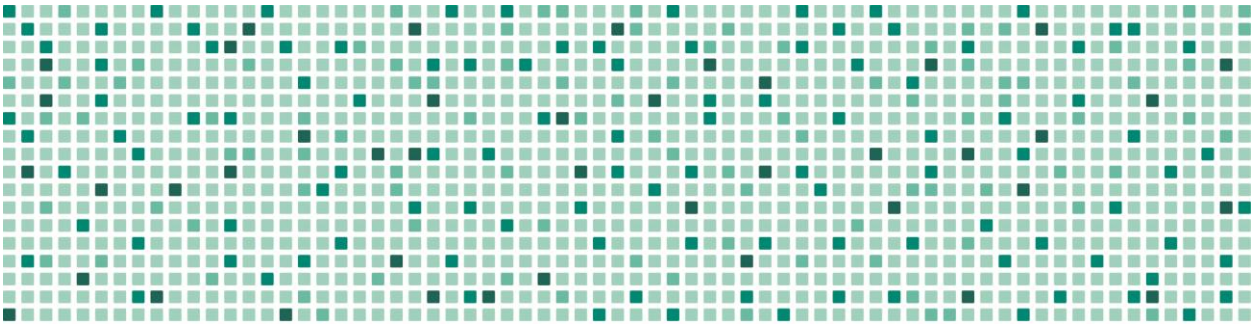

CONTENTS

INTRODUCTION

This report describes the value-added model used by Education Analytics to measure student growth at the teacher level in South Carolina public schools using South Carolina College-and-Career-Ready Assessments (SC READY) test score data. The report is divided into two sections. The first section describes the data set used to produce the value-added estimates. The second section describes the model used to estimate value-added for teachers in South Carolina and presents some properties of the value-added results.

Conceptually, value-added analysis is the use of statistical techniques to isolate the component of measured student knowledge that is attributable to teachers. In practice, value-added models focus on the improvement students make on annual assessments from one year and grade to the next.

The model used in South Carolina for teacher value-added controls for up to two years of prior student achievement in English language arts and mathematics, as well as for a selection of student demographic characteristics. It also controls for average prior student achievement in English language arts and mathematics and average demographic characteristics among students associated with the same group of teachers.

ANALYSIS DATA SET

Before estimation can take place, a substantial amount of work is required to assemble the analysis data sets used to produce the value-added estimates. A separate analysis data set is produced for each grade and subject. In total, ten analysis data sets are produced, covering grades 4 through 8 for SC READY English language arts (ELA) and math in 2018-19.

Each analysis data set includes students who have a posttest in the grade and subject being considered, who have at least one year of pretests in both ELA and math, and who were tested in consecutive grades.

STUDENT-LEVEL VARIABLES

ASSESSMENTS

The test scores used are from the 2016-17, 2017-18 and 2018-19 SC READY assessments. The value-added system produces teacher-level measures for grades 4 through 8 in ELA and math based on performance on the 2018-19 SC READY. The 2018-19 value-added model in ELA uses the 2018-19 ELA score as the posttest, while the 2018-19 value-added model in math uses the

2018-19 math score as the posttest. All value-added models include pretests in both ELA and math. The assessment scores were used to produce overall value-added measures in both math and ELA for each teacher and for each combination of teachers with students in common.

All test scores were linearly normalized to have a mean equal to zero and standard deviation equal to one by grade and subject. Thus, in the value-added analyses, all test scores were measured relative to the state mean, and in units of the statewide standard deviation of test scores across students by grade and subject. The normalization is used to make it easier to interpret estimated coefficients in the value-added models, but it does not affect the statistical properties of the model or the ranking of teacher measures.

Demographic Variables

In addition to controlling for prior achievement, the value-added model also controls for the following student demographic variables:

- Economic status: Economically disadvantaged; Not economically disadvantaged
- English proficiency: English learner, not English learner
- Disability: With disability, Without disability
- Race: African American, Asian, Hispanic, Native American, and White
- Gender: Female, male

Teacher-Student Matching

In order to create value-added models for teachers, it is necessary to link teachers to the students they teach. The source for this information is school rosters pulled from PowerSchool. School districts were given the opportunity to review and make changes to the students that were assigned to teachers in ELA and mathematics courses. Data from the rosters were used to associate students with teachers in the value-added models.

Descriptive Statistics

Tables 1 and 2 describe the sample used for the 2018-2019 school year in Math and ELA, respectively.

