

South Carolina College- and Career-Ready Standards for Mathematics



Support Document

Grades K-5

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K-5 Mathematics Support Document

As support for implementing the *South Carolina College- and Career-Ready Standards for Mathematics*, the standards for each grade K-5 have been grouped into possible units. In the *Table of Contents* below, the titles for those possible units are listed in a column under each grade. To see which standards are addressed in each unit and to read a brief description of the focus for each unit, click on the *Overview of Units* in the [Table of Contents](#). Two units for each grade are now complete and are hyperlinked from/to the Table of Contents. The purpose of this document is to provide guidance as to how all the standards at each grade K-5 may be grouped into units and how those units might look. Since this document is merely guidance, districts should implement the standards in a manner that addresses the district curriculum and the needs of students.

Acknowledgments

“Jean Baptiste Massieu, famous deaf educator, made a statement that is now considered a French proverb. *Gratitude is the memory of the heart*. Indeed, appreciation comes when you feel grateful from the depths of your heart. The head keeps an account of all the benefits you received and gave. But the heart records the feelings of appreciation, humility, and generosity that one feels when someone showers you with kindness.” It is with sincere appreciation that we humbly acknowledge the dedication, hard work and generosity of time provided by the following individuals who are making the K-5 Mathematics Support Document possible. (<http://quotations.about.com/od/ThankYou/a/Gratitude-Quotes.htm>)

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Kindergarten Overview of Units

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Counting and Cardinality	Understanding Relationship of Counting and Quantity	Count and Compare	Composing and Decomposing Numbers	Addition and Subtraction	Patterns and Positions	Two Dimensional and Three Dimensional Geometry	Foundations of Measurement	Understanding Graphs and Data
Standards	Standards	Standards	Standards	Standards	Standards	Standards	Standards	Standards
K.NS.1 K.NS.2 K.NS.3 K.NS.4 K.NS.5 K.NS.6	K.NS.4 K.NS.5	K.NS.4 (c) K.NS.7 K.NS.8	K.NSBT.1 K.ATO.3	K.ATO.1 K.ATO.2 K.ATO.4 K.ATO.5	K.ATO.6 K.G.1	K.G.2 K.G.3 K.G.4 K.G.5	K.MDA.1 K.MDA.2	K.MDA.3 K.MDA.4
Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus
The focus of this unit is for students to understand the relationship between number and quantity. Students will understand that quantities can be counted, represented with a number name and a written numeral is a symbol representing that quantity.	The focus of this unit is for students to develop a sense of quantity and how numbers they count relate to one another. Students will begin to understand the reasonableness of answers, understanding the need to be consistent and accurate when counting.	The main focus of this unit is for students to apply their understanding of counting and quantity to compare sets of objects and written numerals.	This unit focuses on developing the concepts of how numbers can be composed and decomposed in a variety of ways, forming a foundation for number sense in base ten.	The main focus is for students to build on composing and decomposing strategies which make sense to them to develop the conceptual understanding for the operations of addition and subtraction.	The main focus is for students to describe simple repeating patterns and positions of objects. Focus should include simple number patterns.	The main focus is on basic geometrical shapes. Students will develop an understanding of the attributes of 2-dimensional and 3-dimensional shapes. Students will apply number sense to reason about shapes in the environment.	The main focus is for students to develop a foundational understanding of length and weight measurement. This includes students developing the language needed to describe attributes, similarities and differences in the length and weight of a variety of objects.	The focus is for students to be given categories in which to sort and count objects. Students will then represent their findings in object and picture graphs to answer questions about the categories.

Grade One Overview of Units

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
Composing & Decomposing Numbers through 10	Addition and Subtraction Strategies	Understanding Place Value	Applying Place Value Concepts	Comparisons and Data	Geometry and Equal Shares	Measurement and Data
Standards	Standards	Standards	Standards	Standards	Standards	Standards
1.ATO.1 1.ATO.3 1.ATO.5 1.ATO.6 1.ATO.9.b 1.NSBT.1.a	1.ATO.1 1.ATO.2 1.ATO.3 1.ATO.4 1.ATO.6 1.ATO.7 1.ATO.8 1.NSBT.1.(a,c,d)	1.NSBT.1 1.NSBT.2 1.NSBT.3 1.NSBT.5 <i>1.ATO.3</i> <i>1.ATO.5</i> <i>1.ATO.6</i> <i>1.ATO.8</i>	1.NSBT.4 1.NSBT.6 <i>1.ATO.1</i> <i>1.ATO.2</i> <i>1.ATO.3</i> <i>1.ATO.5</i> <i>1.ATO.6</i> <i>1.ATO.8</i>	1.MDA.4 1.MDA.5 1.ATO.1 1.ATO.2 1.ATO.9.b	1.G.1 1.G.2 1.G.3 1.G.4 1.ATO.9	1.MDA.1 1.MDA.2 1.MDA.3 1.MDA.6
Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus
<i>Unit 1</i> will focus on number concepts and relations by composing and decomposing numbers through 10. Students will develop a beginning understanding of addition and subtraction.	<i>Unit 2</i> will focus on representing, solving, and exploring addition and subtraction. Students will extend their understanding of addition and subtraction from <i>Unit 1</i> to developing strategies for adding and subtracting whole numbers through 20. Students will have an understanding of subtraction as an unknown addend problem and determine missing numbers in addition and subtraction equations within 20 using a variety of strategies.	<i>Unit 3</i> will focus on developing an understanding of whole number relationships and place value through 99, including “making a ten”. Students will develop an understanding of the relative magnitude of numbers by comparing two-digit numbers based on the meanings of the tens and ones. They will use that number sense to solve problems.	<i>Unit 4</i> will focus on students developing, discussing, and using efficient, accurate, and generalizable methods to add within 99 and to subtract multiples of 10.	<i>Unit 5</i> will focus and build on the K concepts of sorting and classifying by collecting, organizing and representing data with up to 3 categories using object graphs, picture graphs, t-charts, and tallies. Students will ask/answer questions and draw conclusions based on given graphs (object graphs, picture graphs, t-charts, tallies, bar graphs). Building on Unit 3 comparison knowledge, students will develop comparison statements for a set of data and draw and solve comparison problems.	<i>Unit 6</i> will focus on students identifying, naming, partitioning, and reasoning about attributes of two-dimensional and three-dimensional shapes. Students will compose and decompose plane or solid figures (e.g., combine two triangles to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes.	<i>Unit 7</i> will focus on an understanding of the meaning and processes of measurement, including an understanding of linear measurement as iterating length units. Students will work with both analog and digital clocks as they tell and record time to the nearest hour and half hour. Students will also identify coins and their values.

