Technical Report on the South Carolina School Value-Added Model

Academic Year 2018-2019

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CONTENTS

INTRODUCTION

This report describes the value-added model used by Education Analytics to measure the effectiveness of South Carolina public schools using South Carolina College-and-Career-Ready Assessments (SC READY) test score data. The report is divided into two sections. The first section describes the data set used to produce the value-added estimates. The second section describes the model used to estimate value-added for schools in South Carolina and presents some properties of the value-added results.

Conceptually, value-added analysis is the use of statistical techniques to isolate the component of measured student knowledge that is attributable to schools. In practice, value-added models focus on the improvement students make on annual assessments from one year and grade to the next.

The model used in South Carolina controls for up to two years of prior student achievement in English language arts and mathematics. It also controls for average prior student achievement in English language arts and mathematics at the school and grade level.

ANALYSIS DATA SET

Before estimation can take place, a substantial amount of work is required to assemble the analysis data sets used to produce the value-added estimates. A separate analysis data set is produced for each grade and subject. In total, 10 analysis data sets are produced, covering grades 4 through 8 for SC READY English language arts (ELA) and math in 2018-19.

Each analysis data set includes students who have a posttest in the grade and subject being considered, who have at least one year of pretests in both ELA and math, who had continuous enrollment status in their school, and who were tested in consecutive grades.

STUDENT-LEVEL VARIABLES

ASSESSMENTS

The test scores used are from the 2016-17, 2017-18 and 2018-19 SC READY assessments. The value-added system produces school-level measures for grades 4 through 8 in ELA and math based on performance on the 2018-19 SC READY. The 2018-19 value-added model in ELA uses the 2018-19 ELA score as the posttest, while the 2018-19 value-added model in math uses the 2018-19 math score as the posttest. All value-added models include pretests in both ELA and math.

The assessment scores were used to produce overall value-added measures in both math and ELA for each school and for each grade within each school. In addition, the assessment scores were also used to produce value-added measures for each school and for each grade within each school for students scoring in the lowest quintile within the school and grade on the same-subject prior-year assessment.

All test scores were linearly normalized to have a mean equal to zero and standard deviation equal to 1 by grade and subject. Thus, in the value-added analyses, all test scores were measured relative to the state mean, and in units of the statewide standard deviation of test scores across students by grade and subject. The normalization is used to make it easier to interpret estimates of the value-added models, but it does not affect the statistical properties of the model or the ranking of estimated school effects.

Demographic Variables

In addition to producing overall value-added measures for each school, EA also produced value-added measures for demographic subgroups within each school. These subgroups include:

- Economic status: Economically disadvantaged; Not economically disadvantaged
- English proficiency: English learner, not English learner
- Disability: With disability, Without disability
- Race: African American, Asian, Hispanic, Native American, and White

Value-added measures were produced for each of these subgroups for each school and for each grade within each school in both ELA and math. In addition, value-added measures were produced for each of these subgroups among students in the lowest quintile of prior achievement for each school.

School Enrollment

Only students that have continuous enrollment status at a single school were included in the value-added analysis set. For the purpose of value-added modeling, continuous enrollment is defined as students who were enrolled in the same school on the 45th day of the school year and on the 160th day of the school year, with no break in enrollment.

Descriptive Statistics

Tables 1 and 2 describe the sample used for the 2018-2019 school year in Math and ELA, respectively.

Table 1. Sample for 2019 Math Growth

	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	All
Number of Students	54,221	55,276	55,378	53,075	51,652	269,602
Number of Schools	658	642	352	322	317	926
Number of School Districts	84	84	85	86	84	86
Percent Poverty	64.29	64.00	63.03	60.34	58.29	62.05
Percent English Learners	8.53	7.34	6.01	5.92	6.57	6.88
Percent with Disability	14.95	14.56	13.32	12.86	12.53	13.66
Percent African American	36.70	37.19	36.84	35.83	35.00	36.33
Percent Asian	2.17	2.23	2.22	2.35	2.39	2.27
Percent Hispanic	10.18	10.41	10.54	10.37	9.84	10.27
Percent Native American	0.63	0.69	0.64	0.66	0.69	0.66
Percent White	50.32	49.47	49.76	50.79	52.09	50.46
2019 Posttest Mean	495.30	537.70	539.18	554.40	596.48	n/a
2019 Posttest Standard Deviation	108.35	114.07	116.19	103.63	111.48	n/a
2018 Math Pretest Mean	466.66	493.87	535.44	540.12	557.07	n/a
2018 ELA Pretest Mean	443.48	491.81	528.71	548.66	590.83	n/a
2018 Math Pretest Standard Deviation	117.45	110.27	113.59	118.43	105.66	n/a
2018 ELA Pretest Standard Deviation	100.42	108.15	104.71	113.82	108.24	n/a
2017 Math Pretest Mean*	n/a	457.80	486.46	528.55	540.71	n/a
2017 ELA Pretest Mean*	n/a	435.73	484.41	525.83	548.97	n/a
2017 Math Pretest Standard Deviation*	n/a	112.63	103.63	107.83	110.74	n/a
2017 ELA Pretest Standard Deviation*	n/a	97.90	101.66	103.98	105.18	n/a

*2017 assessment data are included for students who have the additional year of test data.