Table 1. Sample for 2019 Math Growth

	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	All
Number of Students	56,579	57,531	57,594	55,238	53,601	280,537
Number of Schools	124	125	109	103	103	134
Number of School Districts	84	85	85	86	86	86
Percent Poverty	70.61	69.95	72.90	70.11	67.47	70.25
Percent English Learners	9.10	7.84	6.79	6.77	7.27	7.56
Percent with Disability	16.46	16.00	15.34	14.84	14.53	15.45
Percent African American	41.01	41.14	43.44	42.35	40.98	41.79
Percent Asian	2.23	2.31	2.38	2.49	2.48	2.38
Percent Hispanic	10.98	11.16	12.05	11.95	10.99	11.43
Percent Native American	0.70	0.73	0.71	0.74	0.76	0.73
Percent White	52.08	51.13	53.68	55.00	56.72	53.68
Percent Female	52.65	52.32	54.72	55.32	55.16	54.01
2019 Posttest Mean	490.45	533.13	531.97	548.60	589.51	n/a
2019 Posttest Standard Deviation	108.01	113.45	114.71	102.39	110.00	n/a
2018 Math Pretest Mean	461.74	489.63	529.20	533.53	550.61	n/a
2018 ELA Pretest Mean	439.31	487.76	523.45	543.17	584.50	n/a
2018 Math Pretest Standard Deviation	117.36	109.68	112.42	117.35	104.05	n/a
2018 ELA Pretest Standard Deviation	100.38	107.98	104.05	113.21	107.61	n/a
2017 Math Pretest Mean*	n/a	453.79	481.16	523.91	534.42	n/a
2017 ELA Pretest Mean*	n/a	432.22	479.55	521.69	543.45	n/a
2017 Math Pretest Standard Deviation*	n/a	112.14	102.41	106.98	109.20	n/a
2017 ELA Pretest Standard Deviation*	n/a	97.51	101.31	103.40	104.41	n/a

*2017 assessment data are included for students who have the additional year of test data.

Table 2. Sample for 2019 English Language Arts (ELA) Growth

	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	All
Number of Students	56,531	57,511	57,565	55,193	53,570	280,364
Number of Schools	124	125	109	103	103	134
Number of School Districts	84	85	85	86	86	86
Percent Poverty	72.85	72.58	78.60	75.44	71.47	74.22
Percent English Learners	9.55	8.27	7.85	7.88	8.52	8.42
Percent with Disability	17.04	16.73	16.31	15.84	15.19	16.24
Percent African American	42.34	42.66	46.59	45.30	43.10	44.01
Percent Asian	2.34	2.40	2.49	2.64	2.61	2.49
Percent Hispanic	11.44	11.68	13.31	13.22	12.29	12.39
Percent Native American	0.70	0.74	0.77	0.80	0.81	0.76
Percent White	53.66	53.60	57.53	58.49	59.17	56.45
Percent Female	54.37	54.50	58.86	59.07	58.09	n/a
2019 Posttest Mean	504.11	528.34	541.45	591.97	617.27	n/a
2019 Posttest Standard Deviation	126.43	108.73	115.35	115.77	115.38	n/a
2018 ELA Pretest Mean	439.38	488.31	522.53	541.26	582.78	n/a
2018 Math Pretest Mean	462.03	490.20	529.31	531.78	549.34	n/a
2018 ELA Pretest Standard Deviation	100.53	108.22	104.33	113.16	107.92	n/a
2018 Math Pretest Standard Deviation	117.46	110.04	112.35	116.36	103.84	n/a
2017 ELA Pretest Mean*	n/a	432.81	478.89	519.89	541.94	n/a
2017 Math Pretest Mean*	n/a	454.41	481.17	522.53	533.24	n/a
2017 ELA Pretest Standard Deviation*	n/a	97.81	101.45	103.17	104.80	n/a
2017 Math Pretest Standard Deviation*	n/a	112.47	102.32	106.25	108.95	n/a

*2017 assessment data are included for students who have the additional year of test data.

VALUE-ADDED MODEL

THE MODEL FRAMEWORK

The value-added model describes the achievement of student *i* in classroom *c* in year *t* as in equation (1):

$$y_{ict} = \zeta + \lambda_1 y_{ict-1} + \lambda_1^{alt} y_{ict-1}^{alt} + \lambda_2 y_{ict-2} + \lambda_2^{alt} y_{ict-2}^{alt} + \gamma \bar{y}_{ct-1} + \gamma^{alt} \bar{y}_{ct-1}^{alt} + X_{ict}\beta + \bar{X}_{ct}\theta + \alpha_{ct} + \varepsilon_{ict} \quad (1)$$

where

- *c* is the classroom associated with student *i* in year *t*;
- y_{ict}, y_{ict-1}, and y_{ict-2} are the achievement of student *i* in years *t*, *t-1*, and *t*-2;
- y_{ict-1}^{alt} and y_{ict-2}^{alt} are the achievement of student *i* in the subject other than that of y_{ict} (e.g., y_{ict-1}^{alt} is math achievement if y_{ict} is ELA achievement, and vice versa) in years *t*-1 and *t*-2;
- $\bar{y}_{ct-1}$ and $\bar{y}_{ct-1}^{alt}$ are the averages of y_{ict-1} and y_{ict-1}^{alt} among students in the same classroom and grade in year *t* as student *i*;
- X_{ict} is a vector of characteristics of student *i* at time *t*, including gender, race/ethnicity, English language learner status, economic disadvantage, and special education status;
- $\bar{X}_{ct}$ is the average of X_{ict} among students in the same classroom and grade in year *t* as student *i*;
- α_{ct} is the impact on student achievement y_{ict} of classroom *c* at time *t*; and
- ε_{ict} is the impact of non-school, non-classroom factors on student achievement y_{ict} that cannot be explained with the other variables on the right-hand-side of (1).