Grade Two Overview of Units

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Place Value Concepts	Developing Concepts Addition/ Subtraction	Fluency and Word Problems Addition/ Subtraction	Developing an Understanding of Multiplication	Attributes Polygons and Fractional Parts	Measurement Length	Measurement Time and Money	Creating and Understanding Data
Standards	Standards	Standards	Standards	Standards	Standards	Standards	Standards
2.NSBT.1 2.NSBT.2 2.NSBT.3 2.NSBT.4 2.ATO.3	2.NSBT.5 2.NSBT.6 2.NSBT.7 2.NSBT.8 2.MDA.5	2.ATO.2 2.ATO.1 2.ATO.4	2.ATO.4 2.G.2	2.G.1 2.G.3	2.MDA.1 2.MDA.2 2.MDA.3 2.MDA.4	2.MDA.6 2.MDA.7	2.MDA.8 2.MDA.9 2.MDA. 10
Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus
Unit 1 will focus on the development of numeracy and place value concepts for the one’s, ten’s and hundred’s places at a concrete (manipulative) level. Students will then use this conceptual understanding to 1) decompose numbers, 2) expand numbers, and 3) compare numbers.	After students have fully developed place value concepts, students will use those concepts in Unit 2 to add and subtract four 2-digit numbers, and two 3 digit numbers (through 999). Students are expected to use place value strategies, concrete models, and properties of operations to develop their own strategies. Standard algorithms are not to be introduced at this early level of development.	In Unit 2, students developed models for addition and subtraction. Unit 3 requires students to use those strategies to solve one and two step word problems and to demonstrate fluency with addition and subtraction through 20.	Unit 4 requires students to begin the development of multiplication concepts (as repeated addition and as the area of a rectangular array). Students are not expected to use the multiplication sign (X). Students will instead use repeated addition equations. For example,  $3+3+3+3=15$	Students must be able to identify shapes based upon specified attributes (number of angles, number of equal faces)in Unit 5 . They will also use the terms “halves”, “fourths”, “half of” and “fourth of” to describe geometric shapes that have been equally divided. <i>Students are not expected to use the fractional symbols with numerators and denominators.</i>	In Unit 6 , students will use rulers, yardsticks, meter sticks, and measuring tapes to: 1) measure the length of an object in customary and metric units, 2) compare the lengths of two measured objects, and 3) measure the same object using different units of measurements (feet vs. inches) and explain why the measurements differ.	Unit 7 requires students to solve money story word problems in dollars, quarters, dimes, nickels, and pennies. Students must also be able to tell time to the nearest 5 minutes interval on analog clocks.	In previous Unit 6 students learned to measure length using measuring tools; in Unit 8 , students will organize measurement data into line plots marked into whole number units. Students will also collect data to create, represent, and draw conclusions from picture graphs and bar graphs with a single unit scale.

Grade Three Overview of Units

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Conceptual Understanding of Multiplication & Division	Place Value	Addition & Subtraction	Application of Multiplication & Division	Conceptual Understanding of Fractions	Data Analysis	Identification and Classification of Geometric Shapes	Problem Solving with Measurement	Fluency with Multiplication & Division
Standards	Standards	Standards	Standards	Standards	Standards	Standards	Standards	Standards
3.ATO.1 3.ATO.2 3.ATO.3 3.ATO.6	3.NSBT.1 3.NSBT.4 3.NSBT.5	3.NSBT.2 3.ATO.8 3.ATO.9	3.ATO.3 3.ATO.4 3.ATO.5 3.ATO.9	3.NSF.1 3.NSF.2 3.NSF.3 3.G.2	3.MDA.3 3.MDA.4	3.G.1 3.G.3 3.G.4	3.MDA.1 3.MDA.2 3.MDA.5 3.MDA.6 3.ATO.8	3.NSBT.3 3.ATO.7 3.ATO.8
Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus
To develop conceptual understanding, students use concrete objects, drawings and symbols to represent multiplication and division facts and then solve real-world problems.	Students use place value understanding to read, write, round, compare, and order numbers.	Students develop fluency with addition and subtraction of whole numbers through 1,000, and apply these skills in real-world problems.	Building on conceptual understanding from Unit 1 and now with a focus on application, students continue working with multiplication and division facts in a variety of problem solving situations.	Students begin to build fraction sense, working with a variety of models to emphasize unit fractions and explore fraction equivalence.	Students extend their understanding of data analysis to include scaled picture and bar graphs, as well as line plots.	Students continue working with 2-D shapes, categorizing by attributes. Students also identify 3-D shapes based on given 2-D nets.	Students solve real-world problems involving a variety of measurement concepts.	Students apply a variety of strategies to demonstrate fluency and solve real-world problems with multiplication and division facts.

Grade Four Overview of Units

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Place Value, Addition, & Subtraction of Whole Numbers	Algebraic Thinking	Multiplication & Division of Whole Numbers	Fraction Equivalence	Adding, Subtracting, & Multiplying with Fractions	Decimal Concepts	Conversions & Problem Solving with Measurement	Geometric Classifications & Line Symmetry	Angle Measurement
Standards	Standards	Standards	Standards	Standards	Standards	Standards	Standards	Standards
4.NSBT.1 4.NSBT.2 4.NSBT.3 4.NSBT.4	4.ATO.1 4.ATO.2 4.ATO.3 4.ATO.4 4.ATO.5	4.NSBT.5 4.NSBT.6 4.ATO.3 4.ATO.5	4.NSF.1 4.NSF.2 4.NSF.5	4.NSF.3 4.NSF.4 4.NSF.5 4.MDA.4	4.NSF.6 4.NSF.7	4.MDA.1 4.MDA.2 4.MDA.3 4.MDA.8	4.G.1 4.G.2 4.G.3 4.G.4 4.ATO.5	4.MDA.5 4.MDA.6 4.MDA.7
Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus
Students examine the structure and patterns within the base ten system. They then use this knowledge to develop fluency with addition and subtraction of whole numbers.	To extend algebraic reasoning, students employ flexible thinking with multiplication and division to solve a variety of real-world problems.	Students employ a variety of strategies to efficiently multiply and divide multi-digit numbers.	Students strengthen their fraction sense by using a variety of models and strategies, such as the multiplicative identity element in fraction form, to generate and compare equivalent fractions.	Students use a variety of models when adding and subtracting fractions with like denominators and when multiplying a whole number and a fraction to solve real-world problems.	Students write decimals as fractions and use concrete and visual models to compare and order decimal numbers.	Students convert measurements within a single system and solve real-world problems involving a variety of measurement concepts.	Students learn specific geometric attributes, such as parallel and perpendicular lines, and use those attributes to classify shapes. The concept of line symmetry is introduced.	Students create and measure angles using a protractor. They also solve real-world problems involving unknown angle measures.

Grade Five Overview of Units

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Expressions, Equations, and the Coordinate Plane	Place Value	Operations with Whole and Decimal Numbers	Adding and Subtracting Fractions	Multiplying with Fractions	Dividing with Fractions	Classifying 2D Shapes	Perimeter, Area, and Volume	Converting Measurements within a Single System
Standards	Standards	Standards	Standards	Standards	Standards	Standards	Standards	Standards
5.ATO.1 5.ATO.2 5.ATO.3 5.G.1 5.G.2	5.NSBT.1 5.NSBT.2 5.NSBT.3 5.NSBT.4	5.NSBT.5 5.NSBT.6 5.NSBT.7	5.NSF.1 5.NSF.2 5.MDA.2	5.NSF.4 5.NSF.5 5.NSF.6 5.MDA.2	5.NSF.3 5.NSF.7 5.NSF.8	5.G.3 5.G.4	5.MDA.3 5.MDA.4	5.MDA.1
Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus	Unit Focus
Students expand their ability to evaluate numerical expressions that include grouping symbols. Students connect their understanding of numerical expressions to the coordinate plane.	Students work with powers of 10 to extend place value understanding of whole and decimal numbers through thousandths.	Students develop fluency with multiplication and division of multi-digit whole numbers. Students also develop conceptual understanding of operations with decimal numbers.	Students use a variety of models when adding and subtracting fractions with unlike denominators to solve real-world problems.	Students extend their understanding of fractions by multiplying with fractions in a variety of situations to solve real-world problems.	Students divide unit fractions and whole numbers using a variety of models to solve real-world problems.	Students culminate their understanding of two-dimensional figures by classifying them in a hierarchy based on their attributes.	Students investigate volume measurement from concrete exploration to derivation of the formula for right rectangular prisms. Students then differentiate among perimeter, area, and volume when solving real-world problems.	Students convert measurements within a single system by applying operational fluency in problem solving situations.