Table 2. Sample for 2019 English Language Arts (ELA) Growth

	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	All
Number of Students	54,190	55,267	55,344	53,035	51,629	269,465
Number of Schools	658	642	352	322	317	926
Number of School Districts	84	84	85	86	85	86
Percent Poverty	64.28	64.00	63.02	60.33	58.26	62.03
Percent English Learners	8.53	7.34	6.01	5.92	6.57	6.88
Percent with Disability	14.92	14.55	13.31	12.85	12.50	13.64
Percent African American	36.68	37.17	36.82	35.81	34.99	36.32
Percent Asian	2.16	2.23	2.22	2.35	2.39	2.27
Percent Hispanic	10.17	10.41	10.54	10.38	9.84	10.27
Percent Native American	0.64	0.69	0.64	0.66	0.69	0.66
Percent White	50.34	49.48	49.78	50.80	52.09	50.48
2019 Posttest Mean	509.52	531.88	547.85	599.58	625.28	n/a
2019 Posttest Standard Deviation	125.82	108.58	115.57	116.34	115.49	n/a
2018 Math Pretest Mean	466.74	493.88	535.46	540.18	557.12	n/a
2018 ELA Pretest Mean	443.55	491.82	528.74	548.72	590.91	n/a
2018 Math Pretest Standard Deviation	117.43	110.27	113.61	118.43	105.65	n/a
2018 ELA Pretest Standard Deviation	100.39	108.15	104.72	113.82	108.20	n/a
2017 Math Pretest Mean*	n/a	457.82	486.51	528.60	540.75	n/a
2017 ELA Pretest Mean*	n/a	435.74	484.45	525.89	549.03	n/a
2017 Math Pretest Standard Deviation*	n/a	112.62	103.63	107.83	110.73	n/a
2017 ELA Pretest Standard Deviation*	n/a	97.9	101.66	103.96	105.15	n/a

*2017 assessment data are included for students who have the additional year of test data.

VALUE-ADDED MODEL

THE MODEL FRAMEWORK

The value-added model describes the achievement of a student *i* attending school *j* in year *t* as in equation (1):

$$y_{ijt} = \zeta + \lambda_1 y_{ijt-1} + \lambda_1^{alt} y_{ijt-1}^{alt} + \lambda_2 y_{ijt-2} + \lambda_2^{alt} y_{ijt-2}^{alt} + \gamma \bar{y}_{jt-1} + \gamma^{alt} \bar{y}_{jt-1}^{alt} + \alpha_{jt} + \varepsilon_{ijt} \qquad (1)$$

where

- j is the school attended by student i in year t;
- y_{ijt}, y_{ijt-1}, and y_{ijt-2} are the achievement of student i in years *t*, *t-1*, and *t*-2;
- y_{ijt-1}^{alt} and y_{ijt-2}^{alt} are the achievement of student *i* in the subject other than that of y_{ijt} (e.g., y_{ijt-1}^{alt} is math achievement if y_{ijt} is ELA achievement, and vice versa) in years *t*-1 and *t*-2;
- $\bar{y}_{jt-1}$ and $\bar{y}_{jt-1}^{alt}$ are the averages of y_{ijt-1} and y_{ijt-1}^{alt} among students in the same school and grade in year *t* as student *i*;
- α_{jt} is the impact on student achievement y_{ijt} of attending school *j* at time *t*; and
- ε_{ijt} is the impact of non-school factors on student achievement y_{ijt} that cannot be explained with the other variables on the right-hand-side of (1).

The student achievement variables y_{ijt}, y_{ijt-1}, y_{ijt-1}^{alt}, etc. in the model above are assumed to be measured without error. In practice, student achievement is inevitably measured with error. Consequently, we define measured achievement – the actual assessment scores used in the data analysis – using the variables:

- Y_{ijt}, Y_{ijt-1}, and Y_{ijt-2} are the measured achievement of student *i* in years *t*, *t-1*, and *t*-2;
- Y_{ijt-1}^{alt} and Y_{ijt-2}^{alt} are the measured achievement of student *i* in the subject other than that of Y_{ijt} (e.g., Y_{ijt-1}^{alt} is measured math achievement if Y_{ijt} is measured ELA achievement, and vice versa) in years *t*-1 and *t*-2;
- $\bar{Y}_{jt-1}$ and $\bar{Y}_{jt-1}^{alt}$ are the averages of Y_{ijt-1} and Y_{ijt-1}^{alt} among students in the same school and grade in year *t* as student *i*.

We estimate this model separately by subject and grade. This allows the coefficients ζ, λ, and γ to vary by subject and grade. It also produces school growth measures α_{jt} that differ across subjects and grades within schools. We normalize all of the measured assessment score variables Y to have a mean of zero and a standard deviation of one across students by grade and subject.

Estimating the Model Coefficients and Producing Student Growth Residuals

The model described in (1) cannot be estimated in a single step if the school effects α_{jt} are fixed. This is because the average prior achievement scores $\bar{y}_{jt-1}$ and $\bar{y}_{jt-1}^{alt}$ are perfectly correlated with school assignment. Consequently, we estimate the coefficients in (1) in several steps.