In practice, we define classrooms as unique combinations of teachers associated with students. These combinations are defined not only by the set of teachers included in the combination, but also by the proportions of student instruction associated with each teacher in the set. For example, we may define one classroom as including all students associated 70 percent with teacher A and 30 percent with teacher B. We may define another classroom as including all students associated 50 percent with teacher A and 50 percent with teacher B, and a third classroom as including all students associated 100 percent with teacher A.

The student achievement variables y_{ict}, y_{ict-1}, y_{ict-1}^{alt}, etc. in the model above are assumed to be measured without error. In practice, student achievement is inevitably measured with error. Consequently, we define measured achievement – the actual assessment scores used in the data analysis – using the variables:

- Y_{ict}, Y_{ict-1}, and Y_{ict-2} are the measured achievement of student *i* in years *t*, *t-1*, and *t*-2;

- Y_{ict-1}^{alt} and Y_{ict-2}^{alt} are the measured achievement of student i in the subject other than that of Y_{ict} (e.g., Y_{ict-1}^{alt} is measured math achievement if Y_{ict} is measured ELA achievement, and vice versa) in years t-1 and t-2;
- $\bar{Y}_{ct-1}$ and $\bar{Y}_{ct-1}^{alt}$ are the averages of Y_{ict-1} and Y_{ict-1}^{alt} among students in the same classroom and grade in year t as student i.

We estimate this model separately by subject and grade. This allows the coefficients ζ, λ, γ, β, and θ to vary by subject and grade. It also produces classroom growth measures α_{ct} that differ across subjects and grades within classrooms. We normalize all of the measured assessment score variables Y to have a mean of zero and a standard deviation of one across students by grade and subject.

Estimating the Model Coefficients and Producing Student Growth Residuals

The model described in (1) cannot be estimated in a single step if the classroom effects α_{ct} are fixed. This is because the average prior achievement scores $\bar{y}_{ct-1}$ and $\bar{y}_{ct-1}^{alt}$ and average demographics $\bar{X}_{ct}$ are perfectly correlated with classroom assignment. Consequently, we estimate the coefficients in (1) in several steps.

First, over a sample of students with measured scores in all three years (t, t-1, and t-2), we estimate a regression of the posttest score Y_{ict} on the pretest scores Y_{ict-1}, Y_{ict-2}, Y_{ict-1}^{alt}, and Y_{ict-2}^{alt}, the demographic variables X_{ict}, and a full set of classroom fixed effects. We estimate this regression using errors-in-variables (EIV) regression to account for measurement error in the pretests Y_{ict-1}, Y_{ict-2}, Y_{ict-1}^{alt}, and Y_{ict-2}^{alt} using Cronbach's alpha to measure the extent of measurement error of the pretests. (For discussion of errors-in-variables regression, see Fuller, 1987.) This produces estimates of the coefficients on the pretests λ_1, λ_1^{alt}, λ_2, and λ_2^{alt} and the coefficients on the demographics β.

Second, over the same sample of students, we produce a growth residual that controls for student-level prior achievement equal to:

$$q_{ict} = Y_{ict} - \hat{\lambda}_1 Y_{ict-1} - \hat{\lambda}_1^{alt} Y_{ict-1}^{alt} - \hat{\lambda}_2 Y_{ict-2} - \hat{\lambda}_2^{alt} Y_{ict-2}^{alt} - X_{ict}\hat{\beta} \qquad (2)$$

where $\hat{\lambda}_1$, $\hat{\lambda}_1^{alt}$, $\hat{\lambda}_2$, $\hat{\lambda}_2^{alt}$, and $\hat{\beta}$ are the estimates of λ_1, λ_1^{alt}, λ_2, λ_2^{alt}, and β produced in the previous step.

Third, we regress q_{ict} on $\bar{Y}_{ct-1}$, $\bar{Y}_{ct-1}^{alt}$, and $\bar{X}_{ct-1}$ using ordinary least squares, producing estimates of the coefficients on the average pretests γ and γ^{alt} and of the coefficients on the average demographics θ. The residual from this regression, which we denote w_{ict}^{*}, is a growth residual that controls for student-level and classroom-level prior achievement and demographics. It is an estimate of the sum of the classroom effect α_{ct} and the residual term ε_{ict}.

Incorporating Students With Only Two Years of Scores

The estimation approach above produces growth residuals for students with measured scores in all three years (*t*, *t*-1, and *t*-2). To include students with measured scores in the two most recent years only (*t* and *t*-1), we repeat the steps in the previous section above by grade and subject over a sample of students with measured scores in the two most recent years (*t* and *t*-1) using a version of (1) that does not include Y_{ict-2} and Y_{ict-2}^{alt} on the right-hand-side. This sample of students includes all students included in the estimation steps described in the previous section, as well as an additional group of students with measured scores in years *t* and *t*-1 but not in year *t*-2. This produces another set of growth residuals, which we denote $w_{ict}^{\dagger}$, which covers all students with measured scores in the two most recent years (*t* and *t*-1).