Kindergarten Math Unit 1

Counting and Cardinality

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Content Standards with Clarifying Notes

Open Bullets Indicate Clarifying Notes

- **K.NS.1** Count forward by ones and tens to 100.
 - Oral Rote counting, say the count sequence
 - Counting by tens is with decade numbers only (e.g. 10, 20, 30, etc.)
 - Students are not expected to write the numerals for this standard.
- **K.NS.2** Count forward by ones beginning from any number less than 100.
 - Oral Rote counting, say the count sequence from any number less than 100.
 - Students are not expected to write the numerals for this standard.
- **K.NS.3** Read numbers from 0 – 20 and represent a number of objects 0 – 20 with a written numeral.
 - Number reversal (writing a 3 backward) is developmentally appropriate at this stage, however students should be corrected so as not to form misconceptions or incorrect habits.
 - When students write the numeral, the order of the digits is important so as not to confuse the value of the number. For example, 31 is not 13 and students should be corrected so as not to form misconceptions or incorrect habits that would impact their beginning understanding of number meaning.
- **K.NS.4** Understand the relationship between number and quantity. Connect counting to cardinality by demonstrating an understanding that:
 - a. the last number said tells the number of objects in the set (cardinality);
 - b. the number of objects is the same regardless of their arrangement or the order in which they are counted (conservation of number);
 - c. each successive number name refers to a quantity that is one more and each previous number name refers to a quantity that is one less.
- **K.NS.5** Count a given number of objects from 1 – 20 and connect this sequence in a one-to-one manner.
 - Say a number name for each object counted.
- **K.NS.6** Recognize a quantity of up to ten objects in an organized arrangement (subitizing).

- Teacher note: Subitizing is defined as seeing a small amount of objects and knowing how many there are *without counting*. (Carlyle & Mercado 2012), Subitizing should include perceptual and conceptual subitizing. Perceptual subitizing is recognizing the quantity such as 3 without using any formal mathematical processes. Conceptual subitizing is viewing number and number patterns as units of units. (e.g. On a dominoe or a die student sees 4 dots as 2 dots and 2 dots as 1 unit of 4)(Adapted from http://www.nwaea.k12.ia.us/documents/filelibrary/pdf/connections/Subitizing_B2518BBFE8FCF.pdf)

New Academic Vocabulary for This Unit

- | | | | |
|--|------------|------------|------------|
| ● numeral | ● quantity | ● 1 more | ● how many |
| ● number | ● count | ● 1 less | ● amount |
| ● number names
(e.g. one, two, three, etc.) | ● sequence | ● count on | ● set |

Prior Knowledge Required for this Unit

While students may have had some informal exposure to counting in everyday situations, this may be their first formal introduction to counting and cardinality. Frequent and brief opportunities to count will allow for rote counting concepts to develop over time. Development of meaningful counting, understanding the relationship between numbers, etc., requires intentional activities designed to target understandings such as those set forth in K.NS.4. Children should come to the understanding that number symbols signify the meaning of counting. Many students come to kindergarten knowing the rote count sequence, making adults believe they know how to count, but they are just saying the rote count sequence, they do not have any formal meaning for what they are saying (numbers). For example, when shown a number such as 4, the student doesn't know it's a 4, or doesn't know a quantity of 4 objects, but they can verbalize the rote count sequence, "1,2,3,4,5,6,..."

Subsequent Knowledge Related to this Unit

This unit is the foundation for the more complex stages of number sense:

Hierarchical inclusion: Understanding that smaller numbers are part of bigger numbers. Idea of one more and one less

Part/Whole Relationships: Understanding different ways to make up a given number, composing and decomposing numbers, understanding that 6 is made up of 1 & 5, 3 & 3, 2 & 4.

Compensation: Ability to see the parts of a whole and are able to compensate, so if I know that $5+1=6$, then I also know that $4+2=6$ because 4 is one less than 5 and 2 is one more than 1.

Unitizing: Once students begin to gain a solid understanding of the preceding number sense, then they can construct unitizing as they work with larger numbers. Place value

Unit 1 provides the foundation for student understanding that in the count sequence, the next number is one more, and when counting by tens, the next number is “ten more” (or one more group of 10). As students gain this understanding, they will be prepared to compare numbers in Unit 3. However, those understandings are not innate and students should be given intentional learning opportunities. Students should then be able to apply this knowledge to part/whole relationships, compensate to compose and decompose numbers in Units 3 & 4, as well as develop a foundation for place value and operations with numbers in subsequent units as well as subsequent grade levels.

Students will use their fluency with the count sequence and cardinality to compose and decompose numbers in Unit 4, and then again to model addition and subtraction with story problems in Unit 5.

As students are able to subitize, they will use this skill in addition to cardinality to build single-digit fluency to 5 in Unit 5.

Relationship Among Standards in this Unit

The content standards and the process standards work together in this unit to form the foundation which will enable all students to develop the world class knowledge, skills, and life and career characteristics identified in the *Profile of the South Carolina Graduate*.

The standards in Unit 1 are intertwined and work together to help students build their foundation in the key concepts Number Sense and Number Sense in Base Ten . Counting and Cardinality are about understanding and using numbers. These are both essential foundations for Number Sense in Base Ten. This is the first formal introduction for students to the language of mathematics/ numbers. Frequent and brief opportunities to count will allow for rote counting concepts to develop over time. However, children must be given intentional learning opportunities to understand that number symbols signify the meaning of counting.

When taught simultaneously these standards will develop students’ understanding of number names, the count sequence, quantity, written numerals, and how they are all related; together these will provide a foundation for comparing numbers in Unit 3. The main focus of this unit is for students to use and understand numbers in a variety of ways orally and written so as to master kindergarten standards by the end of the year. Students will count objects in a set, and count out a given number of objects so as to make the connection between counting and cardinality while learning the count sequence and developing one-to one correspondence, as well as represent quantities. Students will also

answer quantitative questions by subitizing, counting and producing sets of a given size.

K.NS.1 and K.NS.2 require students to say the count sequence to 100, by ones, tens, and beginning at any number less than 100. As students learn the count sequence, K.NS.5 requires that they say a number name for each object counted in a set of up to 20 objects. This leads to students understanding of cardinality, K.NS.4. Accuracy in counting relies upon knowing the patterns in the number name, correctly saying one number name to one object, and keeping track of counted and uncounted objects so as not to count an object more than once. Young students benefit from regularity and rhythm aspects of counting so as to develop one-to-one correspondence. As students begin to make sense of counting, they will be ready to read and write numbers 0-20 to correctly represent a quantity.

