South Carolina College- and Career-Ready Standards for Mathematics

6th Grade Support Document

South Carolina Department of Education
Office of Standards and Learning
August 2016 - DRAFT

South Carolina College- and Career-Ready Standards for Mathematics
Grade 6 Overview

The Table of Contents below arranges the *South Carolina College- and Career-Ready Standards for Mathematics* for middle school into *Course Coversheets* and *Units*.

- Each middle school *Course Coversheet* organizes the middle school course standards into possible instructional units and provides links to specific middle school course *Units*.
- Each middle school course *Unit* contains:
  - Clarifying notes related to the standards within the unit
  - New academic vocabulary in the unit
  - Prior and subsequent knowledge related to the unit
  - Description of the relationship between the standards in the unit
  - Potential instructional strategies and lessons
  - Resources for the unit
  - Sample formative assessment tasks and questions

DRAFT

South Carolina College- and Career-Ready Standards for Mathematics
Grade 6 Overview

Table of Contents

Unit	Standards	Support Document		
Number System	6.NS.1 6.NS.2 6.NS.3 6.NS.4 6.NS.9	Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Potential Instructional Strategies/Lessons
		New Academic Vocabulary	Subsequent Knowledge Related to this Unit	Resources
			Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions
Ratios and Rates	6.RP.1 6.RP.2 6.RP.3	Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Potential Instructional Strategies/Lessons
		New Academic Vocabulary	Subsequent Knowledge Related to this Unit	Resources
			Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions
Graphing and Rational Numbers	6.NS.5 6.NS.6 6.NS.7 6.NS.8	Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Potential Instructional Strategies/Lessons
		New Academic Vocabulary	Subsequent Knowledge Related to this Unit	Resources
			Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions
Expressions	6.EEI.1 6.EEI.2 6.EEI.3 6.EEI.4	Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Potential Instructional Strategies/Lessons
		New Academic Vocabulary	Subsequent Knowledge Related to this Unit	Resources
			Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

DRAFT

South Carolina College- and Career-Ready Standards for Mathematics
Middle School Support Document
Grade 6 Coversheet

Return to Middle School Overview or Table of Contents

Table of Contents (Continued)

Equations	6.EEI.5 6.EEI.6 6.EEI.7 6.EEI.8 6.EEI.9	Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Potential Instructional Strategies/Lessons
		New Academic Vocabulary	Subsequent Knowledge Related to this Unit	Resources
			Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions
Statistics	6.DS.1 6.DS.2 6.DS.3 6.DS.4 6.DS.5	Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Potential Instructional Strategies/Lessons
		New Academic Vocabulary	Subsequent Knowledge Related to this Unit	Resources
			Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions
Geometry	6.GM.1 6.GM.2 6.GM.3 6.GM.4	Content Standards with Clarifying Notes	Prior Knowledge Required for this Unit	Potential Instructional Strategies/Lessons
		New Academic Vocabulary	Subsequent Knowledge Related to this Unit	Resources
			Relationship Among Standards in this Unit	Sample Formative Assessment Tasks/Questions

DRAFT

South Carolina College- and Career-Ready Standards for Mathematics
Middle School Support Document
Grade 6 Coversheet

Return to Middle School Overview or Table of Contents

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
Number System	Ratios and Rates	Graphing and Rational Numbers	Expressions	Equations	Statistics	Geometry
Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
6.NS.1 6.NS.2 6.NS.3 6.NS.4 6.NS.9	6.RP.1 6.RP.2 6.RP.3	6.NS.5 6.NS.6 6.NS.7 6.NS.8	6.EEI.1 6.EEI.2 6.EEI.3 6.EEI.4	6.EEI.5 6.EEI.6 6.EEI.7 6.EEI.8 6.EEI.9	6.DS.1 6.DS.2 6.DS.3 6.DS.4 6.DS.5	6.GM.1 6.GM.2 6.GM.3 6.GM.4

DRAFT

South Carolina College- and Career-Ready Standards for Mathematics
Middle School Support Document
Grade 6 Coversheet

Return to Middle School Overview or Table of Contents

Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

1. Make sense of problems and persevere in solving them. a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

DRAFT

Unit Title
Number System
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
6.NS.1 Compute and represent quotients of positive fractions using a variety of procedures (e.g., visual models, equations, and real-world situations). ○ Divide positive fractions by fractions using visual models and equations. ○ Solve real-world problems using division of fractions. **6.NS.2** Fluently divide multi-digit whole numbers using a standard algorithmic approach. ○ Divide whole numbers by whole numbers Convert remainders to fractional parts in simplest form and decimal notation **6.NS.3** Fluently add, subtract, multiply and divide multi-digit decimal numbers using a standard algorithmic approach. ○ Perform all operations with decimal notation Modeling equivalent numerical expressions to support the understanding of division of decimal numbers **6.NS.4** Find common factors and multiples using two whole numbers. ○ Compute the greatest common factor (GCF) of two numbers both less than or equal to 100. ○ Compute the least common multiple (LCM) of two numbers both less than or equal to 12. ○ Express sums of two whole numbers, each less than or equal to 100, using the distributive property to factor out a common factor of the original addends. ○ Understand that greatest common factor and least common multiple are ways to discuss number relationships in multiplication and division. ○ Understand the process of prime factorization. ○ Understand the distributive property using sums and its use in adding numbers 1-100 with a common factor. ○ Use LCM and GCF to teach fluency for adding and subtracting of fractions using a standard algorithmic approach. **6.NS.9** Investigate and translate among multiple representations of rational numbers (fractions, decimal numbers, percentages). Fractions should be limited to those with denominators of 2, 3, 4, 5, 8, 10, and 100. ○ Recognize ⅛ as half of ¼ to assist with conversions within all representations. ○ Understand that fractions with a denominator of 3 will generate a repeating decimal (limit repeating decimals to fractions with a denominator of 3).

DRAFT

New Academic Vocabulary for This Unit
● Reciprocal ● Inverse ● Greatest common factor ● Least common multiple ● Prime factorization ● Distributive property ● Rational number

Prior Knowledge Required for this Unit
● Multiplication facts (3.ATO.1, 3.ATO.3) ● Understand the relationship between multiplication and division (5.NSF.3) ● Understand parts of a fraction (3.NSF.1) ● Divide up to four-digit dividends by two-digit divisors (5.NSBT.6) ● Add, subtract, multiply, and divide decimal numbers to hundredths using concrete area models and drawings (5.NSBT.7) ● Firm conceptual understanding of place value (3.NSBT.1, 4.NSBT.1, 5.NSBT.1)
Subsequent Knowledge Related to this Unit
This unit will end direct instruction for operations with whole numbers, fractions, and decimals. To ensure readiness for work with integers in Grade 7, students must be computationally fluent with these operations. In Grade 8, students will be solving multi-step equations where the computational skills will be secondary skills in an algebraic approach. Students will begin multiple representations of rational numbers with limited denominators in Grade 6. That knowledge in Grades 7 and 8 will be extended to include all denominators and repeated decimals in Grade 8. The information taught in this unit will also prepare students for ratios and rates including work with greatest common factor and least common multiple for simplifying rates. This knowledge will be extended in Grade 8 to include work with functions including linear functions where students will analyze slope as the constant rate of change.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop computational skills necessary for work with positive rational numbers.

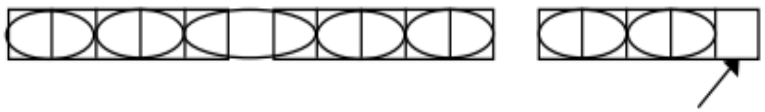
DRAFT

Potential Instructional Strategies/Lessons

- Visual models - Multiple representations of visual models should be used to show multiplication and division of fractions.

Example 1:

Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, "how many $\frac{2}{5}$ are in 3?" One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of $\frac{1}{2}$. Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are $7\frac{1}{2}$ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.



This section represents one-half of two-fifths

Students also write contextual problems for fraction division problems. For example, the problem, $\frac{2}{3} \div \frac{1}{6}$ can be illustrated with the following word problem:

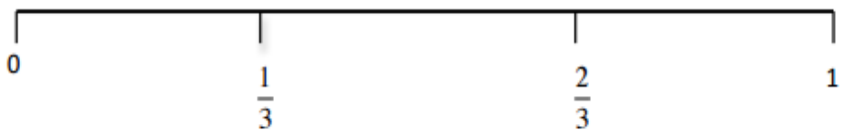
Source: NC DPI 6th Grade Mathematics Unpacked Contents

Example 2:

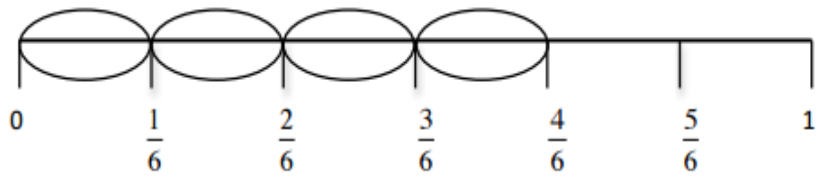
Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make?

This problem can be modeled using a number line.

a. Start with a number line divided into thirds.



b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.

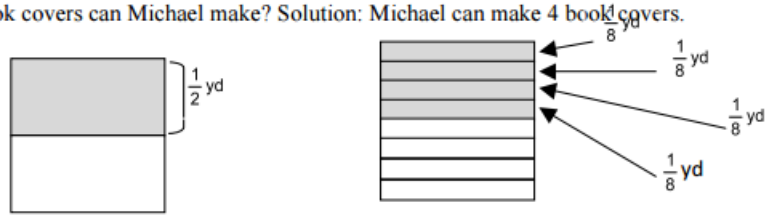


c. Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

Source: NC DPI 6th Grade Mathematics Unpacked Contents

Example 3:

Michael has $\frac{1}{2}$ of a yard of fabric to make book covers. Each book cover is made from $\frac{1}{8}$ of a yard of fabric. How many book covers can Michael make? Solution: Michael can make 4 book covers.



Example 4:

Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: A recipe requires $\frac{2}{3}$ of a cup of yogurt. Rachel has $\frac{1}{2}$ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?