Over a sample of all students with measured scores in years *t* and *t*-1, we create a combined growth residual equal to:

$$w_{ict} = \begin{cases} w_{ict}^{*} & \text{if student } i \text{ has measured scores in years } t, t\text{-}1, \text{ and } t\text{-}2 \\ w_{ict}^{\dagger} & \text{if student } i \text{ has measured scores in years } t \text{ and } t\text{-}1 \text{ only} \end{cases} \quad (3)$$

The growth residual w_{ict} includes all students with scores in years *t* and *t*-1 and controls for student achievement in year *t*-2 when possible. This growth residual is demeaned to have a mean of zero by grade and subject.

Producing Classroom and Teacher Growth Measures by Grade

We produce classroom growth measures by regressing, by ordinary least squares, the combined student growth residual w_{ict} on a full set of clasroom indicator variables. Recall, as noted above, that classroom is defined in practice as a unique combination of teachers. As before, this regression is estimated separately by grade and subject. This produces a fixed effect estimate $\hat{\alpha}_{ct}$ for each classroom by grade and subject. This estimates the impact of classroom *c* on student achievement in year *t* relative to the average classroom impact across students. It is measured in units of standard deviations of achievement across students in the measured posttest Y_{ict}.

From the classroom growth measures by grade, we produce growth measures for each individual teacher by grade by computing a weighted average of classroom growth measures across all classrooms associated with the teacher. The weight used in this average is the number of students associated with the classroom multiplied by the proportion of instruction associated with the teacher among the students in the classroom. We can express this mathematically in equation (4) below:

$$\hat{\alpha}_{jt} = \frac{\sum_c p_{jct} n_{ct} \hat{\alpha}_{ct}}{\sum_c p_{jct} n_{ct}} \quad (4)$$

where $\hat{\alpha}_{jt}$ is the growth measure for teacher *j* in year *t*; n_{ct} is the number of students associated with classroom *c* at time *t*; and p_{jct} is the proportion of instruction of students in classroom *c* that is associated with teacher *j*. For example, if teacher *j* is associated with a classroom with 12 students, and 75 percent of the instruction of those students is associated with teacher *j*, then the weight with which that classroom enters into teacher *j*'s growth measures is equal to 12 × 0.75 = 9. If teacher *j* is associated with another classroom with 6 students, and 100 percent of the instruction of those students is associated with teacher *j*, then the weight with which that classroom enters into teacher *j*'s growth measures is equal to 6 × 1.00 = 6. If those are the only two classrooms associated with teacher *j*, then teacher *j*'s growth measure is equal to 9 / (9 + 6) = 0.60 times the growth measure of the first classroom and 6 / (9 + 6) = 0.40 times the growth measure of the second classroom.

We estimate the variance, corrected for sampling error, of the teacher measures $\hat{\alpha}_{jt}$ using the following equation:

$$\hat{\omega}^2 = Var\left[\hat{\alpha}_{jt}\right] - Mean[\hat{\sigma}_{jt}^2] \qquad (5)$$

where $\hat{\omega}^2$ is the estimate of the variance of $\hat{\alpha}_{jt}$ across teachers that is not the result of randomness in individual student growth and $\hat{\sigma}_{jt}^2$ is the square of the estimated standard error of $\hat{\alpha}_{jt}$. We use this variance estimate for two purposes. First, we use it to produce a shrinkage estimate of each teacher's effect on student achievement using Empirical Bayes shrinkage, using the following formula:

$$\tilde{\alpha}_{jt} = r_{jt}\hat{\alpha}_{jt} \qquad (6)$$

where r_{jt}, the reliability of the value-added measure $\hat{\alpha}_{jt}$, is equal to $\hat{\omega}^2\left(\hat{\omega}^2 + \hat{\sigma}_{jt}^2\right)^{-1}$. The shrinkage has the effect of tempering teacher estimates based on small numbers of students, which are typically overrepresented among the highest and lowest values of $\hat{\alpha}_{jt}$ as a result of randomness in individual student growth, toward the average growth measure of zero. The standard error of the shrunk growth measure $\tilde{\alpha}_{jt}$ is equal to:

$$\tilde{\sigma}_{jt} = r_{jt}^{1/2}\hat{\sigma}_{jt} \qquad (7)$$

Second, we use the variance estimate $\hat{\omega}^2$ to rescale both the unshrunk growth measure $\hat{\alpha}_{jt}$ and the shrunk growth measure $\tilde{\alpha}_{jt}$ to be measured in standard deviations of growth across teachers. This is accomplished by dividing the growth measure by the square root of $\hat{\omega}^2$:

$$\hat{\alpha}_{jt}^{tier} = \hat{\alpha}_{jt}/\hat{\omega} \qquad (8a)$$

$$\tilde{\alpha}_{jt}^{tier} = \tilde{\alpha}_{jt}/\hat{\omega} \qquad (8b)$$

where $\hat{\alpha}_{jt}^{tier}$ and $\tilde{\alpha}_{jt}^{tier}$ are the rescaled growth measures, with the *tier* superscript indicating that the growth measures have been rescaled to occupy "tiers" that are for the most part between -3 and +3. The standard errors of the rescaled growth measures are similarly estimated by dividing their standard errors before rescaling by the standard deviation estimate $\hat{\omega}$.