Unit 1 provides the foundation for the critical learning phase of counting objects. In this unit, common behaviors of students might be: 1-1 correspondence, keeping track of an unorganized pile, noticing when a recount results in a different number, bothered when counting a group results in the same number after more objects have been added or taken away, spontaneously checking or recounting to see if the result is the same, answering “How many” questions, counting out a specific quantity, showing a reaction to estimate while counting, and adjusting estimate while counting to have a closer estimate. Depending where students are in their development, these behaviors might be observed at different times with different students.

The standards in Units 1-4 are essential to the key concepts Number Sense and Base Ten and Algebraic Thinking and Operations in grades K-5. Meaningful, daily experiences with the standards in Unit 1 should be provided to young students throughout the kindergarten year to set a strong foundation for mathematics success. Students will deepen their understanding of counting and cardinality in Unit 2 as they look more closely at quantities.

Teacher Notes:

Students typically develop number sense through the following progression:

Subitizing ability to see small amounts without counting.

Magnitude ability to tell which of two sets has more without counting.

Counting ability to say the counting sequence, which happens before one-to-one correspondence is fully developed.

One-to-One correspondence is saying one number name for each object counted.

Cardinality is when you count a group of objects, and the last number you say tells how many in all. Students who don't have this, recount objects when asked "how many?"

Hierarchical inclusion understanding that smaller numbers are part of bigger numbers. Idea of one more and one less.

Potential Instructional Strategies/Lessons

Kindergarten is the pivotal grade for creating a mathematical classroom which encourages collaboration and builds community. Students who are college-and-career-ready take a productive and confident approach to mathematics by communicating about mathematics to gain understanding.

Students in kindergarten can begin developing their communication skills by:

- Acting out mathematical scenarios/problems
- Frequently exchanging mathematical ideas and problem solving strategies
- Listening to understand one another. This involves thinking about what a person is saying so that you can explain it yourself or to help them explain it more clearly. It is not just being quiet when someone else is talking. Also, children need to listen so that they can ask a question or help the explainer.
- Utilizing *Math Talk* in the classroom before being asked to follow procedures

Kindergarten students will naturally want to do many things on their own, however it is encouraged to have students begin working in pairs or small groups to build reasoning and communication skills.

Differentiation should be considered in the mathematics classroom, even at the kindergarten level where students' background knowledge and experiences vary. Instruction on the count sequence should be scaffolded as students are developmentally ready. (e.g. focus on 1-5, 1-10, then 1-20, etc.)

Possible Strategies:

Students should be provided many, meaningful opportunities to count throughout the day and throughout the kindergarten year. Provide settings which connect mathematical language and symbols to kindergartners' everyday lives. Support the natural ability of young children to mathematize their world.

Number Sense Routines Daily routines that allow children ongoing support with counting, thinking and talking about numbers which help them make sense of numbers and how they work.

A list of possible routines is here <https://gradelevelcommoncoremath.wikispaces.hcpss.org/Routines>

Introductory Lesson/Strategy

Numbers and Me <http://illuminations.nctm.org/Lesson.aspx?id=910>

- An activity which focuses on the use of numbers in everyday situations and school to introduce teacher and students to each other.

Choral counting

- Count the number of chairs in the room
- Count the number of students today
- Count the number of shoes, papers, pencils, crayons, anything in the classroom
- Count the number of empty chairs in the room
- Count the number of snacks provided
- Count the number of napkins/cups/plates needed for snack
- Count around the circle: Students sit in a circle and pass an object as they count.
- Count the days in school using the number line and/or hundreds chart

Choral Counting <https://www.illustrativemathematics.org/content-standards/K/CC/A/1/tasks/360>

The Counting Jar

- Place objects (same objects at beginning of year such as marbles, color tiles, beans, shells, etc.) in a jar and each day, remove the objects from the jar, asking students to count the objects. At first, the teacher might record the written number, and ask students to find that number on a number line or hundred chart. As students become more fluent with counting and one-to-one correspondence, increase the magnitude of the quantity of objects in the jar.

The Counting Jar <https://www.youtube.com/watch?v=QWoRC3KwB2M>

Read Alouds

- Children’s Literature is an effective and engaging way to provide students with the opportunity to understand numbers.

Possible Lesson Using :

Ten Black Dots by Donald Crews <http://www.k-5mathteachingresources.com/support-files/ten-black-dots.pdf>

<https://www.youtube.com/watch?v=h3ePDTStq0> A recorded read aloud

After listening to the story, this lesson engages students in counting out a specific number of dots to create something. They then complete a sentence frame for a page that can be combined to create a class book.

Ten Black Dots <http://www.scholastic.com/teachers/lesson-plan/ten-black-dots-extension-activities>

- Activities to use with the children’s book by Donald Crews.

Organization of Numbers

- Five Frames
- Ten Frames
- Dot Cards
- Rekenreks <http://www.k-5mathteachingresources.com/Rekenrek.html>

The rekenrek is a tool to teach students one -to-one correspondence and then more complex counting such as in groups of 5 and ten. Lessons with the rekenrek support students natural concept development of counting.

Five Frame Fill & Ten Frame Fill <http://illuminations.nctm.org/Activity.aspx?id=3565>

- An interactive game using five and ten frames. Several options in the game allow students to count, match a quantity, and identify numbers with quantities. Can be used individually on a computer or projected on Smart Boards for whole group.

Ten Frames and Dot Cards <http://www.k-5mathteachingresources.com/ten-frames.html>

- A variety of lesson ideas for using ten frames and dot cards. Also provides blackline masters of ten frames and dot cards.

-
- Counting Songs
 - Counting rhymes

All About the Number ____ :

- Following the count sequence, create an anchor chart for each number. Asking students, “ What do we know about the number ____?”

“How can we show the number ___?” “What does it look like as a picture? a word? a symbol?” Post the anchor charts in the classroom to begin building an organic number line with meaning based on what students know about the numbers. As students learn more, they can add to the anchor charts.

Possible Culminating Strategy:

My Counting Book <http://www.k-5mathteachingresources.com/support-files/my-counting-book.pdf>

- Students create their own counting book. A good activity for students to work on over time, especially after reading several counting books as read alouds. Provides opportunity for students to count, draw/represent, and write numbers.

Resources

Teacher Resources:

Number Early Learning Progression <http://nzmaths.co.nz/sites/default/files/Number-Early-Learning-Progression.pdf>

- This chart provides the learning progression for learning numbers and the count sequence.

The Principles of Counting <http://math.about.com/od/counting/a/The-Principles-Of-Counting.htm>

- Explanation of the different aspects of counting such as cardinality, conservation of number, and unitizing.

Mingle and Count game <https://www.teachingchannel.org/videos/mingle-count-a-game-of-number-sense>

- Explains and demonstrates how to play the game

Lesson Planning Resources:

Building Sets of Ten <http://illuminations.nctm.org/Lesson.aspx?id=1649>

- A lesson plan for exploring sets of up to 10 items, and practice writing numbers 0-10.

Tallies, Tens Frames and Baseball Games <http://illuminations.nctm.org/Lesson.aspx?id=3300>

- A lesson plan for reinforcing counting concepts using tally marks, tens frames, and patterning wins and losses for a baseball team. This lesson could be used as a daily support for these concepts. It extends to using operations with the numbers which could be used again in Unit 3 for composing and decomposing numbers and building fluency in Unit 5.