Explanation of Model:

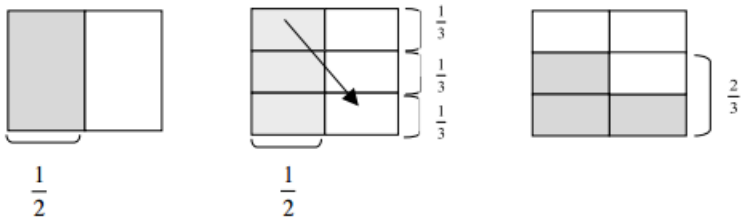
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole) .

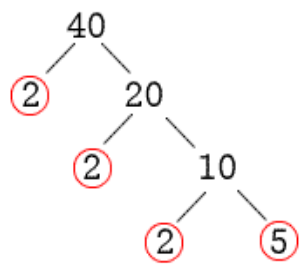
3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so only ¾ of the recipe can be made.



Source: NC DPI 6th Grade Mathematics Unpacked Contents

- Algorithmic approaches to divide multi-digit numbers
- Algorithmic approaches to add, subtract, multiply, and divide decimal numbers

- For division, modeling equivalent numerical expressions will support understanding of moving the decimal point
- Prime factorization

Prime Factor Tree	
• Start by dividing the given number by the smallest prime which is 2. • The factors of the number above are broken down into "branches" as indicated by the line segments. • We are able to divide **40** and its quotient by the prime number 2 three times which means this prime number will have an exponent of 3 in the factorization. • The last quotient after repeated division of 2 is a prime number which is 5. • Upon reaching a prime number as its last quotient in the process, this shows that we are done!	 $40 = 2 \times 2 \times 2 \times 5$ $= 2^3 \times 5$

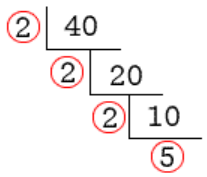
Source: Chili Math

- Upside Down Division (Birthday Cake)

Upside-Down Division

- Now you know why it is called the Upside-Down Division because the division symbol is literally upside-down.

- I start dividing the given number by the smallest prime number which is 2. If that prime evenly divides the number, then I place the quotient below. Continue the process as needed.
- Notice that we are able to perform repeated division of prime number 2, until reaching the prime number 5 as its final whole number quotient (most bottom).
- Present the final factorization as product of exponential numbers having a prime number base in the exponential notation.



$$40 = 2 \times 2 \times 2 \times 5$$

$$= 2^3 \times 5$$

Source: Chili Math

Resources

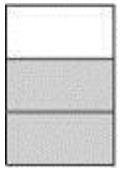
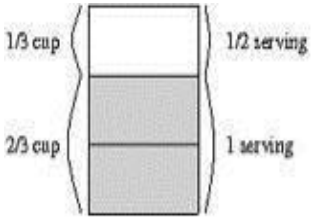
6.NS.1 - This game allows students to practice the division of fractions.
http://www.math-play.com/math-basketball-dividing-fractions-game/math-basketball-dividing-fractions-game.html

6.NS.1 - This performance task requires students to divide fractions, interpret quotients, and support solutions.
http://schools.nyc.gov/NR/rdonlyres/946D93E8-E911-4589-871C-97317E227C3C/141874/NYCDOE_G6_Math_SharemyCandy_FINAL.pdf

6.NS.9 - The included activities encourage students to perform operations with fractions and translate between the multiple representations of rational numbers.
http://empower.terc.edu/pdf/Using_Benchmarks.pdf

This website provides real-world problems associated with this unit.
https://www.illustrativemathematics.org/NS

DRAFT

Sample Formative Assessment Tasks/Questions
6.NS.1: Tonya and Chrissy are trying to understand the following story problem for 1÷2/3. *One serving of rice is* 2/3 *of a cup. I ate 1 cup of rice. How many servings of rice did I eat?* To solve the problem, Tonya and Chrissy draw a diagram divided into three equal pieces, and shade two of those pieces.  Tonya says, "There is one 2/3-cup serving of rice in 1 cup, and there is ⅓ cup of rice left over, so the answer should be 1 ⅓ " Chrissy says, "I heard someone say that the answer is 3/2=1 ½ . Which answer is right?" Is the answer 1 ⅓ or 1 ½ ? Explain your reasoning using the diagram. *Answer* In Tonya's solution of 1 ⅓ , she correctly notices that there is one ⅔ cup serving of rice in 1 cup, and there is cup of rice left over. But she is mixing up the quantities of servings and cups in her answer. The question becomes how many servings is ⅓ cup of rice? The answer is "⅓ cup of rice is 12 of a serving." It would be correct to say, "There is one serving of rice with ⅓ cup of rice left over," but to interpret the quotient 1 ½ the units for the 1 and the units for the ½ must be the same: *There are* 1 ½ *servings in 1 cup of rice if each serving is ⅔ cup.* (Source: Illustrative Mathematics)    Source: Illustrative Mathematics
6.NS.2: Southern Middle School is hosting a football game this Thursday. Band members are selling ads for the game's program. Their goal is to sell $3,462 worth of ads. If the band members sell each ad for $15, determine how many ads they will need to sell to reach their goal. Justify your answer. *Answer* 231 ads - To determine the number of ads to be sold, students should divide the cost of each ad ($15) into the desired sells ($3462). The resulting quotient is 230 with a remainder 12. Since the quotient is 230 with a remainder 12, students need to determine that 231 ads must be sold to reach the goal of $3462 raised.

DRAFT

6.NS.3: Jalyn and her 3 friends bought snacks that cost $2.86, $7.52, $4.38, $2.95, and $3.05. If they share the cost, how much will each student have to pay to divide it evenly? *Answer* $5.19
6.NS.4b: Johnny and Maria like to go to the mall and window shop. Both Johnny and Maria are at the mall at the same time today. Johnny goes to the mall every 5 days while Maria goes to the mall every 6 days. When will Johnny and Maria run into one another again at the mall? (GCF) *Answer* 30 days
6.NS.4a,c: Bob scored 24 points in a basketball game while Jim scored 40 points. Write an equivalent numerical expression to find the sum of these two numbers. Justify your thinking. *Answer* 8(3 + 5) – To determine this answer the student needs to see that both 24 and 40 have a greatest common factor of 8. The student explains that factoring out the eight in both numbers allows them to write an equivalent expression using the distributive property. By factoring out the 8 you are left with adding 3 and 5 in the parentheses.
6.NS.9: Which form is best to use when comparing rational numbers? Explain your rationale. *Answer* Student answers will vary. Be sure their rationale supports the form they select.
6.NS.9: The Dream Team - In this task, students are the head of a basketball team in the NBA. Their three best starters are injured and not available to play in the next game. It is now their task to look at the statistics provided, and decide which five players will start the game. Source: Converting and Ordering Rational Numbers [6th grade]

DRAFT

6.NS.9: Before a game, Jake's batting average was exactly 0.350. That is the decimal representation for number of hits to number of times at bat. During the game, Jake bats 4 times and gets 2 hits. If Jake's batting average after the game is 0.359, how many times had Jake batted before the end of the game? Explain your reasoning.

Answer
64 at bats after the game - To check Jake's batting average after the game with 2 hits in 4 at bats, you can take a row of numbers from the ratio table, add 2 to the first number and 4 to the second, and then evaluate the quotient. For example, if Jake had 7 hits in 20 at bats before the game, then we would have 9 hits in 24 at bats after the game. Since 9/24=3/8=0.375, this is not the correct value. With 14 hits in 40 at bats, Jake would have 16 hits in 44 at bats for a batting average of 16/44=4/11≈0.364 so this is also not correct. With 21 hits in 60 at bats before the game, Jake would have 23 hits in 64 at bats after the game. Since 23/64≈0.359, this could be correct. The next value to calculate would be 28 hits in 80 at bats entering the game so 30 hits in 84 at bats after the game: 30/84=5/14≈0.357, so this is not correct. The more at bats Jake has before the game, the less impact his 2 hits in 4 at bats have on his overall batting average. The only possibility that fits the given information is that Jake had 21 hits in 60 at bats before the game therefore he had 64 at bats at the end of the game.

DRAFT

Unit Title
Ratios and Rates
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
6.RP.1 Interpret the concept of a ratio as the relationship between two quantities, including part to part and part to whole. ○ Ratios compare two quantities ○ Simplify ratios to simplest form **6.RP.2** Investigate relationships between ratios and rates. a. Translate between multiple representations of ratios (i.e., a/b , a : b, a to b, visual models). b. Recognize that a rate is a type of ratio involving two different units. c. Convert from rates to unit rates. ○ When writing a ratio, order of terminology matters ○ Transfer between multiple representations of ratios ○ Understand that operations with ratios are generally performed when the ratio is written in fractional form ○ Exclude complex fractions (i.e. ⅓ / 4) **6.RP.3** Apply the concepts of ratios and rates to solve real-world and mathematical problems. a. Create a table consisting of equivalent ratios and plot the results on the coordinate plane. b. Use multiple representations, including tape diagrams, tables, double number lines, and equations, to find missing values of equivalent ratios. c. Use two tables to compare related ratios. d. Apply concepts of unit rate to solve problems, including unit pricing and constant speed. e. Understand that a percentage is a rate per 100 and use this to solve problems involving wholes, parts, and percentages. f. Solve one-step problems involving ratios and unit rates (e.g., dimensional analysis). ○ Ratios can be used to find missing values in a table ○ Percent is a rate per 100 ○ Ratio reasoning can be used to convert measurement units ○ Include single step dimensional analysis (e.g., converting miles to yards)

DRAFT

New Academic Vocabulary for This Unit
● Rate ● Ratio ● Unit Rate ● Dimensional Analysis (single step)

Prior Knowledge Required for this Unit
● Equivalent fractions (3.NSF.2, 4.NSF.1) ● Simplifying fractions (3.NSF.2, 4.NSF.1) ● Multiplication and division of rational numbers (5.NSF.4, 5.NSF.7)
Subsequent Knowledge Related to this Unit
In Grade 5, students developed a conceptual understanding of writing equivalent fractions with unlike and like denominators using visual models. This knowledge will lead students to an understanding of how to simplify ratios and rates down to unit rates. Students will also need to build conceptual knowledge of rates and ratios due to the abstractness of comparing two things within a fraction. The comparison of two quantities with like units will lead students to an analysis of like and unlike units including complex fractions in Grade 7; additionally, students will extend their understanding to geometry by identifying the relationship between diameter and circumference. Work with ratios and rates will also contribute to students connecting this concept to probability, in Grade 7, by finding the number of like outcomes in a comparison with the number of total outcomes. Finding equivalent ratios using a table will lead students to finding a constant of proportionality in Grade 7 and ultimately the constant rate of change (slope) in Grade 8 Functions. In Grade 8, students will also explore transformations of transversals to transformations of similar figures including side lengths and angles.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among quantities of similar and different units.