First, over a sample of students with measured scores in all three years (*t*, *t*-1, and *t*-2), we estimate a regression of the posttest score Y_{ijt} on the pretest scores Y_{ijt-1}, Y_{ijt-2}, Y_{ijt-1}^{alt}, and Y_{ijt-2}^{alt} and on a full set of school fixed effects. We estimate this regression using errors-in-variables (EIV) regression to account for measurement error in the pretests Y_{ijt-1}, Y_{ijt-2}, Y_{ijt-1}^{alt}, and Y_{ijt-2}^{alt} using Cronbach's alpha to measure the extent of measurement error of the pretests. (For discussion of errors-in-variables regression, see Fuller, 1987.) This produces estimates of the coefficients on the pretests λ_1, λ_1^{alt}, λ_2, and λ_2^{alt}.

Second, over the same sample of students, we produce a growth residual that controls for student-level prior achievement equal to:

$$q_{ijt} = Y_{ijt} - \hat{\lambda}_1 Y_{ijt-1} - \hat{\lambda}_1^{alt} Y_{ijt-1}^{alt} - \hat{\lambda}_2 Y_{ijt-2} - \hat{\lambda}_2^{alt} Y_{ijt-2}^{alt} \qquad (2)$$

where $\hat{\lambda}_1$, $\hat{\lambda}_1^{alt}$, $\hat{\lambda}_2$, and $\hat{\lambda}_2^{alt}$ are the estimates of λ_1, λ_1^{alt}, λ_2, and λ_2^{alt} produced in the previous step.

Third, we regress q_{ijt} on $\bar{Y}_{jt-1}$ and $\bar{Y}_{jt-1}^{alt}$ using ordinary least squares, producing estimates of the coefficients on the average pretests γ and γ^{alt}. The residual from this regression, which we denote w_{ijt}^{*}, is a growth residual that controls for student-level and school-level prior achievement. It is an estimate of the sum of the school effect α_{jt} and the residual term ε_{ijt}.

Incorporating Students With Only Two Years of Scores

The estimation approach above produces growth residuals for students with measured scores in all three years (*t*, *t*-1, and *t*-2). To include students with measured scores in the two most recent years only (*t* and *t*-1), we repeat the steps in the previous section above by grade and subject over a sample of students with measured scores in the two most recent years (*t* and *t*-1) using a version of (1) that does not include Y_{ijt-2} and Y_{ijt-2}^{alt} on the right-hand-side. This sample of students includes all students included in the estimation steps described in the previous section, as well as an additional group of students with measured scores in years *t* and *t*-1 but not in year *t*-2. This produces another set of growth residuals, which we denote $w_{ijt}^{\dagger}$, which covers all students with measured scores in the two most recent years (*t* and *t*-1).

Over a sample of all students with measured scores in years *t* and *t*-1, we create a combined growth residual equal to:

$$w_{ijt} = \begin{cases} w_{ijt}^{*} & \text{if student } i \text{ has measured scores in years } t, t\text{-1, and } t\text{-2} \\ w_{ijt}^{\dagger} & \text{if student } i \text{ has measured scores in years } t \text{ and } t\text{-1 only} \end{cases} \quad (3)$$

The growth residual w_{ijt} includes all students with scores in years t and t-1 and controls for student achievement in year t-2 when possible. This growth residual is demeaned to have a mean of zero by grade and subject.

PRODUCING SCHOOL-AND-GRADE GROWTH MEASURES

We produce school growth measures by regressing, by ordinary least squares, the combined student growth residual w_{ijt} on a full set of school indicator variables. As before, this regression is estimated separately by grade and subject. This produces a fixed effect estimate $\hat{\alpha}_{jt}$ for each school by grade and subject. This estimates the impact of school j on student achievement in year t relative to the average school impact across students. It is measured in units of standard deviations of achievement across students in the measured posttest Y_{ijt}.

We estimate the variance, corrected for sampling error, of the school effects α_{jt} using the following equation:

$$\widehat{\omega}^2 = Var[\hat{\alpha}_{jt}] - Mean[\hat{\sigma}_{jt}^2] \quad (4)$$

where $\widehat{\omega}^2$ is the estimate of the variance of α_{jt} across schools and $\hat{\sigma}_{jt}^2$ is the square of the estimated standard error of $\hat{\alpha}_{jt}$. We use this variance estimate for two purposes. First, we use it to produce a shrinkage estimate of each school effect using Empirical Bayes shrinkage, using the following formula:

$$\tilde{\alpha}_{jt} = r_{jt}\hat{\alpha}_{jt} \quad (5)$$

where r_{jt}, the reliability of the value-added measure $\hat{\alpha}_{jt}$, is equal to $\widehat{\omega}^2\left(\widehat{\omega}^2 + \hat{\sigma}_{jt}^2\right)^{-1}$. The shrinkage has the effect of tempering school estimates based on small numbers of students, which are typically overrepresented among the highest and lowest values of $\hat{\alpha}_{jt}$ as a result of randomness in individual student growth, toward the average growth measure of zero. The standard error of the shrunk growth measure $\tilde{\alpha}_{jt}$ is equal to:

$$\tilde{\sigma}_{jt} = r_{jt}^{1/2}\hat{\sigma}_{jt} \quad (6)$$

Second, we use the variance estimate $\widehat{\omega}^2$ to rescale both the unshrunk growth measure $\hat{\alpha}_{jt}$ and the shrunk growth measure $\tilde{\alpha}_{jt}$ to be measured in standard deviations of growth across schools. This is accomplished by dividing the growth measure by the square root of $\widehat{\omega}^2$:

$$\hat{\alpha}_{jt}^{tier} = \hat{\alpha}_{jt}/\widehat{\omega} \quad (7a)$$

$$\tilde{\alpha}_{jt}^{tier} = \tilde{\alpha}_{jt}/\widehat{\omega} \quad (7b)$$

where $\hat{\alpha}_{jt}^{tier}$ and $\tilde{\alpha}_{jt}^{tier}$ are the rescaled growth measures, with the *tier* superscript indicating that the growth measures have been rescaled to occupy "tiers" that are for the most part between -3 and +3. The standard errors of the rescaled growth measures are similarly estimated by dividing their standard errors before rescaling by the standard deviation estimate $\widehat{\omega}$.

Producing Multi-Grade School Growth Measures

To produce multi-grade school growth measures, we average, by subject, the rescaled, unshrunk growth measures $\hat{\alpha}_{jt}^{tier}$ across grades within each school, using the the number of students associated with the grade and subject within the school as a weight. This produces an unshrunk schoolwide growth measure for a given subject. We compute a multi-grade sampling-error-adjusted variance estimate $\widehat{\omega}^2$ across these schoolwide measures by subject as in equation (4), and then shrink and rescale them using $\widehat{\omega}^2$ as in equations (5) through (7) above. This produces shrunk, rescaled schoolwide growth measures in math and English language arts.

Producing School-and-Grade Subgroup Growth Measures

In addition to producing overall value-added measures for each school, we also produce growth measures for subgroups within schools, using the steps described below. Suppose we can categorize students into *S* subgroups, with each subgroup *s* corresponding to a number between 1 and *S*, and let each school *j* have an impact of α_{jst} among students in subgroup *s*. We estimate α_{jst} by averaging the growth residual w_{ijt} across all students in subgroup *s* in school *j*; this estimate, denoted $\hat{\alpha}_{jst}$, is computed separately by subject and grade. This is equivalent to regressing w_{ijt} on a full set of school-subgroup interactions. The error from this regression is equal to $w_{ijt} - \hat{\alpha}_{jst}$. We estimate separately by subgroup this error's variance, which we denote as $\hat{\sigma}_{e(s)}^2$, and estimate the standard error of the value-added measure $\hat{\alpha}_{jst}$ using $\hat{\sigma}_{e(s)}/\sqrt{n_{jst}}$, where n_{jst} is the number of students in subgroup *s* in school *j*.

We take an additional step when estimating $\hat{\alpha}_{jst}$ when the subgroups *s* are defined in some way by whether a student is in the lowest quintile of measured prior achievement Y_{ijt-1} within their school by subject and grade. In this case, measurement error in Y_{ijt-1} biases the estimate $\hat{\alpha}_{jst}$ upward when the subgroup *s* is made up of students in the lowest quintile and downward when the subgroup *s* is made up of students not in the lowest quintile. To reduce the impact of this bias, we demean the estimates $\hat{\alpha}_{jst}$ that include students in the lowest quintile by the mean, weighted by n_{jst}, across schools among all the estimates $\hat{\alpha}_{jst}$ that include students in the lowest quintile. Similarly, we demean the estimates $\hat{\alpha}_{jst}$ that do not include students in the lowest quintile by the mean, weighted by n_{jst}, across schools among all the estimates $\hat{\alpha}_{jst}$ that do not include students in the lowest quintile.

After estimating the subgroup growth estimates $\hat{\alpha}_{jst}$, we shrink them using a multivariate shrinkage estimate that takes the correlation of growth across subjects within schools into account. This shrinkage is executed as in equation (8):

$$\tilde{\alpha}_{jt}^{sub} = \mu_{jt}^{sub} + \Omega^{sub}\left(\Omega^{sub} + \hat{\Sigma}_{jt}^{sub}\right)^{-1}\left(\hat{\alpha}_{jt}^{sub} - \mu_{jt}^{sub}\right) \qquad (8)$$

where

$$\tilde{\alpha}_{jt}^{sub} = \begin{bmatrix} \tilde{\alpha}_{j1t} \\ \tilde{\alpha}_{j2t} \\ \vdots \\ \tilde{\alpha}_{jSt} \end{bmatrix}; \qquad \hat{\alpha}_{jt}^{sub} = \begin{bmatrix} \hat{\alpha}_{j1t} \\ \hat{\alpha}_{j2t} \\ \vdots \\ \hat{\alpha}_{jSt} \end{bmatrix}; \qquad \mu_{jt}^{sub} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_S \end{bmatrix};$$