Children’s Literature such as; Mouse Count by Ellen Stoll Walsh, Ten Black Dots by Donald Crews, How Many Snails? by Paul Giganti, Jr., Ollie

and Me, $1 \times 2 \times 3$ by Shirley Hughes, Ten Flashing Fireflies by Philemon Sturges

Howard County Kindergarten Math Wiki <https://gradecommoncoremath.wikispaces.hcpss.org/kindergarten+home>

- This website has a wide variety of daily number routines, vocabulary cards, lesson plans/activities, and assessment tasks including rubrics for proficiency.

Counting by Tens <https://www.illustrativemathematics.org/content-standards/K/CC/A/1/tasks/754>

- Kinesthetic and auditory lesson to demonstrate the count sequence to 100 when counting by tens.

Number Line Up <https://www.illustrativemathematics.org/content-standards/K/CC/A/2/tasks/401>

- Kinesthetic lesson for students to physically identify numbers and put them in order to form a “human number line”.

Children’s Books for Math <http://www.the-best-childrens-books.org/math-for-kids.html>

- This website has lists of children’s books for different concepts in math that make learning the concepts engaging and relatable to small children.

Interactive Resources

Counting Superhero Song by Harry Kindergarten music <https://www.youtube.com/watch?v=4RhBhzFJ4Ww>

- A counting song and video to practice and reinforce counting by ones to 100.

Find the Numbers 0-5 or 5-10 <https://www.illustrativemathematics.org/content-standards/K/CC/A/tasks/403>

- Game which students can play in groups of 2 or 3 to practice identifying numbers and matching quantities.

Glencoe Virtual Manipulatives http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

- An interactive library of virtual manipulatives which allow teachers to choose grade level, manipulative (tools) and backgrounds such as storyboards, workmats, and game boards for students to practice. Could be used at individual computers or on an interactive whiteboard. At the kindergarten level, students should first use concrete manipulatives before using digital manipulatives.

National Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/grade_g_1.html

- A library of virtual manipulatives which allow teachers to engage students in visualizing problems. At the kindergarten level, students should first use concrete manipulatives before using digital manipulatives.

Teddy Numbers <http://www.topmarks.co.uk/learning-to-count/teddy-numbers>

- An interactive counting game that provides practice for young students to count objects by dragging and dropping items onto a teddy bear.

Underwater Counting <http://www.topmarks.co.uk/learning-to-count/underwater-counting>

- An interactive counting game that provides practice for young students to count objects in an underwater scene and click on the corresponding numeral.

Bones <http://colaborativelearning.pbworks.com/w/page/31904645/Kindergarten%20Math>

- An interactive game for students to count to find the location on the hundreds board where the dog bones are hidden. Reinforces using a blank hundreds board and knowing count sequence from 1-100 by ones and tens.

Sample Formative Assessment Tasks/Questions

Assessing Counting Sequences Part 1; <https://www.illustrativemathematics.org/content-standards/K/CC/A/1/tasks/448>

Assessing Counting Sequences Part II <https://www.illustrativemathematics.org/content-standards/K/CC/A/2/tasks/449>

Assessing Reading Numbers to 20 <https://www.illustrativemathematics.org/content-standards/K/CC/A/tasks/450>

Assessing Sequencing Numbers <https://www.illustrativemathematics.org/content-standards/K/CC/A/tasks/451>

Assessing Writing Numbers <https://www.illustrativemathematics.org/content-standards/K/CC/A/3/tasks/452>

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Understanding Relationship of Counting and Quantity

Content Standards with Clarifying Notes*Open Bullets Indicate Clarifying Notes*

- **K.NS.4** Understand the relationship between number and quantity. Connect counting to cardinality by demonstrating an understanding that:
 - a. the last number said tells the number of objects in the set (cardinality);
 - b. the number of objects is the same regardless of their arrangement or the order in which they are counted (conservation of number);
 - c. each successive number name refers to a quantity that is one more and each previous number name refers to a quantity that is one less.
- **K.NS.5** Count a given number of objects from 1-20 and connect this sequence in a one-to-one manner.

New Academic Vocabulary for This Unit

- | | | | |
|--|------------|------------|------------|
| ● numeral | ● quantity | ● 1 more | ● amount |
| ● number | ● count | ● 1 less | ● sequence |
| ● digit | ● how much | ● count on | ● accuracy |
| ● number names
(e.g. one, two, three, etc.) | ● how many | ● set | ● estimate |

Prior Knowledge Required for this Unit

In Unit 1 students learn the count sequence from 1-100 by ones and tens. In this unit, students continue to build on the count sequence and number names to deepen their understanding that number names and number symbols relate to a quantity and all of these together are the meaning of counting. In Unit 1 students begin to develop one-to-one correspondence, however students will develop this skill up to different

magnitudes at their own developmental pace. While one student might have one-to-one correspondence for a small set of objects, another student might be developing one-to-one with a larger set of objects. The same applies to the count sequence, it cannot be forced in a given amount of time.

Subsequent Knowledge Related to this Unit

As students deepen their understanding of numbers and make meaning of counting and cardinality, they will develop beyond rote counting and one-to-one correspondence to more complex phases of meaningful counting. Unit 2 and Unit 3 should be a continuation of Unit 1, building on the development of individual students. Unit 1 provided the foundation for the critical learning phase of counting objects.

Unit 2 provides more time for student understanding that in the count sequence, the next number is one more, and when counting by tens, the next number is “ten more” (or one more group of 10). As students gain this understanding, they will be prepared to compare numbers in Unit 3. However, those understandings are not innate and students should be given intentional learning opportunities. Students should then be able to apply this knowledge to part/whole relationships, compensate to compose and decompose numbers in Units 3 & 4, as well as develop a foundation for place value and operations with numbers in subsequent units as well as subsequent grade levels.

Students will use their fluency with the count sequence and cardinality to compose and decompose numbers in Unit 4, and then again to model addition and subtraction with story problems in Unit 5.

As students are able to subitize, they will use this skill in addition to cardinality to build single-digit fluency to 5 in Unit 5.

Relationship Among Standards in this Unit

The content standards and the process standards work together in this unit to form the foundation which will enable all students to develop the world class knowledge, skills, and life and career characteristics identified in the *Profile of the South Carolina Graduate*.

The standards in this unit work together to deepen students understanding of counting and cardinality, as well as provide more time for students to understand quantities. Building on the foundational standards in Unit 1, the standards in this unit focus on students making meaning of their counting skills. As students develop one-to-one correspondence, they should notice the necessity to keep track when counting, thus students should be able to keep track of an unorganized pile when counting (conservation of number). As students deepen their understanding that the last number said tells the number of objects in the set (cardinality) they should take notice when a recount of objects results in a different number or be bothered when counting a group results in the same number after more objects have been added or taken away.

This is the beginning stage of students reasoning about their counting. As students begin to make meaningful connections between the objects they count, cardinality, and the rote count sequence, they might spontaneously check or recount to see if the result is the same. With K.NS.4 and K.NS.5 students will begin to answer “How many?” questions, as well as be able to count out a specific quantity. Depending where students are in their development, these behaviors might be observed at different times with different students. Providing students with real world, meaningful counting experiences will lend itself to students reasoning about their counting.