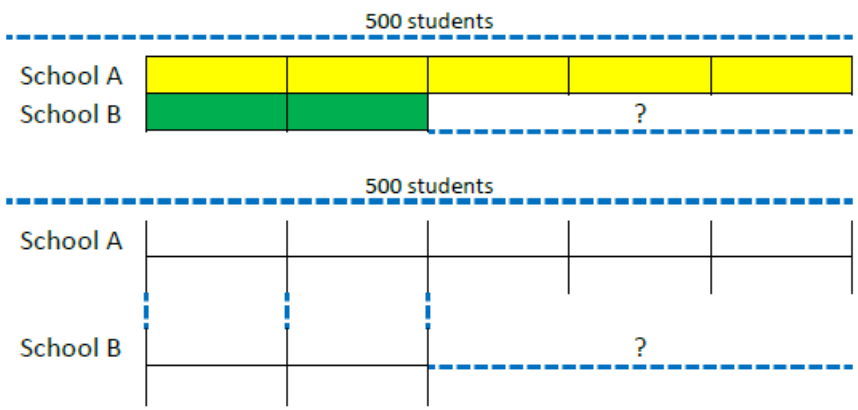
DRAFT

Potential Instructional Strategies/Lessons

- **Visual Models** - Multiple representations of visual models should be used to show equivalencies among ratios and different missing values.

Tape diagrams

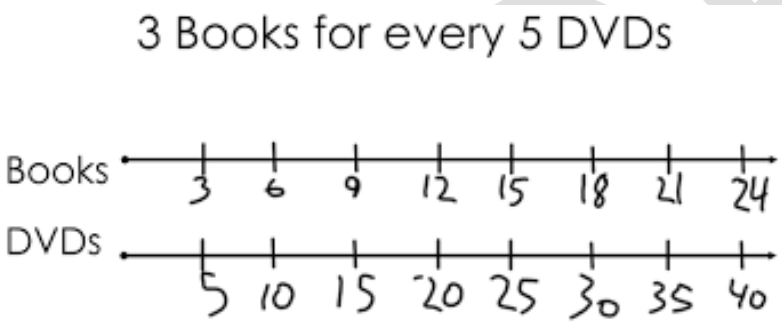
Comparison Model (part-part): School A has 500 students, which is 2 $^1/_2$ (which is equal to $^5/_2$) times as many students as School B. How many more students attend School A than School B?



Tables

Girls	Boys
3	2
6	4
9	6
12	8

Double Number Lines



Equations

The equations generated during the ratio unit will be unique in that they follow the form of $y = mx$ or $px = q$ (these notations represent the same thing). The intercept (location the line crosses on the y-axis) will always be zero.

Miles traveled = 20 · number of hours **or** $y = 20x$

- **Dimensional Analysis Tables**

Starting amount	Equal amounts		End Amount
24 inches	1 foot	=	feet
	12 inches		
24 ~~inches~~	1 foot	=	2 feet
	12 ~~inches~~		

Source: Chemistry Land

Resources
6.RP.2 - This video discusses the concepts of ratios, rates, and unit rates. http://mathantics.com/index.php/section/lesson/ratiosandrates 6.RP.2 - This site allows students to practice simplifying ratios and identifying equivalent ratios while racing a dirt bike around a track. http://www.mathplayground.com/ASB_Index.html 6.RP. 3 - This task requires students to use proportional reasoning to compare the size of a typical hamburger with that of the largest hamburger ever made. http://www.yummymath.com/2015/big-burger-2/ 6.RP.1-3 - This site includes a full lesson plan, activity instructions, and accompanying worksheets for ratio/rates competition stations. http://www.uen.org/Lessonplan/preview.cgi?LPid=23491 Conceptual Foundations for Ratios and Proportions http://elemmath.jordandistrict.org/files/2012/06/CF1.docx

DRAFT

Sample Formative Assessment Tasks/Questions

6.RP.1: Pianos and pipe organs contain keyboards, a portion of which is shown below.

a. What is the ratio of black keys to white keys in the picture above?
b. If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys?
c. If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

Answer

a. 5:7

b.

black keys	5	10	15	20	25
white keys	7	14	21	28	35

c.

black keys	5	10	100
total keys (black keys + white keys)	12	24	240

Source: The New York City Department of Education

DRAFT

6.RP.2: There are 12 boys and 16 girls in a classroom. Which represents the simplified ratio of girls to students in the classroom?
a. 3 to 4
b. 4 to 3
c. 4 to 7
d. 7 to 4

Answer
C, to determine the answer to this question the students need to read the question and work left to right with setting up the ratio.
The numerator of the ratio is 16 for girls and the denominator can be calculated by adding the number of boys and girls together to get 28.
This gives me the ration 16/28. I can simplify this down to 4/7 by dividing by the greatest common factor of 4.

6.RP.3: Dianne went for a ride on her new scooter. She traveled 450 meters in 36 seconds. Which statements are true? Select all that apply.
a. She traveled 12.5 meters every second.
b. Every 75 seconds she traveled 6 meters.
c. Every 8 seconds she traveled 100 meters.
d. She traveled 1 meter in 0.8 second.
e. Every 24 seconds, she traveled 300 meters.

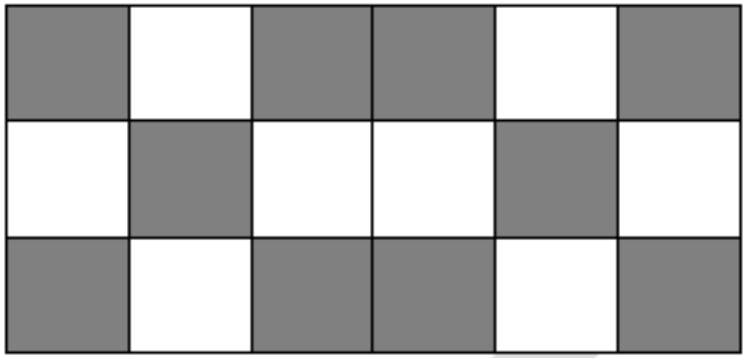
Answer
A, C, E - To determine the answer to this question, students must determine the unit rate that represents Dianne's speed. Since she is traveling at a rate of 12.5 meters for every one second, answer choice A is correct. Answer choice B is incorrect because it did not maintain the correct order in the ratio; it would be 75 meters for every 6 seconds. Answer choice C is correct because 12.5 meters multiplied by 8 is 100 meters, and 1 second multiplied by 8 is 8 seconds. By multiplying each part of the rate by the same factor we maintain an equivalent ratio. Answer choice D is incorrect because the rate was simplified incorrectly. A correct statement would read that, "She traveled 1 meter in 0.08 seconds." Answer choice E is correct because 12.5 meters multiplied by 24 is 300 meters, and 1 second multiplied by 24 is 24 seconds.

6.RP.1: Longest NHL Matches Ever - This task requires students to use their knowledge of elapsed time and proportional reasoning to answer questions about record setting sporting events.

Source: Yummy Math

DRAFT

6.RP.1 and 6.RP.2: The new floor in the school cafeteria is going to be constructed of square tiles that are either gray or white and in the pattern that appears below:



Part A: What is the ratio of gray tiles to white tiles?
Answer _____
Part B: What is the ratio of white tiles to the total number of tiles in the pattern?
Answer _____
Part C: If the total cost of the white tiles is $12, what is the unit cost per white tile?
Answer $_____

Answer
Part A: 10 to 8, 5:4, or other equivalent ratio. The correct answer is a ratio of 10 gray tiles to 8 white tiles, or simplified, the ratio will be 5 gray tiles to 4 white tiles.
Part B: 8 to 18, 4:9, or other equivalent ratio. The correct answer is a ratio of 8 white tiles to 18 total tiles, or simplified, the ratio will be 4 white tiles to 9 tiles, in total.
Part C: $1.50 per white tile. Counting the tiles by color in the pattern above, it is found that there are 8 white tiles. If 8 white tiles cost $12, then the cost per white tile is $1.50.

DRAFT

Unit Title
Graphing and Rational Numbers
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*

6.NS.5 Understand that the positive and negative representations of a number are opposites in direction and value. Use integers to represent quantities in real-world situations and explain the meaning of zero in each situation.

- Include temperature, elevation, credits/debits
- Include vertical and horizontal number lines

6.NS.6 Extend the understanding of the number line to include all rational numbers and apply this concept to the coordinate plane.

a. Understand the concept of opposite numbers, including zero, and their relative locations on the number line.
b. Understand that the signs of the coordinates in ordered pairs indicate their location on an axis or in a quadrant on the coordinate plane.
c. Recognize when ordered pairs are reflections of each other on the coordinate plane across one axis, both axes, or the origin.
d. Plot rational numbers on number lines and ordered pairs on coordinate planes.

- Include vertical and horizontal number lines, plot all rational numbers
- Understand the effects of reflections on the ordered pairs (e.g., the ordered pair (2, 3) reflected about the y-axis becomes (-2, 3))

6.NS.7 Understand and apply the concepts of comparing, ordering, and finding absolute value to rational numbers.

a. Interpret statements using equal to (=) and not equal to (≠).
b. Interpret statements using less than (<), greater than (>), and equal to (=) as relative locations on the number line.
c. Use concepts of equality and inequality to write and to explain real-world and mathematical situations.
d. Understand that absolute value represents a number's distance from zero on the number line and use the absolute value of a rational number to represent real-world situations.
e. Recognize the difference between comparing absolute values and ordering rational numbers. For negative rational numbers, understand that as the absolute value increases, the value of the negative number decreases.