$$\Omega^{sub} = \begin{bmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1S} \\ \omega_{21} & \omega_{22} & \cdots & \omega_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{S1} & \omega_{S2} & \cdots & \omega_{SS} \end{bmatrix}; \qquad \hat{\Sigma}_{jt}^{sub} = \begin{bmatrix} \hat{\sigma}_{j1t}^2 & 0 & \cdots & 0 \\ 0 & \hat{\sigma}_{j2t}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\sigma}_{jSt}^2 \end{bmatrix};$$

the parameter $\tilde{\alpha}_{jst}$ is the shrunk estimate of α_{jst}; μ_s is the mean in α_{jst} within subgroup *s* across schools *j*; and ω_{s,s^*} is the covariance across schools *j* between α_{jst} and α_{js^*t}. For discussion of multivariate shrinkage, see Longford (1999).

The structure of Ω^{sub} depends on whether the subgroups are defined using only one criterion (for example, if subgroup is defined by race/ethnicity and only by race/ethnicity, or by lowest quintile status and only by lowest quintile status) or by the interaction of two criteria (which, in this application, is always an interaction between lowest quintile status and a second criteria, such as race/ethnicity). If the subgroups are defined by only one criterion, then we assume that all of the diagonal elements of Ω^{sub}, which correspond to variances of α_{jst} across schools, equal the same value ω_V, while all of the off-diagonal elements of Ω^{sub}, which correspond to covariances of α_{jst} and α_{js^*t} across schools where $s \neq s^*$, equal the same value ω_C.

If, on the other hand, the subgroups are defined by the interaction of two criteria (which, as noted before, is always lowest-quintile status and another criterion), then we assume that the entries of Ω^{sub} take one of four possible values: ω_V, when the element corresponds to a variance; ω_L, when the element corresponds to a covariance between α_{jst} and α_{js^*t} where $s \neq s^*$ but both *s* and *s** are made up of students with the same lowest quintile status; ω_O, when the element corresponds to a covariance between α_{jst} and α_{js^*t} where $s \neq s^*$ but both *s* and *s** are made up of students who meet the same criteria other than that of lowest quintile; and ω_N, where $s \neq s^*$ and the students in *s* and *s** have neither the same lowest-quintile status nor meet the same criteria other than that of lowest quintile.

In practice, we use estimates of μ_{jt}^{sub} and Ω^{sub} to implement the shrinkage described in equation (8). The means μ_s are estimated using the mean of the estimates $\hat{\alpha}_{jst}$ across schools *j* within subgroup *s*, weighted by n_{jst}. The elements of Ω^{sub} are estimated as in equation (9):

$$\omega_X = \frac{\sum_s \sum_{s^*} \sum_j I(s,s^*,X)\sqrt{n_{jst}\, n_{js^*t}}\left(\hat{\alpha}_{jst}-\hat{\mu}_s\right)\left(\hat{\alpha}_{js^*t}-\hat{\mu}_{s^*}\right)}{\sum_s \sum_{s^*} \sum_j I(s,s^*,X)\sqrt{n_{jst}\, n_{js^*t}}} \tag{9}$$

where X is V, C, L, O, or N; $I(s, s^*, X)$ is a variable that equals 1 if the combination of s and s^* applies to the variance or covariance ω_X as defined above and 0 otherwise; and μ_s is the estimate of μ_s. If the variances and covariances estimated using (9) produce an estimated Ω^{sub} matrix that is not positive semidefinite, we constrain the estimates of ω_X with a set of business rules to ensure a positive semidefinite Ω^{sub}. When the subgroups s are determined by only one criterion, we constrain ω_C to be no greater than 0.99 times ω_V. When the subgroups s are determined by the interaction of lowest-quintile status and another criteria, we constrain ω_N to be at least zero, ω_L and ω_O to be at least as great as ω_N, and ω_V to be at least as great as ω_L + ω_O - ω_N.

After shrinkage, we adjust all of the post-shrinkage subgroup measures $\tilde{\alpha}_{jst}$ to be consistent with the overall measures by adding to them $\tilde{\alpha}_{jt} - \sum_s \left(n_{jst}/n_{jt}\right) \tilde{\alpha}_{jst}$, where $\tilde{\alpha}_{jt}$ is the shrunk overall measure for school j. We also rescale the subgroup measures to conform to the rescaled overall measures by dividing them by $\hat{\omega}$, the square root of the ω^2 variance measure estimated for the overall measures using equation (4). We denote these measures $\tilde{\alpha}_{jst}^{tier}$.

The steps above are implemented separately by subject and grade for each set of subgroups. For example, the steps above are implemented once when producing subgroup measures by economic status, and then are entirely implemented again when producing subgroup measures by lowest quintile, with new estimates of α_{jst} μ_{jt}^{sub}, Ω^{sub}, etc.