The standards in Units 1-4 are essential to the key concepts Number Sense and Base Ten and Algebraic Thinking and Operations in grades K-5. Meaningful, daily experiences with the standards in Unit 2 should be provided to young students to set a strong foundation for mathematics success. Students will build upon their understanding of counting and cardinality in Unit 3 as they count and compare quantities.

Potential Instructional Strategies/Lessons

Kindergarten is the pivotal grade for creating a mathematical classroom which encourages collaboration and builds community. Students who are college-and-career-ready take a productive and confident approach to mathematics by communicating about mathematics to gain understanding. Students in kindergarten can begin developing their communication skills by:

- Acting out mathematical scenarios/problems
- Frequently exchanging mathematical ideas and problem solving strategies
- Listening to understand one another. This involves thinking about what a person is saying so that you can explain it yourself or to help them explain it more clearly. It is not just being quiet when someone else is talking. Also, children need to listen so that they can ask a question or help the explainer.
- Utilizing *Math Talk* in the classroom before being asked to follow procedures

Kindergarten students will naturally want to do many things on their own, however it is encouraged to have students begin working in pairs or small groups to build reasoning and communication skills.

Students should be provided many, meaningful opportunities to count throughout the day. Provide settings which connect mathematical language, quantities, and symbols to kindergartners’ everyday lives. Support the natural ability of young children to mathematize their world. Students should be provided with daily, meaningful opportunities to continue counting and demonstrating one-to-one correspondence with concrete objects, in addition to being asked to reason about the quantities they count as well as count sets of objects (organized and unorganized). While students should be provided opportunities to answer questions about “How Many?”, they should not be expected to answer how many altogether? How many left? Those questions will be addressed later with operations in Units 4 & 5.

Possible Introductory Lesson

Counting Stories

Standard: K.NS.4 and K.NS.5

Objective: Students will be able to act out number stories using manipulatives to represent the objects in the stories. After students have acted out the stories, they will be able to answer questions about “How Many?” Students will also be able to count the objects in their stories and tell the corresponding number to the quantity counted, justifying their count.

Materials:

Provide each student with

*connecting cubes (or counters)

*blank paper or blank story setting backgrounds (such as a barn, lake, tree, garden, classroom)

Glencoe Virtual Manipulatives This website has virtual story setting backgrounds

http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html

Procedure:

1. Tell students a counting story such as,
“Three children are playing on the slide. Two children are playing in the sandbox. Count the children.”
2. Provide time for students to act out the story placing their counters on the story setting or blank piece of paper.
3. Tell students another counting story, “A mama bear and a daddy bear are walking in the woods with their two babies. Count the bears.
4. Provide time for students to act out the story placing their counters on the story setting or blank piece of paper.
5. Repeat telling students stories to act out with the counters.
6. Allow students to make up their own counting stories to have the class act out.

Possible Extensions:

For students who are ready, ask them to write the number that matches how many counters they put out for each object in the story. For example, *“Three children are playing on the slide. Two children are playing in the sandbox. Count the children.”* Children would write 3 and write 2 to represent the children playing, along with placing their counters on the paper. They could also write 5 for the number of children they counted.

Possible Instructional Strategies:

- **Read Alouds and Counting Books**

Book: *The Very Hungry Caterpillar* by Eric Carle <http://www.k-5mathteachingresources.com/support-files/the-very-hungry-caterpillar.pdf>

<https://www.youtube.com/watch?v=vkYmvxPOAJI> The author, Eric Carle reads the book aloud.

Standard: K.NS.4 and K.NS.5

Objective: Students will be able to count a set of objects and match the quantity with a numeral.

Materials:

*Copy of the book The Very Hungry Caterpillar to read aloud to the class. If possible, a copy for each pair of students or small group working together as well to refer back to story.

*Counters

Procedure:

1. Build students background knowledge by asking, “What are some foods you like to eat? How much of that food would you eat if you are very hungry?”
2. Tell students you are going to read aloud the story, The Very Hungry Caterpillar by Eric Carle 2 times, the first time you just want them to listen to what is happening in the story.
3. Ask students, “What happened in the story?”
4. Tell students you are going to read the story a second time and this time you want students to listen for the quantity or how many fruits/foods the caterpillar eats on the different days of the week.
5. Ask students to share what they heard for how many fruits the caterpillar ate on different days of the week, refer back to the pictures in the book as necessary for students to see the pictures.
6. Tell students they are going to solve a problem today about the caterpillar’s eating. Show the following problem and read it aloud to students:*The Very Hungry Caterpillar was very hungry so he ate... One apple, two pears, three plums, four strawberries, and five oranges. How many pieces of fruit did he eat?*
7. Provide partners or small groups with counters and ask them to show the number of each fruit with their counters.
8. As students represent the number of fruits with their counters ask them, “How many apples did the caterpillar eat? How many pears? How many _____?” Checking to see if students orally count and use one-to-one correspondence, matching quantities with the objects.
9. Wrap up the lesson by discussing with students how they represented the number of fruits the caterpillar ate. “How does the number you say relate to the quantity of objects you put out?”

Possible Extensions:

- Provide students with a blank BLM with the phrase, “On (day of the week), the caterpillar ate (number) pieces of (name of food). Allow students to insert a day of the week, number, and name of food, then illustrate the page. Combine all the students’ pages into a class book.

- **Grab Bag Counting**

Standard: K.NS.5

Objective: Students will be able to count a set of objects and match the quantity with a numeral.

Materials:

Provide students (individually or in small groups) with

*brown paper bags filled with up to 20 objects to count (pennies, beans, unifix cubes, buttons, anything that can be counted).

*Numeral Cards 1-20 with a picture of the quantity in an organized arrangement one one side of the card and the numeral on the other

Procedure:

1. Student grabs a handful of objects out of the bag, putting the objects on the table in front of them.
2. Student counts the objects
3. Student matches a numeral card with the quantity and verifies the match by using one-to-one correspondence with the cubes and number picture on the back side of the card.
4. Students repeat steps 1-3 taking turns grabbing, counting, matching and verifying.

Possible Extensions:

1. Increase the number of objects in the Grab Bags
2. Have students who are ready write the numerals instead of matching with the card. In order for teacher to verify, have students also draw the correct quantity with the written numeral.

- **Pick a Number**

Standard: K.NS.4 and K.NS.5

Objective: Students will be able to count out a specific number of objects

Materials: Provide students (individually or in small groups) with:

*counting cubes

*paper bags

*Numeral cards 0-20

Procedure:

1. Student picks a number card out of the brown bag
2. Student counts out the number of cubes to match the numeral card drawn and verifies the count by a one-to-one correspondence with

the picture on the back of the numeral card.

3. Repeat steps 1-2 for students to take turns.

Possible Extensions:

1. Include higher numbers beyond 20 on the cards and in the bags.
2. In pairs, have one student write a numeral and the partner student counts out the correct number of objects.

● **Favorite Numbers Routine :**

1. Ask students to tell the class their favorite number. Teacher writes the student's name and favorite number on an index card. (Or students write it if able)
2. Teacher places all of the favorite numbers in a jar, and draws one out each day.
3. Each day, when teacher draws out a favorite number, he/she tells the class whose number it is and what number it is.
4. Then teacher asks students to count out the number of objects with the class.
5. As favorite number cards are drawn, teacher can post in classroom to begin creating an organic number line.