Producing Teacher Team Growth Measures by Grade

In addition to the teacher growth measures described above, we also produce growth measures for each team of teachers by grade. A team of teachers is defined as a set of teachers with shared students. The growth measure of a team of teachers is based on the growth of the set of students who are associated to at least some degree with all of the teachers in the team and who are not associated with any other teachers outside of the team. The difference between a classroom and a team of teachers (as defined for the purposes of the model) is that a classroom is defined both by a set of teachers with shared students and by the proportion of instruction of those shared students associated with each teacher, while a team of teachers is defined only by a set of teachers with shared students regardless of the proportion of instruction of those students associated with each teacher. The growth measures for each team of teachers is computed using a weighted average of the classroom growth measures across all of the classrooms associated with the team of teachers, using the number of students associated with the classroom as a weight.

After producing growth measures for teacher teams from averaging classroom growth measures, we produce shrunk and/or tiered estimates of the effects of teacher teams using equations (6) through (8) as described in the previous section. When implementing equations (6) through (8) to shrink and tier the teacher team growth estimates, we use the same variance estimate $\widehat{\omega}^2$ as that used to shrink and tier the teacher growth estimates. This variance estimate is employed to ensure that teacher and teacher-team estimates are identical when based on exactly the same group of students.

Producing Multi-Grade Teacher and Teacher Team Growth Measures

To produce multi-grade teacher growth measures, we compute a weighted average, by subject, of the rescaled, shrunk growth measures $\tilde{\alpha}_{jt}^{tier}$ across grades by teacher. The weight used is the number of students associated with that teacher by grade and subject, multiplied by the average proportion of instruction of those students associated with that teacher by grade and subject. Similarly, we produce multi-grade teacher team growth measures by averaging the rescaled, shrunk teacher team growth measures across grades by teacher team, using the number of students associated with that teacher team by grade and subject as a weight.

PROPERTIES OF THE VALUE-ADDED RESULTS

COEFFICIENT ESTIMATES

The coefficients estimated in the value-added model are presented in Tables 3 through 6. To interpret these coefficients, note that both pretest and posttest are measured using scores that have been standardized to have a mean of zero and a standard deviation of one. For example, note, in the model of 2019 fourth-grade math, that the coefficient on the 2018 math pretest is 0.843 (see Table 3). This means that, at the student level, a one standard deviation increase in the 2018 third-grade math assessment is associated with a 0.843 standard deviation increase in the 2019 fourth-grade math assessment.

It is important to keep in mind the standard errors of the coefficients when interpreting them. A span of 1.96 standard errors in both the positive and negative directions provides a 95 percent confidence range for a coefficient. Continuing with the previous example, note that, in the model of 2019 fourth grade math, the estimated coefficient on 2018 math pretest has a standard error of 0.007. This means that, while our best estimate of the standardized impact on 2018 third-grade math achievement on 2019 fourth grade math achievement is 0.843, a 95 percent confidence interval for this estimated impact ranges from 0.829 to 0.857.

Note that coefficients are presented below for models that include two lags of prior achievement, as well as for models that include only one lag of prior achievement. The coefficients from models that include two lags of prior achievement are employed when measuring the growth of students for whom two lags of prior achievement are available. The coefficients from models that include only one lag of prior achievement are employed when measuring the growth of students for whom only one lag of prior achievement is available.

Table 3. Coefficient Estimates in Double-Lag Model, 2018-19 SC READY Math

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
Variable	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**
2018 Math Pretest	0.843	0.007	0.674	0.011	0.615	0.011	0.653	0.012	0.733	0.017
2018 ELA Pretest	0.04	0.007	0.053	0.012	0.078	0.011	0.075	0.011	0.054	0.014
2018 cls. avg. Math Pretest	-0.131	0.018	-0.118	0.017	-0.160	0.018	-0.095	0.013	-0.122	0.019
2018 cls. avg. ELA Pretest	0.119	0.019	0.139	0.019	0.152	0.021	0.095	0.015	0.174	0.021
2017 Math Pretest	n/a	n/a	0.186	0.012	0.190	0.010	0.214	0.010	0.194	0.014
2017 ELA Pretest	n/a	n/a	0.028	0.012	0.037	0.010	0.003	0.009	-0.063	0.012
Students with Disabilities	-0.043	0.007	0.006	0.007	0.007	0.007	0.025	0.007	0.009	0.008
Native American	-0.028	0.028	0.019	0.024	-0.004	0.025	-0.054	0.024	0.001	0.026
Hispanic	-0.027	0.011	0.043	0.009	0.034	0.009	-0.033	0.009	0.036	0.010
African American	-0.043	0.006	0.017	0.005	-0.021	0.005	-0.065	0.005	0.046	0.006
Asian/Pacific Islander	0.085	0.016	0.127	0.014	0.098	0.014	0.095	0.014	0.097	0.014
Female	-0.067	0.005	0.079	0.004	0.057	0.004	-0.018	0.004	0.042	0.005
English Learners	0.028	0.012	0.036	0.011	0.047	0.011	0.009	0.011	0.022	0.012
Economically Disadvantaged	-0.054	0.006	-0.039	0.005	-0.050	0.005	-0.022	0.005	-0.019	0.005
Students with Disabilities cls. avg.	0.056	0.030	0.062	0.030	0.018	0.029	0.038	0.020	0.023	0.029
Native Am. cls. avg.	0.038	0.152	-0.550	0.171	-0.582	0.190	0.041	0.125	-0.368	0.186
Hispanic cls. avg.	0.109	0.055	-0.040	0.058	-0.256	0.056	-0.016	0.040	-0.003	0.066
Afr. American cls. avg.	-0.067	0.020	-0.029	0.020	-0.109	0.020	0.014	0.015	0.008	0.021
Asian/Pacific Is. cls. avg.	0.022	0.101	-0.002	0.103	0.059	0.108	0.107	0.073	0.317	0.120
Female cls. avg.	-0.051	0.029	-0.038	0.029	-0.007	0.029	-0.012	0.021	0.007	0.031
English Learners cls. avg.	-0.061	0.061	0.039	0.071	0.155	0.078	0.047	0.054	0.128	0.084
Economically Disadvantaged cls. avg.	-0.120	0.025	0.014	0.025	-0.085	0.028	-0.142	0.019	-0.049	0.028