Producing Multi-Grade Subgroup Growth Measures

Subgroup growth measures are not only produced by school, grade, and subject, but also as multi-grade measures by school and subject. After averaging w_{ijt} by school, grade, subject, and subgroup, we rescale the resulting unshrunk estimate $\hat{\alpha}_{jst}$ by dividing it by the $\hat{\omega}$ estimate corresponding to the same grade and subject to produce the rescaled estimate $\hat{\alpha}_{jst}^{tier}$. These are averaged across grades within school, subject, and subgroup using n_{jst} as a weight to produce an unshrunk schoolwide estimate by subject and subgroup. After these are produced, all of the steps in the previous section that follow producing $\hat{\alpha}_{jst}$ and its standard error from the residuals w_{ijt} are applied to the schoolwide subgroup measures: demeaning $\hat{\alpha}_{jst}$ when one of the criteria is lowest-quintile status; estimating μ_{jt}^{sub} and Ω^{sub} using equation (9) and shrinking $\hat{\alpha}_{jst}$ using equation (8); adjusting the shrunk measures to be consistent with the overall results; and rescaling the results by dividing by the $\hat{\omega}$ estimate corresponding to schoolwide estimates in the same subject.

Producing Final School Measures

The final school measures are based on the shrunk, rescaled overall estimates $\tilde{\alpha}_{jt}^{tier}$ and their subgroup analogues $\tilde{\alpha}_{jst}^{tier}$. We create multi-subject measures by computing a weighted

average of the shrunk, rescaled measures across math and English Language Arts, using the number of students associated with the measure (which will typically be equal or very close to equal across the two subjects) as a weight. We produce final school scores by computing the average of a school's multi-grade overall measure and its multi-grade subgroup measure for students in the lowest quintile of prior achievement. These are computed at the multi-subject level by combining the overall and lowest-quintile multi-subject measures, as well as separately for English language arts and math by combining the overall and lowest-quintile single-subject measures. Finally, all measures are rescaled to a 0-to-40 scale for reporting by multiplying by (20/2.25) and adding 20.

PROPERTIES OF THE VALUE-ADDED RESULTS

COEFFICIENT ESTIMATES

The coefficients estimated in the value-added model are presented in Tables 3 through 6. To interpret these coefficients, note that both pretest and posttest are measured using scores that have been standardized to have a mean of zero and a standard deviation of one. For example, note, in the model of 2019 fourth-grade math, that the coefficient on the 2018 math pretest is 0.882 (see Table 3). This means that, at the student level, a one standard deviation increase in the 2018 third-grade math assessment is associated with a 0.882 standard deviation increase in the 2019 fourth-grade math assessment.

It is important to keep in mind the standard errors of the coefficients when interpreting them. A span of 1.96 standard errors in both the positive and negative directions provides a 95 percent confidence range for a coefficient. Continuing with the previous example, note that, in the model of 2019 fourth grade math, the estimated coefficient on 2018 math pretest has a standard error of 0.007. This means that, while our best estimate of the standardized impact on 2018 third-grade math achievement on 2019 fourth grade math achievement is 0.882, a 95 percent confidence interval for this estimated impact ranges from 0.868 to 0.896.

Note that coefficients are presented below for models that include two lags of prior achievement, as well as for models that include only one lag of prior achievement. The coefficients from models that include two lags of prior achievement are employed when measuring the growth of students for whom two lags of prior achievement are available. The coefficients from models that include only one lag of prior achievement are employed when measuring the growth of students for whom only one lag of prior achievement is available.

Table 3. Coefficient Estimates in Double-Lag Model, 2018-19 SC READY Math

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
Variable	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**
2018 Math Pretest	0.882	0.007	0.667	0.011	0.613	0.011	0.656	0.012	0.708	0.015
2018 ELA Pretest	0.026	0.007	0.070	0.012	0.102	0.011	0.081	0.010	0.082	0.013
2018 sch. avg. Math Pretest	-0.121	0.050	-0.127	0.043	-0.250	0.069	-0.074	0.037	-0.088	0.064
2018 sch. avg. ELA Pretest	0.207	0.050	0.154	0.046	0.344	0.071	0.150	0.041	0.167	0.070
2017 Math Pretest	n/a	n/a	0.180	0.012	0.192	0.010	0.233	0.010	0.215	0.014
2017 ELA Pretest	n/a	n/a	0.034	0.013	0.032	0.010	-0.006	0.009	-0.074	0.011

Table 4. Coefficient Estimates in Single-Lag Model, 2018-19 SC READY Math

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
Variable	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**
2018 Math Pretest	0.882	0.007	0.812	0.005	0.789	0.006	0.876	0.006	0.903	0.007
2018 ELA Pretest	0.026	0.007	0.124	0.005	0.130	0.006	0.076	0.006	0.020	0.006
2018 sch. avg. Math Pretest	-0.121	0.050	-0.168	0.044	-0.299	0.071	-0.198	0.037	-0.136	0.065
2018 sch. avg. ELA Pretest	0.207	0.050	0.197	0.047	0.402	0.073	0.269	0.041	0.217	0.071