Possible Culminating Lesson

The Right Number of Elephants (adapted from Teaching Number Sense in Kindergarten by Chris Confer)

Standards: K.NS.4 and K.NS.5 and Mathematical Processes

Objective: Students will be able to think about numbers of items that might typically be found in the world, such as the number of legs on an animal, the number of pancakes on a plate, or earrings on a person. Students will be able to engage in mathematical discussions about reasonable and unreasonable numbers of things.

Materials:

*Book, The Right Number of Elephants by Jeff Sheppard

*Half sheets of paper containing a descriptive phrase, (pancakes on a plate, feet on an animal, birds in a tree, hair braids, tires on a truck, people in a car, apples in a bowl, books on a table, fingers on a person, buttons on a shirt, etc)

* BLM that says, "The right number of _____ is" (1 per pair of students)

Procedure:

1. Read the story, The Right Number of Elephants by Jeff Sheppard to the class.
2. Discuss how the author of the story wrote the story to be funny, but that there are reasonable or unreasonable numbers of things and it's our job as mathematicians to decide when a number is "the right number" of objects.
3. Choosing different parts of the story, discuss why the number in the story was the "right" or not right number for those objects.

4. Explain to students they will be illustrating “the right number” of one of the phrases and then you will compile them into a class book.
5. Provide pairs of students with one of the descriptive phrase papers and BLM that says, “The right number of _____ is _____.” instructing students to illustrate and write the number.
6. Combine all pages into a class book and read aloud, then place in classroom library.

Resources

Teacher Resources

Number Early Learning Progression <http://nzmaths.co.nz/sites/default/files/Number-Early-Learning-Progression.pdf>

- This chart provides the learning progression for learning numbers and the count sequence.

The Principles of Counting <http://math.about.com/od/counting/a/The-Principles-Of-Counting.htm>

- Explanation of the different aspects of counting such as cardinality, conservation of number, and unitizing.

Number Talks with Tens Frames and Dot Cards <http://www.cleanvideosearch.com/media/action/yt/watch?v=62epCIFdRa0>

- A video showing a number talk around numbers on tens frames and dot cards

Lesson Planning Resources

K.NS.3 & K.NS.5 Building Sets of Ten <http://illuminations.nctm.org/Lesson.aspx?id=1649>

- A lesson plan for exploring sets of up to 10 items, and practice writing numbers 0-10.

K.NS.3 Building Numbers to Five <http://illuminations.nctm.org/Lesson.aspx?id=1616>

- Detailed lesson plan from NCTM for students to work on building and understanding numbers to 5.

Howard County Kindergarten Math Wiki <https://gradekcommoncoremath.wikispaces.hcpss.org/kindergarten+home>

- This website has a wide variety of daily number routines, vocabulary cards, lesson plans/activities, and assessment tasks including rubrics for proficiency.

Department of Education, Virginia Number and Number Sense Module

http://www.doe.virginia.gov/instruction/mathematics/elementary/number_sense_module/nns_gradeK.pdf

- This is a lesson plan module for Kindergarten teaching number and number sense, it has several lesson ideas and plans.

Children’s Books for Math <http://www.the-best-childrens-books.org/math-for-kids.html>

- This website has lists of children’s books for different concepts in math that make learning the concepts engaging and relatable to small children.

Interactive Resources

K.NS.4 and K.NS.5 Find the Numbers 0-5 or 5-10 <https://www.illustrativemathematics.org/content-standards/K/CC/A/tasks/403>

- Game which students can play in groups of 2 or 3 to practice identifying numbers and matching quantities.

K.NS.4 and K.NS.5 The Gingerbread Man Game <http://www.topmarks.co.uk/learning-to-count/gingerbread-man-game>

- Interactive game for students to practice counting, match numeral to quantity, and order sets

K.NS.4 and K.NS.5 Ladybird Counting <http://www.topmarks.co.uk/learning-to-count/ladybird-spots>

- Interactive game for students to count, read numerals and number words, and match numeral with quantity by dragging and dropping spots on ladybugs.

Sample Formative Assessment Tasks/Questions

Dots and Numeral Card Match: <https://cgpsmathematicsk-5.wikispaces.com/file/view/Elementary+FAL+packet+for+fall+2011+1.pdf>

Apples in a Bag:

<http://www.cpalms.org/Public/PreviewResourceAssessment/Preview/36568>

Books and Bookmarks:

<http://www.cpalms.org/Public/PreviewResourceAssessment/Preview/36564>

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Composing and Decomposing Numbers Through 10

Content Standards with Clarifying Notes*Open Bullets Indicate Clarifying Notes*

- **1.ATO.6** Demonstrate:
 - a. addition and subtraction through 20;
 - b. fluency with addition and related subtraction facts through 10.
 - Students should use strategies such as counting on, making 10, decomposing a number leading to a 10, using the relationship between addition and subtraction, creating equivalent but easier or known sums, doubles plus or minus one, counting back, and the commutative property. Understand the role of 0 in addition and subtraction. Fluency is defined as efficient, accurate, and flexible. Phases of operational understanding: construct operational meaning, develop reasoning strategies, and work toward quick recall.
 - Teacher Note: The NCTM Principles and Standards for School Mathematics defines **computational fluency** as having efficient, flexible, and accurate methods for computing. Computation fluency of “mathematical fluency” with whole numbers is an essential guide for school mathematics and forms the foundation for many higher level math concepts.
- **1.ATO.5** Recognize how counting relates to addition and subtraction.
 - Teachers will need to assist students in developing the relationship between counting and the operations of addition and subtraction. For example, skip counting forward or backward by 10. The teacher could use skip counting or counting on. May want to define counting on. The concept of missing addend may be used for subtraction. (Related to CC.1.OA.5 and CC1.NBT.6)
 - Teacher Note: Use number lines as a visual of the number sequence.
- **1.ATO.9** Create, extend, and explain using pictures and words for:
 - a. Repeating patterns (e.g., AB, AAB, ABB, and ABC type patterns);
 - b. Growing Patterns (between 2 and 3 terms/figures)
 - Teacher Note: Patterns can be found in physical and geometric situations as well as in numbers. Introduction to recognizing linear patterns through looking at the number sequence. A plus 1 growing pattern is found through decomposing numbers to find the combinations within.