- Limit inequalities to simple statements of comparison (e.g., comparing a loss of 5 yards to a loss of 3 yards)
- Limit comparing values with $<, >, =, \neq$
- Recognize absolute value is a distance and not the opposite of the number
- Understand that distance is always positive (it is the direction that changes)

6.NS.8 Extend knowledge of the coordinate plane to solve real-world and mathematical problems involving rational numbers.

a. Plot points in all four quadrants to represent the problem.
b. Find the distance between two points when ordered pairs have the same x-coordinates or same y-coordinates.

DRAFT

c. Relate finding the distance between two points in a coordinate plane to absolute value using a number line.
- Recognize the x-axis as a horizontal number line and the y-axis as a vertical number line
- Plot points that involve all rational numbers
- Limit distance between points to horizontal distances (having the same x-coordinates) or vertical distances (having the same y-coordinates)
- Recognize the final value of a distance between two points results in a positive value

New Academic Vocabulary for This Unit

- Integers
- Quadrant
- Opposite
- Zero pair
- Additive inverse
- Absolute value
- Inequality (including $\neq$)

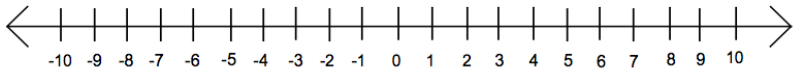
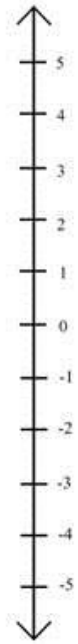
DRAFT

Prior Knowledge Required for this Unit
● Coordinate system (5.G.1, 5.G.2) ● Comparison (3.NSBT.5, 4.NSF.2, 5.NSBT.3)
Subsequent Knowledge Related to this Unit
In Grades 3, 4, and 5, students compare whole numbers, fractions, and decimals using <, >, or =. In Grade 6, students will compare all rational numbers using <, >, =, or ≠. In Grade 7, students will extend knowledge of inequalities to include $\leq$ *and* $\geq$. In Grade 8 and high school courses, students will continue to compare numbers to include irrational numbers, complex numbers, and imaginary numbers. In Grade 5, students are introduced to the coordinate system and graphing in Quadrant I of the coordinate plane. This knowledge will lead students to an understanding of how to plot points in all four quadrants. With the lack of graphing in Grade 7 on the coordinate plane, students need to build a strong foundation of the coordinate system (i.e., x- and y-axes, origin, ordered pairs, four quadrants, graphing x-values before y- values). Content in this unit is linked to constant of proportionality in Grade 7, transformational geometry, graphing linear equations and systems of linear equations, and finding the distance between two points using the Pythagorean Theorem in Grade 8. In High School mathematics courses, students will extend this knowledge to graph nonlinear functions as well as finding the distance between two points using the distance formula and the midpoint formula. Students will also use the knowledge of graphing points to include imaginary and complex values.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among positive and negative representations of rational numbers.

DRAFT

Potential Instructional Strategies/Lessons

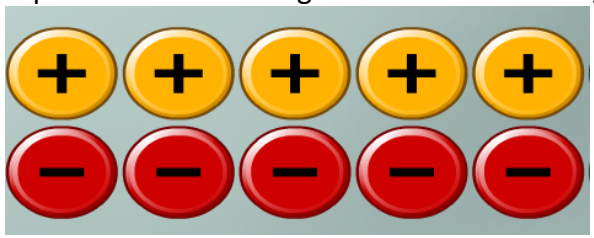
- **Representing Integers**
  - **Number Lines -** Number lines are an effective tool for thinking about positive and negative integers and their relationship to zero. The use of horizontal and vertical numbers lines also supports student thinking on a coordinate plane.

Horizontal	Vertical
	

Source: Utah District Consortium

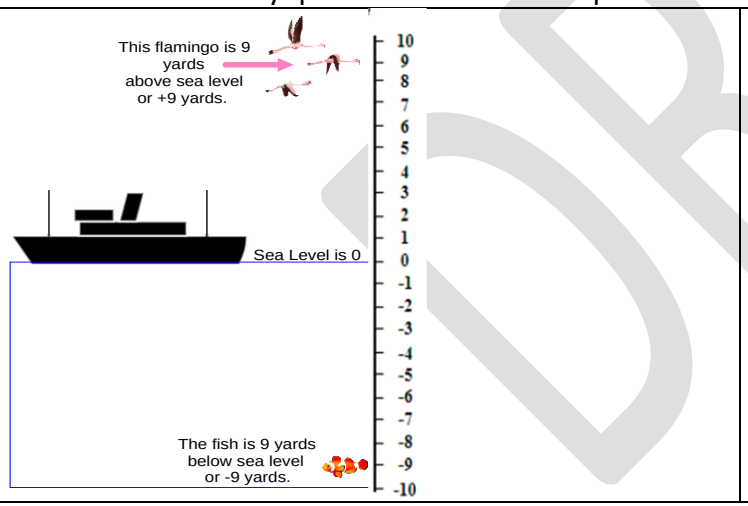
DRAFT

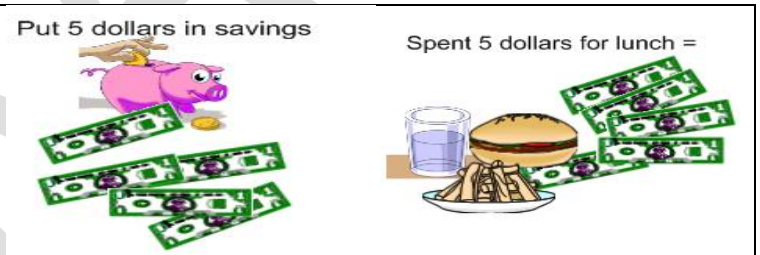
- **Counters/Chips -** Counters can be used to develop understanding of how to represent integers. Using this model requires an understanding of opposites. The model uses one color of chips (yellow) to represent positive integers and another color (red) to represent negative integers. Because each chip represents 1 unit, either positive or negative, a red chip and a yellow chip are thought of as opposites. Combining two opposite chips makes zero. The actual object being used is not as important as the distinction that one item represents negative values and the other positive values. This is a fairly abstract concept and some students struggle with assigning a representation to a negative value. However, it helps them see zero in a new and necessary way.



Source: Houghton Mifflin Harcourt

- **Representing Zero in Real-World Contexts** - Zero can represent various ideas contextually. For example, it can represent sea level when measuring elevation. It can also represent a balance between credits and debits. Thinking about what zero represents in a real-world situation allows students to identify quantitative relationships between numbers.





The net worth for the $5.00 in savings and the $5.00 spent is zero.

Source: Utah District Consortium

Potential Discussion Points for Helping Students Think About the Role of Zero in Real-World Contexts

SITUATION	NEGATIVE	ZERO	POSITIVE
Game/Sports: Golf/ Football	Below par / loss of yards	Par/line of scrimmage	Above par / gain of yards
Business	Loss (In the red)	Holding own	Profit (In the black)
Bank Accounts: Checkbooks	Charge- credit card Loans- interest paid / negative balance	Zero balance	Savings / Interest earned / Positive balance
Time and Time Zones	Past / Yesterday	Present / Midnight	Future / Tomorrow
Daylight Savings	Fall behind	Standard time	Spring ahead
Geologic or Historic Time	Before Common Era (B.C.)	Theoretical, but nonexistent year "0"	Common Era (C.E.)
Gauges/Dipsticks for Oil	Oil is low	Correct amount	Over filled
Tires	Flat	Correct pressure	Over inflated
Blood Pressure	Low blood pressure	Correct Pressure	High blood pressure
Eyes-Vision	-3.75	20/20	+3.75
Temperature-Vertical Time Line	Below Zero	Zero	Above zero
Elevation-Altitude	Below sea level	Sea level	Above sea level
Buildings	Basement / Lower levels	Ground floor	Attic / Upper floor

Source: Utah District Consortium

DRAFT

Resources
Online Graphing Calculator: https://www.desmos.com/calculator 6.NS.7 - This jeopardy game allows students to compare rational numbers http://www.math-play.com/Comparing-Rational-Numbers/comparing-rational-numbers.html 6.NS.6 - This game allows students to compare integers. http://www.xpmath.com/forums/arcade.php?do=play&gameid=61 6.NS.6 - This activity allows students to understand the concept of opposite numbers and absolute value. https://www.ixl.com/math/grade-7/absolute-value-and-opposite-integers 6.NS.8 - This activity allows students to locate points on the coordinate plane. http://www.math-play.com/Coordinate%20Plane%20Game/Graphing-points-in-the-coordinate-plane.html

Sample Formative Assessment Tasks/Questions
6.NS.5: Denver, CO is called the "Mile High City" because its elevation is 5280 feet above sea level. Someone tells you that the elevation of Death Valley, CA is -282 feet. a. Is Death Valley located above or below sea level? Explain. b. How many feet higher is Denver than Death Valley? *Answer* a. Because the elevation is negative, Death Valley is located below sea level. Places that are above sea level have positive elevations. A vertical number line can be used to display this understanding. b. Death Valley is 282 feet below sea level, so it takes 282 feet to get to sea level. Denver is 5280 feet above sea level, so we add that distance to the 282 feet we already have. 282 + 5280 = 5562. Denver is 5562 feet higher above Death Valley. Source: Illustrative Mathematics

DRAFT

6.NS.6:

a. For each set of points below, draw and label a set of coordinate axes and plot the points:

   i. (2, 3), (1, −5), (−3, 0), (-4, $3\frac{1}{2}$)
   ii. (70, 70), (0, 0), (−20, −35), (55, -45)
   iii. ($\frac{1}{8}$, $\frac{7}{8}$), ($\frac{-5}{2}$, 0.25), ($-1\frac{1}{4}$, $-\frac{1}{8}$), (0.5, $-\frac{3}{4}$)

b. Would it be reasonable to use the same scale for plotting each set of points from part (a)? Explain your reasoning.

c. How do the points impact your decision of scale for the axes?