Table 4. Coefficient Estimates in Single-Lag Model, 2018-19 SC READY Math

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
Variable	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**
2018 Math Pretest	0.843	0.007	0.823	0.006	0.782	0.006	0.845	0.007	0.910	0.008
2018 ELA Pretest	0.040	0.007	0.097	0.005	0.107	0.006	0.079	0.006	-0.005	0.007
2018 cls. avg. Math Pretest	-0.131	0.018	-0.148	0.017	-0.190	0.018	-0.163	0.013	-0.172	0.019
2018 cls. avg. ELA Pretest	0.119	0.019	0.157	0.019	0.179	0.021	0.161	0.015	0.232	0.021
Students with Disabilities	-0.043	0.007	-0.022	0.007	-0.031	0.007	0.006	0.007	-0.009	0.008
Native American	-0.028	0.028	0.020	0.025	-0.011	0.026	-0.066	0.025	0.012	0.026
Hispanic	-0.027	0.011	0.035	0.009	0.019	0.009	-0.048	0.009	0.032	0.010
African American	-0.043	0.006	0.011	0.006	-0.040	0.006	-0.078	0.005	0.053	0.006
Asian/Pacific Islander	0.085	0.016	0.132	0.014	0.095	0.014	0.086	0.014	0.111	0.015
Female	-0.067	0.005	0.084	0.004	0.040	0.004	-0.037	0.004	0.048	0.005
English Learners	0.028	0.012	0.042	0.011	0.048	0.011	0.016	0.011	0.037	0.012
Economically Disadvantaged	-0.054	0.006	-0.047	0.005	-0.055	0.005	-0.026	0.005	-0.024	0.005
Students with Disabilities cls. avg.	0.056	0.030	0.041	0.029	0.029	0.029	0.042	0.020	0.021	0.029
Native Am. cls. avg.	0.038	0.152	-0.509	0.159	-0.599	0.187	0.046	0.127	-0.440	0.185
Hispanic cls. avg.	0.109	0.055	-0.051	0.055	-0.215	0.055	0.036	0.039	-0.029	0.065
Afr. American cls. avg.	-0.067	0.020	-0.039	0.020	-0.107	0.021	0.012	0.015	-0.020	0.021
Asian/Pacific Is. cls. avg.	0.022	0.101	0.015	0.099	0.058	0.105	0.236	0.071	0.282	0.113
Female cls. avg.	-0.051	0.029	-0.05	0.029	-0.012	0.029	-0.036	0.021	-0.006	0.030
English Learners cls. avg.	-0.061	0.061	0.050	0.067	0.085	0.075	-0.012	0.052	0.125	0.080
Economically Disadvantaged cls. avg.	-0.120	0.025	0.00	0.025	-0.121	0.028	-0.156	0.019	-0.026	0.027