Table 5. Coefficient Estimates in Double-Lag Model, 2018-19 SC READY ELA

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
Variable	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**
2018 Math Pretest	0.034	0.006	0.070	0.010	0.066	0.011	0.077	0.011	0.039	0.014
2018 ELA Pretest	0.888	0.006	0.604	0.011	0.617	0.011	0.637	0.010	0.735	0.012
2018 sch. avg. Math Pretest	0.111	0.034	-0.011	0.028	-0.048	0.044	-0.036	0.032	0.051	0.038
2018 sch. avg. ELA Pretest	-0.094	0.034	0.040	0.030	0.039	0.045	0.066	0.035	-0.109	0.042
2017 Math Pretest	n/a	n/a	-0.040	0.011	-0.007	0.009	-0.022	0.009	-0.017	0.013
2017 ELA Pretest	n/a	n/a	0.316	0.012	0.281	0.010	0.259	0.009	0.194	0.011

Table 6. Coefficient Estimates in Single-Lag Model, 2018-19 SC READY ELA

	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8	
Variable	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**	**Coeff.**	**SE**
2018 Math Pretest	0.034	0.006	0.066	0.005	0.079	0.006	0.082	0.006	0.047	0.006
2018 ELA Pretest	0.888	0.006	0.865	0.005	0.859	0.006	0.854	0.005	0.898	0.006
2018 sch. avg. Math Pretest	0.111	0.034	0.003	0.028	-0.056	0.047	-0.034	0.034	0.078	0.040
2018 sch. avg. ELA Pretest	-0.094	0.034	0.030	0.030	0.052	0.049	0.063	0.037	-0.135	0.044

Variance and Reliability of Value-added Measures

Tables 7 and 8 present information about the variance and reliability of the value-added estimates by grade and subject. The first three rows present the three terms in equation (4). The first row is the variance across schools in the value-added estimates $\hat{\alpha}_{jt}$, which includes both the variance in the impacts of schools on student achievement as well as variance in the sampling error with which those impacts are estimated. The second row is an estimate of the variance of that sampling error, which is equal to the average of the squared standard errors across the value-added estimates. The third row is an estimate of the variance across schools in α_{jt}, the actual impacts of schools on student achievement. This is estimated by subtracting the second row from the first row, as described in equation (4).

The fourth row of Tables 7 and 8 is an estimate of the standard deviation across schools in their impacts on student achievement and is equal to the square root of the third row. The fifth row is an estimate of the reliability of the value-added measures. It is equal to the proportion of the variance in the value-added measures that is due to actual differences in the impacts of schools rather than variance from sampling error. It is computed by dividing the third row by the first row. For example, note that the reliability of value-added in eighth-grade ELA is .864. This means that 86.4% of the differences in our measures of value-added in eighth-grade ELA are the results of differences in the actual impacts of schools rather than in randomness in individual student growth across schools. Finally, the last row of Tables 7 and 8 present the number of schools used to compute the variance of school value-added.

The results in the first four rows of Tables 7 and 8 are measured in units of standard deviations of student achievement the posttest. For example, note that the estimated standard deviation of value-added in fourth-grade math is 0.194. This means that the standard deviation across schools in their impacts on student achievement is estimated to be 19.4% the size of the standard deviation across students in student achievement in fourth grade math.

Table 7. Variance and Reliability of Math Value-Added

	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8
Variance of estimates ($Var[\hat{\alpha}_{jt}]$)	0.041	0.033	0.032	0.011	0.026
Noise variance ($Mean[\hat{\sigma}_{jt}^2]$)	0.003	0.003	0.001	0.001	0.001
Estimated variance ($\hat{\omega}^2$)	0.037	0.030	0.031	0.010	0.025
Estimated standard deviation ($\hat{\omega}$)	0.194	0.174	0.175	0.098	0.157
Reliability ($\hat{\omega}^2$ / $Var[\hat{\alpha}_{jt}]$)	0.918	0.923	0.957	0.889	0.948
Number of schools	658	642	352	322	317

Table 8. Variance and Reliability of ELA Value-Added

	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8
Variance of estimates ($Var[\hat{\alpha}_{jt}]$)	0.019	0.013	0.013	0.008	0.009
Noise variance ($Mean[\hat{\sigma}_{jt}^2]$)	0.003	0.002	0.001	0.001	0.001
Estimated variance ($\hat{\omega}^2$)	0.016	0.011	0.012	0.007	0.008
Estimated standard deviation ($\hat{\omega}$)	0.125	0.104	0.108	0.085	0.088
Reliability ($\hat{\omega}^2$ / $Var[\hat{\alpha}_{jt}]$)	0.834	0.821	0.902	0.869	0.864
Number of schools	658	642	352	322	317

We also compute variance and reliability of the multi-grade value-added measures, which are presented in Table 9. These can be interpreted in the same way as the single-grade estimates, with the exception that the first four rows are measured in units of standard deviations of the single-grade value-added measures across schools rather than in units of standard deviations of student achievement. This is because the single-grade value-added measures are normalized before being aggregated into the multi-grade value-added measures.