Answer

a. *Click the link beside the set of ordered pairs.*

   i. (2, 3), (1, −5), (−3, 0), (-4, $3\frac{1}{2}$) LINK TO GRAPH
   ii. (70, 70), (0, 0), (−20, −35), (55, -45) LINK TO GRAPH
   iii. ($\frac{1}{8}$, $\frac{7}{8}$), ($\frac{-5}{2}$, 0.25), ($-1\frac{1}{4}$, $-\frac{1}{8}$), (0.5, $-\frac{3}{4}$) LINK TO GRAPH

b. No, it would not be reasonable to use the same scale for the axes. The range between values within the sets is too large.

c. You would get a more accurate representation of the relationship between points. For example, you could use a scale of: 1 for (i); 10 for (ii); $\frac{1}{4}$ for (iii).

DRAFT

6.NS.7: The record low temperatures (in Celsius) in Gaffney, SC for one week include: Monday ($5°C$), Tuesday ($-1°C$), Wednesday ($-6°C$), Thursday ($-2°C$), Friday ($3°C$), Saturday ($7°C$), and Sunday ($0°C$).

a. List these temperatures in order from least to greatest.
b. On a spring day in Portland, OR, the low temperature is 13 degrees below zero (in Celsius), and the low temperature in Nashville, TN was 4 degrees below zero (in Celsius). Daniela wrote Nashville was colder because -4 < -13. Is Daniela correct? Explain your answer.
c. Antarctica has the coldest temperature ever recorded: -89 degrees Celsius. The average temperature on Jupiter is approximately -145 degrees Celsius. Which is warmer, the average temperature on Jupiter or the coldest temperature on Earth? Write an inequality to support your answer.

Answer

a. $-6°C, -2°C, 0°C, 3°C, 5°C, 7°C$ (Students may want to plot the temperatures on a number line to assist with ordering values.)
b. Daniela is incorrect. $13°C$ is less than $-4°C$
c. -145 < -89; therefore, the coldest temperature on Earth is warmer than the average temperature of Jupiter.

6.NS.8: Ms. Johnson is making a map of a local amusement park by plotting locations on a coordinate plane, where one unit represents 100 feet, and the origin represents the ticket stand. Ms. Johnson plots the Rapid Racer Roller Coaster at (-6, -3) and the Wet and Wild Water Coaster at (-10, -3).

What is the distance between the two rides on the coordinate plane? What does this distance represent in real life?

Answer

The rides are four units away from one another on the coordinate plane. This is equivalent to 400 feet apart in the actual amusement park.

DRAFT

Unit Title
Expressions
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
6.EEI.1 Write and evaluate numerical expressions involving whole-number exponents and positive rational number bases using the Order of Operations. ○ Perform arithmetic operations, including those involving whole-number exponents, using order of operations, including expressions with and without parentheses. **6.EEI.2** Extend the concepts of numerical expressions to algebraic expressions involving positive rational numbers. a. Translate between algebraic expressions and verbal phrases that include variables. b. Investigate and identify parts of algebraic expressions using mathematical terminology, including term, coefficient, constant, and factor. c. Evaluate real-world and algebraic expressions for specific values using the Order of Operations. Grouping symbols should be limited to parentheses, braces, and brackets. Exponents should be limited to whole-numbers. ○ Read, write, and evaluate expressions in which letters (variables) stand for numbers. ○ Distinguish the difference between an algebraic and numerical expression. ○ Identify parts of an expression using mathematical terms. ○ View one or more parts of an expression as a single entity. ○ Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. **6.EEI.3** Apply mathematical properties (e.g., commutative, associative, distributive) to generate equivalent expressions. ○ Generate equivalent numeric and algebraic expressions. ○ Understand the following mathematical properties of addition: commutative, associative, additive identity, and additive inverse ○ Understand the following mathematical properties of multiplication: commutative, associative, multiplicative identity, multiplicative inverse, and distributive **6.EEI.4** Apply mathematical properties (e.g., commutative, associative, distributive) to justify that two expressions are equivalent. ○ Justify equivalency of numeric and algebraic expressions.

DRAFT

New Academic Vocabulary for This Unit
● Base ● Coefficient ● Constant ● Defining the variable ● Equivalent expressions ● Exponent ● Grouping symbols ● Like terms ● Substitute ● Term

Prior Knowledge Required for this Unit
● Order of Operations (5.ATO.1) ● Translate numerical expressions (5.ATO.2) ● Represent the problem using an equation with a variable as the unknown quantity (4.ATO.3)
Subsequent Knowledge Related to this Unit
In subsequent math courses, students will evaluate and simplify more complex numerical and algebraic expressions to include all rational numbers and their operations. In Grade 7, students will extend the use of grouping symbols to include fraction bars and solve linear equations using the distributive property. In Grade 8, students will use integer exponents and explore the concepts of perfect squares, perfect cubes, square roots, and cube roots. In High School courses, students will work with more complex expressions.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among numeric and algebraic expressions and mathematical properties.

DRAFT

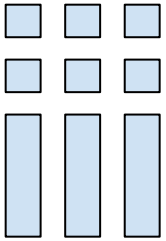
Potential Instructional Strategies/Lessons

- Visual Models

Example 1: The expression 10x + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3). The factors (dimensions) of this figure would be 5(2x + 3).

10x	15

Example 2: Students use their understanding of multiplication to interpret 3 (2 + x) as 3 groups of (2 + x). They use a model to represent x, and make an array to show the meaning of 3(2 + x). They can explain why it makes sense that 3(2 + x) is equal to 6 + 3x. An array with 3 columns and x + 2 in each column:



Source: NC DPI 6th Grade Mathematics Unpacked Contents

DRAFT

Resources
6.EEI.1, 6.EEI.2, 6.EEI.3, 6.EEI.4 - This module set provides tasks, guided practice, collaborative work, homework, and assessments for each of the standards listed. http://www.nps.k12.nj.us/IRC/site/handlers/Grade6ExpressionsandEquationsTeacherModule-moduleinstanceid=12279&dataid=9560&FileName=Grade6ExpressionsandEquationsTeacherModule.pdf.pdf 6.EEI.2, 6.EEI.3, 6.EEI.4 - This performance task has students consider algebraic expressions and mathematical properties as they solve problems involving a grocery list and quilt. http://schools.nyc.gov/NR/rdonlyres/F7DD52E7-FD7E-44DC-B099-90301CAC9025/140802/NYCDOE_G6_Math_GroceryShopping_FINAL1.pdf%20Janel
Sample Formative Assessment Tasks/Questions
6.EEI.1 and 6.EEI.2: Write an expression that is equivalent to 64 using all of the following numbers and symbols once in the expression. **7, 7, 7, ^{2}(exponent of 2),+, ÷, ( )** *Answer* $(7 \div 7 + 7)^2$; The first thing that needed to be recognize by the student is that they must use all of the numbers and symbols when making their expression. The order of operations inside the parentheses should have the student doing the division between the first 2 sevens first which gives a quotient of 1, next you would need to add 1 and 7 to get a total of 8. After the parentheses are done the squaring of 8 will give you and answer of 64 for the expression using all of the necessary symbols.

DRAFT

6.EEI.3: Which of the following expressions is NOT equivalent to the expression $2x+6$? Why? If you were to correct the ones that are NOT equivalent what should it be?

a. $x + x + 2 + 2 + 2$
b. $2(x + 3)$
c. $2x + 2^3$
d. $4 + x + 2 + x$

Answer

a. $x + x + 2 + 2 + 2 = 2x + 6$
b. $2(x + 3) = 2x + 6$
c. $2x + 2^3 = 2x + 8$
d. $4 + x + 2 + x = 2x + 6$

The equivalent expressions are a, b, and d. When you combine like terms, they generate 2x+6.

6.EEI.3 and 6.EEI.4: Write the expression 16 + 14 as a product of two factors. Use properties of multiplication and addition to justify your answer.

16 + 14	
The GCF = 2	Step 1: Find the greatest common factor (GCF) of 16 and 14.
2(8 + 7)	Step 2: Factor that number out using the distributive property
2(7 + 8)	Step 3: Commutative property of addition (OPTIONAL)

DRAFT

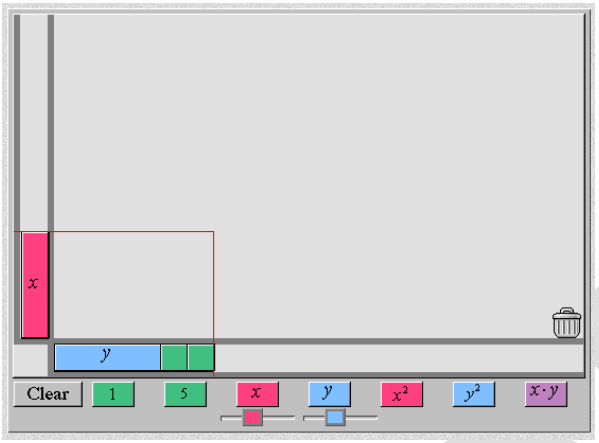
Unit Title
Equations
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
6.EEI.5 Understand that if any solutions exist, the solution set for an equation or inequality consists of values that make the equation or inequality true. ○ Understand that with an equation you will have one solution and with an inequality you will have more than one solution. ○ Verify the solution to equations or solution set to inequalities to determine if it satisfies the equation or inequality. **6.EEI.6** Write expressions using variables to represent quantities in real-world and mathematical situations. Understand the meaning of the variable in the context of the situation. ○ Understand that expressions do not contain equal signs. **6.EEI.7** Write and solve one-step linear equations in one variable involving nonnegative rational numbers for real-world and mathematical situations. ○ Understand that equations contain equal signs. ○ Understand that the expressions on either side of the equal sign must be equivalent to one another. ○ Use inverse operations to solve one-step linear equations one variable involving nonnegative rational numbers (include fractions and decimals). **6.EEI.8** Extend knowledge of inequalities used to compare numerical expressions to include algebraic expressions in real-world and mathematical situations. a. Write an inequality of the form $x > c$ or $x < c$ and graph the solution set on a number line. b. Recognize that inequalities have infinitely many solutions. ○ Understand that if x is greater than or less than c, then c will not be in the solution set. ○ Understand that solutions for x include rational numbers. **6.EEI.9** Investigate multiple representations of relationships in real-world and mathematical situations. a. Write an equation that models a relationship between independent and dependent variables. b. Analyze the relationship between independent and dependent variables using graphs and tables. c. Translate among graphs, tables, and equations ○ Recognize that independent values represent an input (x-coordinates) and dependent values represent an output (y-coordinates).