Table 5. Coefficient Estimates in Double-Lag Model, 2018-19 SC READY ELA

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
Variable	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**
2018 Math Pretest	0.036	0.007	0.073	0.011	0.066	0.011	0.075	0.011	0.066	0.016
2018 ELA Pretest	0.871	0.007	0.589	0.012	0.598	0.011	0.620	0.010	0.700	0.013
2018 cls. avg. Math Pretest	0.122	0.014	0.028	0.012	0.033	0.013	0.015	0.011	0.059	0.013
2018 cls. avg. ELA Pretest	-0.176	0.015	-0.048	0.014	-0.068	0.014	-0.025	0.012	-0.079	0.013
2017 Math Pretest	n/a	n/a	-0.045	0.011	-0.001	0.010	-0.020	0.009	-0.029	0.013
2017 ELA Pretest	n/a	n/a	0.318	0.012	0.273	0.010	0.253	0.009	0.198	0.011
Students with Disabilities	-0.013	0.007	-0.010	0.007	-0.007	0.007	-0.039	0.007	-0.039	0.007
Native American	0.013	0.027	0.020	0.024	0.029	0.025	-0.034	0.024	0.050	0.025
Hispanic	-0.015	0.011	0.021	0.009	0.034	0.009	-0.006	0.008	0.025	0.010
African American	-0.045	0.006	-0.041	0.005	-0.033	0.005	-0.03	0.005	-0.012	0.006
Asian\Pacific Islander	0.022	0.016	0.031	0.014	0.043	0.014	0.091	0.014	0.015	0.014
Female	0.043	0.005	0.034	0.004	0.059	0.004	0.026	0.004	0.071	0.004
English Learners	0.047	0.012	0.013	0.011	0.007	0.011	-0.004	0.011	0.028	0.012
Economically Disadvantaged	-0.026	0.006	-0.033	0.005	-0.026	0.005	-0.022	0.005	0.001	0.005
Students with Disabilities cls. avg.	-0.060	0.022	0.036	0.021	-0.065	0.019	-0.036	0.017	-0.062	0.019
Native Am. cls. avg.	-0.253	0.128	-0.273	0.128	-0.285	0.126	-0.053	0.105	-0.331	0.122
Hispanic cls. avg.	0.078	0.043	0.054	0.041	-0.066	0.036	0.022	0.032	0.130	0.039
Afr. American cls. avg.	-0.027	0.015	0.018	0.014	0.053	0.014	0.068	0.013	0.078	0.014
Asian/Pacific Is. cls. avg.	0.143	0.079	0.109	0.076	0.199	0.072	0.00	0.062	0.084	0.076
Female cls. avg.	0.061	0.023	0.023	0.021	0.021	0.019	0.046	0.017	0.034	0.019
English Learners cls. avg.	-0.104	0.047	-0.046	0.050	0.021	0.043	-0.030	0.037	-0.097	0.044
Economically Disadvantaged cls. avg.	-0.125	0.019	-0.084	0.018	-0.138	0.019	-0.120	0.016	-0.055	0.018

Table 6. Coefficient Estimates in Single-Lag Model, 2018-19 SC READY ELA

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
Variable	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**
2018 Math Pretest	0.036	0.007	0.063	0.006	0.081	0.006	0.077	0.006	0.058	0.007
2018 ELA Pretest	0.871	0.007	0.842	0.006	0.817	0.006	0.826	0.006	0.863	0.007
2018 cls. avg. Math Pretest	0.122	0.014	0.070	0.012	0.050	0.013	0.034	0.012	0.087	0.014
2018 cls. avg. ELA Pretest	-0.176	0.015	-0.097	0.014	-0.087	0.015	-0.050	0.012	-0.111	0.013
Students with Disabilities	-0.013	0.007	-0.046	0.007	-0.070	0.007	-0.081	0.007	-0.058	0.007
Native American	0.013	0.027	0.024	0.025	0.024	0.025	-0.029	0.024	0.040	0.025
Hispanic	-0.015	0.011	0.010	0.010	0.024	0.009	-0.019	0.009	0.016	0.010
African American	-0.045	0.006	-0.048	0.006	-0.043	0.005	-0.039	0.005	-0.026	0.006
Asian/Pacific Islander	0.022	0.016	0.024	0.014	0.040	0.014	0.078	0.014	0.007	0.014
Female	0.043	0.005	0.039	0.004	0.061	0.004	0.016	0.004	0.067	0.004
English Learners	0.047	0.012	-0.003	0.011	-0.004	0.011	-0.027	0.011	0.021	0.012
Economically Disadvantaged	-0.026	0.006	-0.044	0.005	-0.034	0.005	-0.026	0.005	-0.003	0.005
Students with Disabilities cls. avg.	-0.06	0.022	0.009	0.021	-0.099	0.019	-0.044	0.017	-0.079	0.019
Native Am. cls. avg.	-0.253	0.128	-0.235	0.122	-0.295	0.129	-0.027	0.105	-0.391	0.123
Hispanic cls. avg.	0.078	0.043	0.025	0.041	-0.056	0.037	0.059	0.031	0.126	0.039
Afr. American cls. avg.	-0.027	0.015	0.020	0.014	0.057	0.015	0.086	0.013	0.089	0.014
Asian/Pacific Is. cls. avg.	0.143	0.079	0.145	0.075	0.148	0.072	0.026	0.061	0.123	0.074
Female cls. avg.	0.061	0.023	0.047	0.022	0.030	0.020	0.054	0.017	0.043	0.019
English Learners cls. avg.	-0.104	0.047	-0.038	0.049	-0.009	0.043	-0.055	0.035	-0.135	0.043
Economically Disadvantaged cls. avg.	-0.125	0.019	-0.105	0.018	-0.172	0.019	-0.162	0.016	-0.08	0.018

VARIANCE AND RELIABILITY OF VALUE-ADDED MEASURES

Tables 7 and 8 present information about the variance and reliability of the value-added estimates by grade and subject. The first three rows present the three terms in equation (5). The first row is the variance across teachers in the value-added estimates $\hat{\alpha}_{jt}$, which includes both the variance in the impacts of teachers on student achievement as well as variance in the sampling error with which those impacts are estimated. The second row is an estimate of the variance of that sampling error, which is equal to the average of the squared standard errors across the value-added estimates. The third row is an estimate of the variance across teachers in the actual impacts of teachers on student achievement. This is estimated by subtracting the second row from the first row, as described in equation (5).