Table 9. Variance and Reliability of Multi-Grade Value-Added

	MATH, ELEM.	MATH, MIDDLE	ELA, ELEM.	ELA, MIDDLE
Variance of estimates ($Var[\hat{\alpha}_{jt}]$)	0.613	0.450	0.715	0.576
Noise variance ($Mean[\hat{\sigma}_{jt}^2]$)	0.043	0.025	0.103	0.047
Estimated variance ($\hat{\omega}^2$)	0.570	0.424	0.612	0.529
Estimated standard deviation ($\hat{\omega}$)	0.755	0.651	0.782	0.727
Reliability ($\hat{\omega}^2$ / $Var[\hat{\alpha}_{jt}]$)	0.931	0.943	0.856	0.919
Number of schools	666	323	666	323

NEUTRALITY

CORRELATION WITH DEMOGRAPHIC VARIABLES

While the value-added model employed in South Carolina controls only for prior achievement and does not explicitly control for demographic variables such as economic disadvantage, English language learner, disability, or ethnicity, the correlations between the value-

added measures and demographic variables were generally very low. In other words, school value-added and average student characteristics were not substantively empirically related. These correlations are presented in Tables 10 and 11.

Table 10. Correlations between Student Demographics and Math Value-Added

Characteristic	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Elem.	Middle
Poverty	-0.06	-0.02	-0.06	-0.08	0.00	-0.05	-0.07
English Learner	0.06	0.06	-0.01	0.12	0.08	0.06	0.09
Disability	0.05	0.06	0.00	-0.04	-0.05	0.02	-0.09
African American	-0.12	-0.04	-0.12	-0.11	0.02	-0.10	-0.10
Asian	-0.01	0.00	-0.02	0.04	0.12	-0.03	0.08
Hispanic	0.07	0.04	-0.06	0.08	0.05	0.06	0.03
Native American	0.01	-0.05	-0.07	0.07	-0.08	-0.05	0.01
White	0.09	0.02	0.15	0.07	-0.05	0.08	0.08

Table 11. Correlations between Student Demographics and ELA Value-Added

Characteristic	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	Elem.	Middle
Poverty	-0.13	-0.10	-0.14	-0.09	-0.08	-0.10	-0.15
English Learner	0.01	0.08	-0.08	-0.07	-0.01	0.04	-0.06
Disability	0.03	0.08	-0.12	-0.10	-0.13	0.08	-0.21
African American	-0.12	-0.08	0.00	0.05	0.05	-0.09	0.03
Asian	0.11	0.07	0.11	0.08	0.04	0.10	0.12
Hispanic	0.02	0.08	-0.12	-0.08	0.02	0.04	-0.06
Native American	-0.05	-0.09	-0.09	-0.03	-0.10	-0.11	-0.07
White	0.10	0.04	0.04	-0.03	-0.06	0.06	-0.02

Correlation with Average Prior Proficiency

The correlation between value-added and prior proficiency at the school and grade level (e.g., the correlation between a school's value-added in fourth-grade math and proficiency rate in third-grade math) is close to zero. This is an expected result given that average prior achievement, measured using average pretest score among the students associated with the school and grade in the current year, is explicitly controlled for in the model. At the school level, there is a small positive correlation between a school's overall value-added and the proficiency rate at the school in the same subject in the previous year. This is an effect of a correlation between value-added and prior achievement across grades within a school (for example, a correlation between sixth-grade value-added in the current year and proficiency in grades other than grade five in the previous year). The correlations are presented in Table 12.

Table 12. Correlations between Prior Attainment and Value-Added

SUBJECT	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8	ELEM.	MIDDLE
Math	0.03	0.02	0.06	-0.01	0.00	0.07	0.12
ELA	0.05	0.02	0.06	0.00	0.02	0.12	0.15

CORRELATION BETWEEN MATH AND ELA

There were substantive positive correlations between math and ELA value-added within each school. Schools that were high value-added in math were also more often than not high value-added in ELA. This implies that schools with a higher-than-average impact in mathematics also had a higher-than-average impact in English language arts. These correlations are presented in Table 13.

Table 13. Correlations in Value-Added between Subjects

	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8	ELEM.	MIDDLE
2019 Math/ELA	0.56	0.46	0.53	0.33	0.35	0.50	0.37

CORRELATION BETWEEN VALUE-ADDED IN AND OUT OF THE LOWEST QUINTILE

An important differential-effects measure produced as part of value-added in South Carolina is value-added for students in the lowest quintile of prior achievement by school, grade, and subject. As a part of the multivariate shrinkage steps described in equations (8) and (9), we compute both the covariance across schools between school value-added in and out of the lowest quintile (denoted above as ω_C), as well as the variance of school value-added across schools within (or not within) the lowest quintile (denoted above as ω_V). The ratio of these two is a measure of the correlation between value-added in and out of the lowest quintile, adjusted for sampling error. These are presented in Table 14 below. The results in Table 14 suggest that school value-added is substantially correlated between a school's lowest quintile of prior achievement and a school's highest four quintiles of prior achievement.

Table 14. Correlations in Value-Added In and Out of the Lowest Quintile

	GRADE 4	GRADE 5	GRADE 6	GRADE 7	GRADE 8	ELEM.	MIDDLE
2019 Math	0.58	0.64	0.59	0.45	0.51	0.57	0.45
2019 ELA	0.70	0.65	0.84	0.65	0.83	0.67	0.79

REFERENCES

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