DRAFT

New Academic Vocabulary for This Unit
● Independent variable ● Dependent variable ● Inverse operation ● Solution set ● Infinitely many solutions

Prior Knowledge Required for this Unit
● Represent algebraic thinking using expressions and equations. (5.ATO.2, 5.ATO.3) ● Understand grouping symbols (5.ATO.1) ● Understand the use of the distributive property (3.ATO.5, 6.EEI.3, 6.EEI.4)
Subsequent Knowledge Related to this Unit
Standards in this unit are all necessary to develop the computational skills needed for work within the real number system including solving multi-step linear equations and inequalities in Grade 7. In Grade 8, students are asked to solve multi-step equations and inequalities with variables on both sides of the equation. In Grade 7, the simplifying of expressions includes a fraction bar as a grouping symbol as well as whole number exponents. While in Grade 8, this extends to include integer exponents. Additionally, in Grade 8, students will apply their understanding of exponents to develop an understanding of scientific notation. In high school mathematics courses, students will extend their knowledge of solving equations and inequalities to include nonlinear equations and inequalities. Students will graph their solutions on the coordinate plane. Also, students will extend their understanding of exponents to include rational exponents in high school.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among equations, inequalities, their solution sets, and the multiple representations by which these solution sets can be expressed (graphs, tables, and equations).

DRAFT

Potential Instructional Strategies/Lessons
• **Real-world Connections** - When writing and solving equations and inequalities, the use of real world examples should be used to develop students' understanding. • **Vocabulary** - Strategies should include methods that ensure students understand the phrase "at least" indicates that the number given is actually the smallest amount that the value can be; "at most" would indicate the largest number that the value can be. Strategies can be used to help students distinguish between the phrases "less" and "less than" when writing expressions, equations, and inequalities. • **Algebra Tiles**    Source: National Library of Virtual Manipulatives
Resources
Inequalities Graph Generator: http://www.mathwarehouse.com/number-lines/graph-inequality-on-number-line.php Inequalities Lesson Plans: http://betterlesson.com/user/228515/27231/115892/michelle-schade/curriculum Inequalities Videos: https://www.khanacademy.org/math/algebra-basics/core-algebra-linear-equations-inequalities/core-algebra-linear-inequalities/v/inequalities One-Step Linear Equations Lesson: http://www.purplemath.com/modules/solvelin.htm One-Step Linear Equations Videos: http://www.virtualnerd.com/algebra-1/linear-equations-solve/one-step/one-step-solutions

DRAFT

Sample Formative Assessment Tasks/Questions
6.EEI.5: Individuals must be over 54 inches tall to ride the Intimidator thrill ride at Carowinds. a. Represent the height, *h*, requirement using an inequality. b. GIve examples of 3 heights that would be allowed on the ride and 3 heights that would not be allowed on the ride. *Answer* a. $h > 54$ b. Answers will vary: Three possible heights in inches - 55, 60, 72 Three unacceptable heights in inches - 53, 48, 37
6.EEI.6: Hector had a balance of $237 dollars in his checking account. After buying a pair of shoes that cost d dollars, he had $183 in his account. Write an expression to represent this situation. *Answer* $237 - d = 183$
6.EEI.7: The Shuler family is taking a trip to Disney World, and they must travel 450 miles from Columbia, SC. Tom Shuler drives at an average rate of 60 miles per hour. a. Write an expression for the number of hours, *h*, it will take the Shulers to reach Disney World. b. Solve your equation to determine the number of hours they spend traveling. c. If they leave at 7:00 am and stop every 75 miles for a five minute break, at what time will they reach Disney World. *Answer* a. $\frac{450}{60} = h$ b. It will take the Shuler family 7.5 hours to reach Disney World. c. 7.5 hours is equivalent to 7 hours and 30 minutes. 7 hours and 30 minutes past 7:00am is 2:30pm. If they stop every 75 miles, they will make 6 stops. Since each stop takes the family 5 minutes, they will arrive at Disney World at 3:00pm.
6.EEI.8: Natasha wants to treat her friends to the movies. The movie tickets cost $11. She can spend under $131. Write an inequality to represent how many people she can treat to the movies. *Answer* $11x < 131$

DRAFT

6.EEI.9: Describe is the relationship between the two variables. Write an equation and graph the relationship between the two variables.

x	1	2	3	4
y	3.5	7	10.5	14

Answer
As the value of *x* increases by 1, the value of *y* increases by 3.5.
$y = 3.5x$

DRAFT

Unit Title
Statistics
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*

6.DS.1 Differentiate between statistical and non-statistical questions.

- Recognize that a statistical question can be answered by collecting data.
- Recognize that the answer to a statistical question is based on data.
- Recognize that there will likely be a variety of answers to a statistical question as opposed to one single answer.

6.DS.2 Use center (mean, median, mode), spread (range, interquartile range, mean absolute value), and shape (symmetrical, skewed left, skewed right) to describe the distribution of a set of data collected to answer a statistical question.

- Recognize that the mean is affected by outliers.
- Recognize that the median and mode are resistant to outliers.
- Recognize range as the difference between the minimum and maximum values.
- Recognize interquartile range as the difference between the lower (first) and upper (third) quartiles.
- Recognize that the skew of a graph analyzes the symmetry or lack of symmetry of a set of data.
- Recognize that if data is not skewed, then it is symmetric about the mean.
- Recognize that being skewed left (negative skew) means the cluster or peak of data is to the right of the mean.
- Recognize that being skewed right (positive skew) means the cluster or peak of data is to the left of the mean.
- In the above standard, students are expected to calculate the mean absolute deviation.
- The mean absolute deviation (MAD) of a set of data is the average distance between each data value and the mean.
- Understand that mean absolute deviation can be determined by finding the mean of the ranges between each data value and the mean of the data set (mean absolute value).
- The larger the MAD, the *greater variability* there is in the data (the data is more spread out).
- The larger the MAD, the *less reliable* the mean is as an indicator of the values within the set.

6.DS.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

- Recognize that measure of center and measures of central tendencies are synonymous to one another.

DRAFT

6.DS.4 Select and create an appropriate display for numerical data, including dot plots, histograms, and box plots.

- Recognize that box plots and box and whisker plots are synonymous to one another.
- Understand that box plots are created and display values from the Five-Number Summary: minimum, maximum, lower quartile (first), upper quartile (third), and median.
- Understand that the lower quartile (first) is the median of the lower set of data.
- Understand that the upper quartile (third) is the median of the upper set of data.
- Understand that a box plot divides the data into fourths (quartiles).
- Recognize that the 50 percent of the data is located within the box.
- Recognize that dot plots and line plots are synonymous to one another.
- Recognize the distinct differences between bar graphs and histograms.
- Bar graphs display qualitative data and each bar represents one value.
- Histograms display qualitative and quantitative data and each bar presents a range (band) of values.

6.DS.5 Describe numerical data sets in relation to their real-world context.

a. State the sample size.
b. Describe the qualitative aspects of the data (e.g., how it was measured, units of measurement).
c. Give measures of center (median, mean).
d. Find measures of variability (interquartile range, mean absolute deviation) using a number line.
e. Describe the overall pattern (shape) of the distribution.
f. Justify the choices for measure of center and measure of variability based on the shape of the distribution.
g. Describe the impact that inserting or deleting a data point has on the measures of center (median, mean) for a data set.

- Recognize that mean absolute value and mean absolute deviation are synonymous to one another.

DRAFT

New Academic Vocabulary for This Unit
• statistical • non-statistical • measures of center • mean • median • mode • spread • range • interquartile range • quartile (upper and lower) • mean absolute value • mean absolute deviation • shape • symmetrical (normal distribution) • skewed left (negative skew) • skewed right (positive skew) • dot plot/line plot • histogram • box plot/box and whisker plot • sample size • measures of variability • data set • distribution • maximum value • minimum value

DRAFT

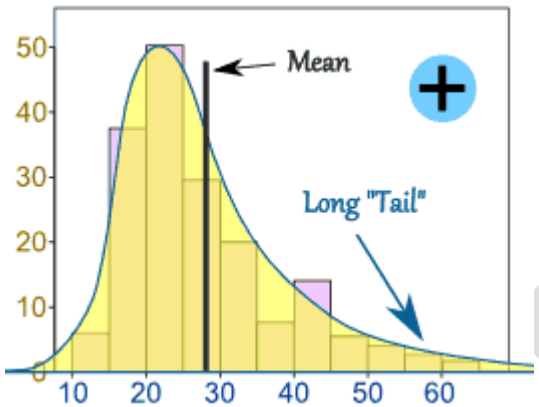
Prior Knowledge Required for this Unit
• Collect, organize, and represent data (2.MDA.9, 3.MDA.3) • Organize data on a line plot (3.MDA.4, 4.MDA.4, 5.MDA.2) • Add and subtract whole numbers (4.NSBT.4) • Divide (5.NSBT.6, 6.NS.2)
Subsequent Knowledge Related to this Unit
In Grade 7, students are asked to investigate the difference between random and non-random sampling. Students will visually compare the center, spread, and overlap of two displays which are graphed on the same scale. Students will also compare numerical measures of center and variability from two random samples to draw inferences about the data. In Grade 8, the structure moves to graphing bivariate data and determining a line of best fit to show a linear association. Students will investigate the comparison of bivariate categorical data in two-way tables. In high school mathematics courses, students will begin to identify linear, quadratic, and exponential relationships among data values. Students will also determine the regression equations and correlation coefficients for linear, quadratic, and exponential relationships to determine which model best represent a set of data. Also in high school, students will further exploration of skewness and mean absolute deviation to determine the relationship between the mean and median and the standard deviation and variance of a set of data. Students will also determine whether a data value is an outlier based on an algebraic formula.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among interpreting, organizing, and analyzing data.