The fourth row of Tables 7 and 8 is an estimate of the standard deviation across teachers in their impacts on student achievement and is equal to the square root of the third row. The fifth row is an estimate of the reliability of the value-added measures. It is equal to the proportion of the variance in the value-added measures that is due to actual differences in the impacts of teachers rather than variance from sampling error. It is computed by dividing the third row by the first row. Finally, the last row of Tables 7 and 8 present the number of teachers used to compute the variance of teacher value-added.

The results in the first four rows of Tables 7 and 8 are measured in units of standard deviations of student achievement the posttest. For example, note that the estimated standard deviation of value-added in fourth-grade math is 0.227. This means that the standard deviation across teachers in their impacts on student achievement is estimated to be 22.7% the size of the standard deviation across students in student achievement in fourth grade math.

Table 7. Variance and Reliability of Math Value-Added

	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8
Variance of estimates ($Var[\hat{\alpha}_{jt}]$)	0.061	0.052	0.041	0.016	0.038
Noise variance ($Mean[\hat{\sigma}_{jt}^2]$)	0.009	0.007	0.003	0.003	0.003
Estimated variance ($\hat{\omega}^2$)	0.051	0.046	0.038	0.013	0.035
Estimated standard deviation ($\hat{\omega}$)	0.227	0.214	0.194	0.116	0.188
Reliability ($\hat{\omega}^2$ / $Var[\hat{\alpha}_{jt}]$)	0.847	0.875	0.923	0.821	0.916
Number of teachers	2,212	1,941	982	991	939

Table 8. Variance and Reliability of ELA Value-Added

	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8
Variance of estimates ($Var[\hat{\alpha}_{jt}]$)	0.031	0.022	0.017	0.012	0.013
Noise variance ($Mean[\hat{\sigma}_{jt}^2]$)	0.010	0.007	0.003	0.003	0.003
Estimated variance ($\hat{\omega}^2$)	0.021	0.016	0.013	0.009	0.010
Estimated standard deviation ($\hat{\omega}$)	0.146	0.125	0.116	0.094	0.098
Reliability ($\hat{\omega}^2$ / $Var[\hat{\alpha}_{jt}]$)	0.689	0.696	0.808	0.755	0.753
Number of teachers	2,401	2,158	1,172	1,126	1,092

NEUTRALITY

CORRELATION WITH DEMOGRAPHIC VARIABLES

The correlations between the value-added measures and demographic variables were generally very low. In other words, teacher value-added and average student characteristics were not substantively empirically related. This is expected given that the value-added model explicitly controls for average student demographics at the classroom level. These correlations are presented in Tables 9 and 10.

Table 9. Correlations between Student Demographics and Math Value-Added

CHARACTERISTIC	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8	OVERALL
Poverty	-0.01	0.00	0.00	-0.03	-0.01	-0.01
English Learners	0.00	0.00	0.00	0.03	0.00	0.01
Disability	0.01	0.01	-0.05	-0.05	0.00	-0.02
African American	-0.01	0.00	-0.01	-0.02	-0.02	-0.01
Asian/Pacific Islander	-0.01	-0.01	-0.02	0.01	0.02	0.00
Hispanic	0.00	0.00	-0.02	0.01	-0.01	0.00
Native American	-0.02	-0.02	-0.05	0.01	-0.02	-0.02
White	0.01	0.00	0.03	0.02	0.02	0.02
Female	-0.03	0.01	0.03	0.01	0.02	0.01

Table 10. Correlations between Student Demographics and ELA Value-Added

CHARACTERISTIC	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8	OVERALL
Poverty	-0.01	-0.01	-0.03	-0.02	0.00	-0.02
English Learners	-0.01	0.00	0.00	-0.02	-0.04	-0.01
Disability	0.02	0.02	-0.03	0.00	-0.02	-0.01
African American	0.00	0.00	0.00	0.02	0.03	0.01
Asian/Pacific Islander	0.01	0.02	0.01	0.00	-0.03	0.00
Hispanic	0.00	0.00	-0.02	-0.02	-0.02	-0.01
Native American	-0.01	-0.03	-0.04	0.00	-0.05	-0.03
White	0.01	0.00	0.01	-0.01	-0.02	0.00
Female	0.00	0.00	0.04	0.00	0.05	0.02

CORRELATION WITH AVERAGE PRIOR PROFICIENCY

The correlation between value-added and prior proficiency (e.g., the correlation between the growth of a teacher's students between third and fourth grade and the third-grade proficiency rate from which those students began their growth) is close to zero. This is an expected result

given that average prior achievement, measured using average classroom pretest score, is explicitly controlled for in the model. The correlations are presented in Table 11.

Table 11. Correlations between Prior Attainment and Value-Added

Subject	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Overall
ELA	0.01	0.02	0.04	0.02	0.02	0.02
Math	0.03	0.02	0.02	0.06	0.04	0.04

Correlation between Math and ELA

There were substantive positive correlations between math and ELA value-added. Teachers that were high value-added in math were also more often than not high value-added in ELA. This implies that teachers with a higher-than-average impact in mathematics also had a higher-than-average impact in English language arts. These correlations are presented in Table 12.

Table 12. Correlations in Value-Added between Subjects

	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Overall
2019 Math/ELA	0.57	0.57	0.47	0.37	0.59	0.57

REFERENCES

Fuller, W. (1987). *Measurement Error Models*, John Wiley and Sons.