DRAFT

Potential Instructional Strategies/Lessons

- **Skewness –** The mean, median and mode are all measures of the center of a set of data. The skewness of the data can be determined by how these quantities are related to one another.

Positive skew is when the long tail is on the positive side of the peak. It is also referred to as "skewed to the right".

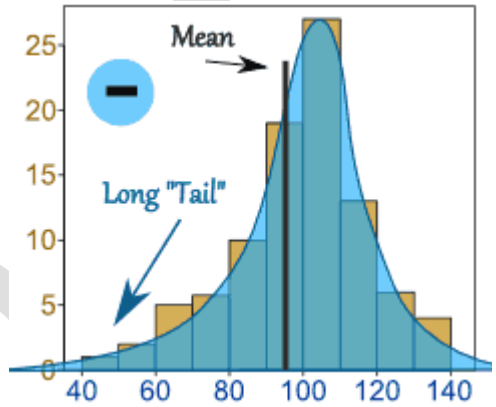


Source: MathIsFun

In this situation the mean and the median are both greater than the mode. As a general rule, most of the time for data skewed to the right, the mean will be greater than the median. In summary, for a data set skewed to the right:

- Always: mean greater than mode
- Always: median greater than mode
- Most of the time: mean greater than median

Negative skew is when the long tail is on the negative side of the peak. It is also referred to as "skewed to the left".

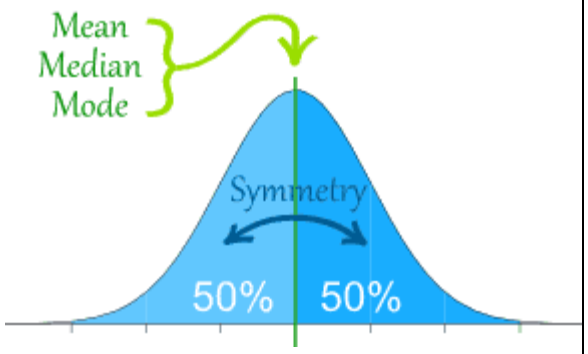


Source: MathIsFun

In this situation the mean and the median are both less than the mode. As a general rule, most of the time for data skewed to the left, the mean will be less than the median. In summary, for a data set skewed to the left:

- Always: mean less than mode
- Always: median less than mode
- Most of the time: mean less than median

A ***normal distribution*** is not skewed. For a normal distribution, the left and right tails are equally balanced, meaning that they have about the same length. It is symmetrical and the mean is exactly at the peak. For normal distributions, the mean is approximately equa to the median.



Source: MathIsFun

For more information, please click the following links: Skewness and Normal Distribution

DRAFT

- **Technology** - Students can use technology to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations.
  - **Box Plot Tool** - http://illuminations.nctm.org/ActivityDetail.aspx?ID=77
  - **Histogram Tool** - http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Resources

6.DS.1 – This video helps students identify the differences between statistical and non-statistical questions.
https://www.khanacademy.org/math/probability/statistical-studies/statistical-questions/v/statistical-questions

6.DS.1 – This site allows students to examine whether a question is or is not statistical.
https://www.illustrativemathematics.org/content-standards/tasks/703

6.DS.2 – The site gives real world examples on helping to understand the meaning of mean and absolute deviation.
http://www.mathsisfun.com/data/mean-deviation.html

6.DS.3 – On this site students will examine 2010 Census population data for counties in New Jersey and answer questions about the data set's measure of center and variation.
http://www.census.gov/schools/index/teach/Math-and-Statistics/Middle-School-Math-Activities/grade-6/measures-of-center-and-variation--how-many-people-live-in-new-je.html

6.DS.4 – This activity provides a class-wide activity for students to build a box plot based on student-produced data.
http://illuminations.nctm.org/Lesson.aspx?id=1688

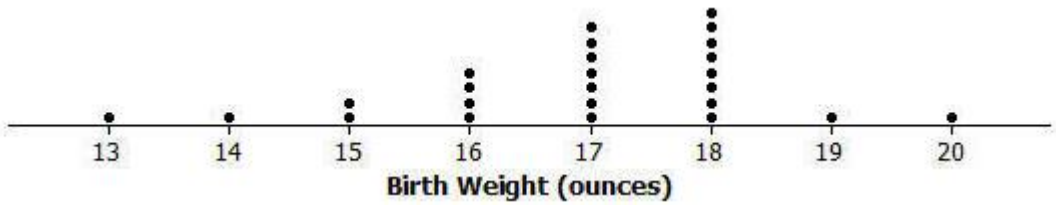
6.DS – Tasks to support all standards in this key concept.
http://www.insidemathematics.org/assets/problems-of-the-month/through%20the%20grapevine.pdf

DRAFT

Sample Formative Assessment Tasks/Questions
6.DS.1: Rion loves to wear patterned socks. She can empty the socks from her drawer and see all of her socks at once. Identify the statistical questions someone could ask Rion about her socks: a. What is the typical height of the socks in her drawer? b. What size does Rion wear in socks? c. If Rion grabbed 8 different socks what are the odds that she grabbed socks belonging to 4 matching pairs? d. How are the socks distributed by color? e. How many socks are in her drawer? f. Write your own statistical question related to Rion's collection of socks. *Answer* a. Statistical Question b. Non-statistical Question c. Statistical Question d. Statistical Question e. Non-statistical Question f. Student answers will vary. Sample: What is the average number of polka dots on the socks in her drawer?

DRAFT

6.DS.2: The dot plot below displays the 25 birth weights, in ounces, of all the Labrador Retriever puppies born at Kingston Kennels in the last six months.



a. Describe the distribution of birth weights for puppies born at Kingston Kennels in the last six months. Be sure to describe shape, center and variability.
b. What is a typical birth weight for puppies born at Kingston Kennels in the last six months? Explain why you chose this value.

Answer

a. The distribution of birth weights is centered at approximately 17 (median = 17 ounces, mean = 16.92 ounces), and the interquartile range is 2 ounces and the mean absolute deviation (MAD) is 1.149 ounces. The distribution has a longer tail for lower values (that is, it is skewed left).
b. A typical weight is one that is about 17 ounces. A student might also choose 18 ounces, the mode of this data distribution.

Source: Illustrative Mathematics

DRAFT

6. DS.3: Consider the data shown in the dot plot of the six trait scores for organization for a group of students.

a. How many students are represented in the data set?
b. What are the mean and median of the data set? What do these values mean? How do they compare?
c. What is the range of the data? What does this value mean?

Answer
a. 19 students are represented in the data set.
b. The mean of the data set is 3.5. The median is 3. The mean indicates that is the values were equally distributed, all students would score a 3.5. The median indicates that 50% of the students scored a 3 or higher; 50% of the students scored a 3 or lower.
c. The range of the data is 6, indicating that the values vary 6 points between the lowest and highest scores

Source: NCDPI 6th Grade Mathematics Unpacked Content

DRAFT

6.DS.4, 6.DS.5: Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the whiteboard. The data set is listed below in order from least to greatest.

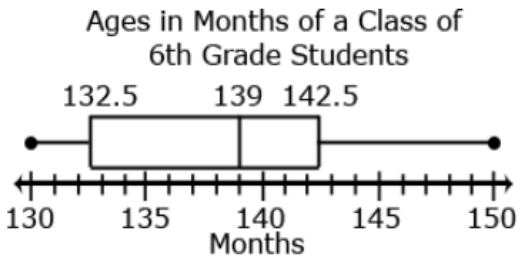
130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

a. Create a data display.
b. What are some observations that can be made from the data display?

Answer

a.

Sample



Minimum – 130 months
Quartile 1 (Q1) – (132 + 133) ÷ 2 = 132.5 months
Median (Q2) – 139 months
Quartile 3 (Q3) – (142 + 143) ÷ 2 = 142.5 months
Maximum – 150 months

b. This box plot shows that
- ¼ of the students in the class are from 130 to 132.5 months old
- ¼ of the students in the class are from 142.5 months to 150 months old
- ½ of the class are from 132.5 to 142.5 months old
- The median class age is 139 months.

Source: NCDPI 6th Grade Mathematics Unpacked Content

DRAFT

6.DS.2, 6.DS.5: In this task, students study historical Super Bowl data to reflect on average (mean, median, and mode) losing scores, winning scores, and range of scores. They are asked to judge which of these measurements seem the most meaningful and explain their reasoning.

Source: Yummy Math

DRAFT

Unit Title
Geometry
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*

6.GM.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

- Find the total areas of trapezoids, kites, hexagons, etc. by dividing the shape into rectangles and triangles.
- Derive the formula for determining the area of a triangle from the decomposition of a rectangle.

6.GM.2 Use visual models (e.g., model by packing) to discover that the formulas for the volume of a right rectangular prism ($V = lwh$, $V = Bh$) are the same for whole or fractional edge lengths. Apply these formulas to solve real-world and mathematical problems.

- Use knowledge of finding the area of a rectangle and apply that to the visual model understanding of finding the volume of right rectangular prism.
- Practice with visual models using fractional side lengths so that students' interpretation of fractional side lengths multiplied together for volume is correct (e.g., ½ x ½ x ½ = ⅛).

6.GM.3 Apply the concepts of polygons and the coordinate plane to real-world and mathematical situations.

a. Given coordinates of the vertices, draw a polygon in the coordinate plane.
b. Find the length of an edge if the vertices have the same *x*-coordinates or same *y*-coordinates.

- Recognize that edge lengths are vertical or horizontal distances in the coordinate plane.
- Understand that when finding the edge length on a coordinate plane the length of the edge is always counting the spaces from point *a* to point *b* instead of counting the dots.
- Understand that the edge length on a coordinate plane is a positive value regardless of the direction.

6.GM.4 Unfold three-dimensional figures into two-dimensional rectangles and triangles (nets) to find the surface area and to solve real-world and mathematical problems.

- Discover surface area of three-dimensional shapes by calculating the total area of all the faces that make up the net. Students are not using surface area formulas.
- Recognize surface area as the area of the lateral surfaces and bases of a figure.
- Recognize that the lateral surfaces of a figure are the faces around the figure (sides).
- Understand that the figures used should be limited to rectangular and triangular prisms.

DRAFT

New Academic Vocabulary for This Unit
• Composing, Decomposing • Vertex, Vertices • Surface area • Volume • Nets • Face • Edge • Lateral surfaces

Prior Knowledge Required for this Unit
• Relationship of 3D shapes and the nets that make them (3.G.4) • Area of Rectangles (4.MDA.3) • Plotting points in coordinate plane (6.NS.6) • Volume of rectangular prisms (5.MDA.3)
Subsequent Knowledge Related to this Unit
In Grade 7, students will understand that the concepts of volume and surface area are related to three dimensional shapes. Also in Grade 7, students will decompose cubes, rectangular prisms, and right triangular prisms into rectangles and triangles to derive the formulas for volume and surface area as well as use the formulas for area, volume, and surface area appropriately. In Grade 8, students use the Pythagorean Theorem to find the diagonal distance between two points on a coordinate plane. Also in Grade 8, students will begin to determine the volume of non-polyhedrons, including cylinders, cones, and spheres, and they will determine the surface area of cylinders. In high school mathematics courses, students will determine the area and volume of regular polygons and polyhedrons respectively using the apothem and knowledge of trigonometric ratios. Students will also determine the surface area and volume of pyramids and spheres. Students will couple their understanding of volume and surface area with probability to determine the likelihood of an object being located in a given area.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among the area, surface area, and volume of two- and three-dimensional shapes.

DRAFT

Potential Instructional Strategies/Lessons

• **Boxes or containers** – cereal boxes, TV dinner boxes, oatmeal containers, etc. enables students to see nets
• **Dynamic Geometric Software** – use software/applications to model surface area and volume in 3D figures
  ○ Geometer's Sketchpad
  ○ GeoGebra
  ○ NCTM's Illuminations Dynamic Paper Tool
  ○ NCTM's Illuminations Cube
  ○ Annenberg Learner
  ○ Shodor
• **Discover the Area of a Triangle** – Allow students the opportunity to discover that the area of a triangle is one-half the area of some rectangle with the same length as the base of the triangle and the same width as the height of that same triangle.
  ○ Discovering the Area Formula for Triangles

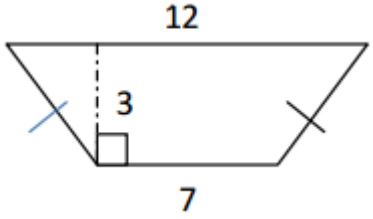
DRAFT

Resources
6.GM.1 – This site is a three-act problem for students to determine area of rectangles. http://threeacts.mrmeyer.com/bubblewrap/ 6.GM.1 – This site allows students to determine the area and missing values of rectangles and squares and have their solutions checked to assess their understanding. http://www.mathgoodies.com/lessons/vol1/challenge_unit1.html 6.GM.1 – This site provides a video for students to review their understanding of the area of a right triangle and connect it to their knowledge of area of a rectangle. https://www.youtube.com/watch?v=RNnY--1jScg 6.GM.2 – This site provides sample questions for students to extend their understanding of the volume of rectangular prisms to real world situations. http://www.opusmath.com/common-core-standards/6.g.2-find-the-volume-of-a-right-rectangular-prism-with-fractional-edge-lengths 6.GM.3 – This site provides a sample task for students to apply their understanding of area of rectangles to the coordinate plane. https://www.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1997 6.GM.1-6.GM.4 – This site has activities for all standards covered in this unit. https://www.georgiastandards.org/Georgia-Standards/Frameworks/6th-Math-Unit-5.pdf

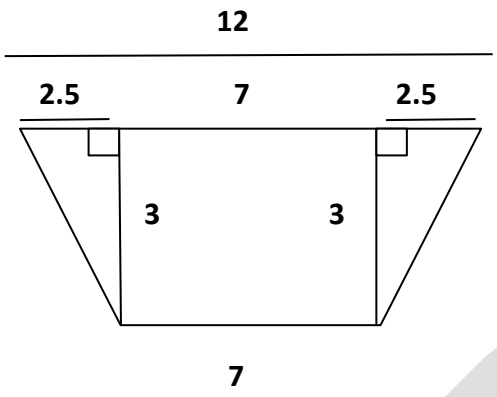
DRAFT

Sample Formative Assessment Tasks/Questions

6.GM.1: Find the area of the trapezoid shown below using the formulas for rectangles and triangles.



Sample Answer

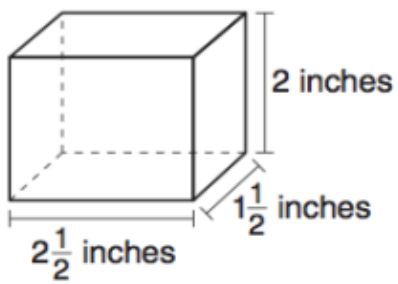


The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units^2 The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there are a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be ½ (2.5 units)(3 units) or 3.75 units^2. Using this information, the area of the trapezoid would be: 21 units^2 + 3.75 units^2 + 3.75 units^2 = 28.5 unit^2

Alternate Solution: Students can compose the two triangles on the ends of the trapezoid to make one rectangle measuring 2.5 units by 3 units. The area of this composed rectangle is 7.5 units^2. Add this to the area of the rectangle in the center (21 units^2) for a total of 28.5 units^2.

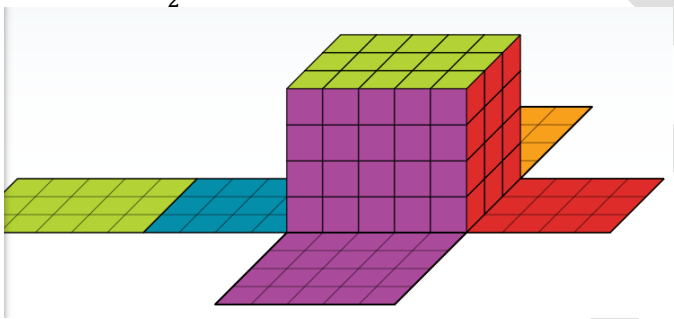
DRAFT

6.GM.2: Lorraine plans to fill this box with cubes of the same size. Each cube has side lengths of $\frac{1}{2}$ inch. How many cubes can inside this box without empty space?



Answer

Since each cube has side lengths of $\frac{1}{2}$ inch, then the number of cubes to represent to the dimensions of the box can be determined by dividing their measurement by $\frac{1}{2}$. The length of the box would be 5 cubes ($2\frac{1}{2} \div \frac{1}{2}$). The width of the box would be 3 cubes ($1\frac{1}{2} \div \frac{1}{2}$). The height of the box would be 4 cubes ($2 \div \frac{1}{2}$). The model below shows how the box would look packed with $\frac{1}{2}$ inch cubes.



The number of cubes can be determined by using the volume formula below.

$V = lwh$

$V = 5 \times 3 \times 4$

$V = 60$ cubes

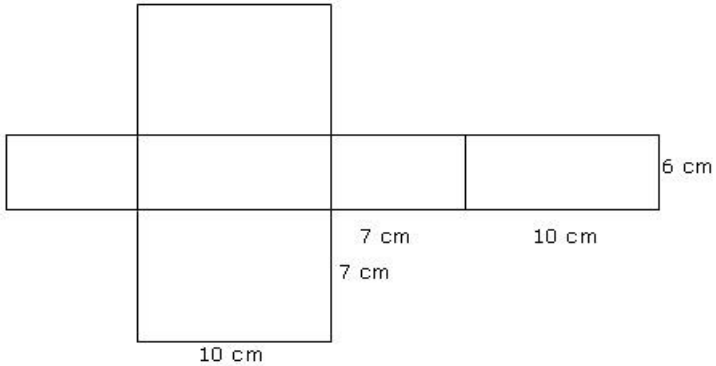
DRAFT

6.GM.3: On a baseball diamond the distance from one base to the other are equivalent. If home plate is located at the origin on a coordinate grid, first base is located at the point (15,0), and third base is located at the point (0,15). What is the location of second base based upon the appropriate information given? What is the distance from first base to second base?

Answer
Students should know that the distance from each base on a baseball diamond is equivalent and that a baseball diamond is the shape of a square. The distance from home plate would be 15 because you are moving from the origin 15 spaces to first base. Based upon this information you would continue from the first base point (0,15) and count upward 15 spaces and land at the location of second base being the point (15, 15). Finally the distance from first base to second base would be 15 due to the properties of a square where all sides are equal.

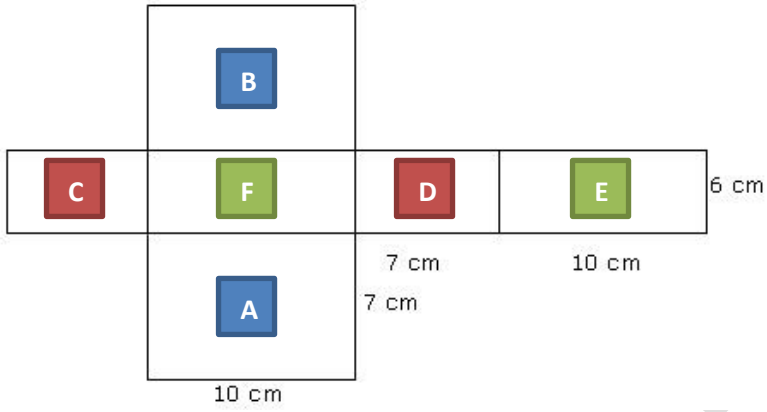
DRAFT

6.GM.4: The picture below is the net of a cereal box with the given dimensions. Determine the surface area of the prism.



Answer

To find the surface area of the net you would find the area of all six of the rectangles and then add them all up to find the total surface area of the net. The area of section A is found by multiplying 7 × 10 getting an area of 70 cm^2. The area of section B is 70 cm^2, which is found by multiplying 7 × 10. Sections C and D have an area of 42 cm^2, which is found by multiplying 6 × 7. The area of sections E and F is 60 cm^2, which is found by multiplying 6 × 7. After finding the area of all of the small rectangles you add 70 +70 +42 +42 +60 + 60 to get a total area of 344 cm^2 for the surface area of the rectangular prism.



DRAFT