- **1.NSBT.1** Extend the number sequence to:
 - a. count forward by ones to 120 starting at any number;
 - b. count by fives and tens to 100, starting at any number;
 - c. read, write and represent numbers to 100 using concrete models, standard form, and equations in expanded form;
 - d. read and write in word form numbers zero through nineteen, and multiples of ten through ninety.
 - Individually and in sequence
 - One purpose for counting by groups (“unitizing”) is to make counting more efficient.
 - Teacher Note: Standard to be thought of as ongoing. Start with number sequence through 10 within Unit 1
- **1.ATO.1** Solve real-world/story problems using addition (as a joining action and as a part-part-whole action) and subtraction (as a separation action, finding parts of the whole, and as a comparison) through 20 with unknowns in all positions.
 - Students should use spoken words, concrete objects, drawings such as tape diagrams, pictorial models, length-based models (i.e., connecting cubes), number lines, and number sentences to solve story problems involving strategies of adding to, taking from, putting together, taking apart, and comparing, with the unknown as any one of the terms . In the problem such as $2 + 4 = []$; $3 + [] = 7$; and $5 = [] - 3$. Students may use a variety of basic fact strategies such as composing a 10 and decomposing a number leading to 10. Students should explain the problem-solving strategy with spoken words, concrete objects, pictorial models, and number sentences.
 - Addition and subtraction have been separated into four categories; join problems, part-part-whole problems, separate problems, and compare problems. Each category has three numbers, and any one of the three numbers can be the unknown in a story problem.
 - Teacher Note: **Joining** action-involves three quantities; an initial amount, a change amount (the part being added or joined), and the resulting amount (the amount after the action is over). **Part-Part-Whole** action-involves two parts that are combined into one whole. There is no meaningful distinction between the two parts within a part-part-whole situation, so there is no need to have a different problem for each parts as the unknown. **Separation action** involves three quantities; the initial amount as the whole or the largest amount, a change, and result amounts. **Compare problems** involve the comparison of two quantities, and the third amount is the difference between the two amounts. (Adapted from Van de Walle)
 - Teacher Note: Unit 1 provides an introduction to the concepts within this standard.
- **1.ATO.3** Apply Commutative and Associative Properties of Addition to find the sum (through 20) of two or three addends.
 - Students should use concrete, pictorial, and verbal representations of the commutative property and associative property of addition when solving. It is not important that students know the property name, but the concept the property provides.
 - Teacher Note: Within this standard Unit 1 focuses primarily on *understanding* the properties to find sums (through 10) of two addends.

New Academic Vocabulary for This Unit

- | | | | | |
|----------------|-----------------------|-------------------|------------------|-------------|
| ● 5-group | ● partner houses | ● sum | ● plus 1, plus 2 | ● fact |
| ● extras | ● switch the partners | ● equal to | ● minus | ● compare |
| ● break apart | ● addend | ● equation | ● take-away | ● fewer |
| ● decompose | ● set | ● number sentence | ● decrease | ● less than |
| ● symbol | ● pair | ● expression | ● counting on | ● more than |
| ● combinations | ● doubles | ● plus sign | ● counting all | ● greater |
| ● partners | ● equal | ● plus | ● counting back | ● represent |

Prior Knowledge Required for this Unit

Students should be able to count groups of objects; telling how many in all (cardinality) and have an understanding of one-to-one correspondence. In Kindergarten students are introduced to the concepts of addition and subtraction and begin developing an understanding through modeling situations within 10 using objects, fingers, mental images, drawings, acting out situations, verbal explanations, expressions, and equations. Fluency within 5 was expected at the Kindergarten level.

Subsequent Knowledge Related to this Unit

In Grade 1, students begin to develop an understanding of place value as working with numbers to 10 continues from Kindergarten. Repeated Kindergarten experiences in Unit 1 extends the conceptual algebraic thinking bridging an understanding of subtraction as an unknown addend problem and determining missing numbers in addition and subtraction equations within 20 required in Unit 2. It takes time for children to build understanding of such problems, but this is crucial, and they need to see them for each number through 10. Later in Unit 2, students will extend their thinking to using strategies for adding and subtracting.

In Unit 1, students begin developing a deeper understanding of the concepts of addition and subtraction begun in Kindergarten. Through a variety of meaningful and engaging experiences, students develop an understanding of the number combinations working towards mastery of

understanding the meaning of addition and subtraction and applying operations to solve problems in context.

Understanding the operations is essential, as well as understanding number concepts and relations. Decomposing and composing numbers to ten builds a foundation towards understanding number concepts and relations. At first children find the partner combinations embedded within one-digit numbers. Then in Unit 2 students build on understanding the partner combinations embedded within one-digit numbers, and begin to apply Commutative and Associative Properties of Addition to find the sum (through 20) of two addends.

Students will continue to apply the properties of addition to find the sum (through 20) in Units 2 and 4. Once students are able to understand and apply place value concepts, applying the Commutative and Associative Properties of Addition to find and the sum (through 20) with 3 addends will progress later on in Units 2 and 4 as well.

Once students have a foundational understanding of numbers and operations, Unit 3 moves on to understanding place value, finding the tens and ones in two-digit numbers. Students should progress quickly from subitizing and unit counting in kindergarten to visualizing numbers in groups of 5 and 10. Visualizing quantities in groups of 5s and 10s prepares children to understand place value and helps them with mental computation which comes later in 2nd grade.

It is essential for students to understand the operations before beginning to practice any math fact fluency. Unit 1 spends time breaking whole numbers apart into combinations working towards automaticity with math facts. As students progress to Unit 2 they will begin to apply strategies and are then ready to begin to commit those facts to memory later in 2nd grade.

Teacher note: Commutative and Associative Properties of Addition to find the sum (through 20) of two or three addends is taught and applied in first grade. Second grade expectations are for students to use the properties to add and subtract within 99 fluently.

Relationship Among Standards in this Unit

The various indicators in the unit were combined to develop number concepts and understanding the operations of addition and subtraction through decomposing and composing numbers through 10. The standards in Unit 1 relate to the key concept Algebraic Thinking and Operations. The standards included in first grade are the ultimate goal for students to master by the end of the year. Units 1-2 set the foundation for understanding and applying place value concepts in Units 3 and 4. Within standard 1.ATO.1, Unit 1 will focus on understanding number concepts and relations as a foundation towards solving real-world story problems using addition and subtraction through 10.

Unit 1 focuses on the 1-more and 1-less pattern, first with counting numbers, then with finding partners, and finally with addition and

subtraction. Students in first grade are working with some of the same number concepts and operations within 10 from Kindergarten. However, in first grade they are progressing from modeling situations to solving and representing equations. Basic fact instruction for fluency begins with conceptual understanding and many opportunities to develop strategic thinking. Understanding the relationship between the operations; addition, and subtraction, is critical. A variety of practice methods build automaticity and quick recall. The foundation for conceptual understanding begins in Unit 1. The standards are intertwined within each other, students will develop an understanding of the 1-more and 1-less pattern first with counting numbers, then with finding partners, and finally with addition and subtraction computations.

As students decompose and compose numbers through 10, direct their focus on thinking about a number as a whole, with partners that form combinations within. When students find all possible combinations of numbers through 10, (e.g., 5 has 4 sets of combinations, $4+1$, $3+2$, $2+3$, $1+4$), they begin recognizing how counting relates to addition and subtraction (1.ATO.5), and then will begin to develop an understanding of how to apply that knowledge and use strategies to solving real-world/story problems using addition (*as a joining action* and *as a part-part-whole action*) and subtraction (as a separation action, finding parts of the whole, and as a comparison) through 20 with unknowns in all positions (1.ATO.1). The exploration of patterns and properties guides students towards demonstrating fluency with addition and related subtraction facts through 10. (1.ATO.6)

Students will discuss patterns as decomposition equations are recorded. Patterns across the decompositions will be recognized as well as recognizing growing patterns within the number sequence (Introduction to 1.ATO.9.b). Our Base Ten numeral system is a system of patterns, and these patterns become visible throughout the exploration of addition and subtraction facts. Math facts are predictable because of these patterns. **Teacher Note:** “Once students understand that our number system is a system of patterns, they begin to recognize patterns in math facts that will help them make sense of, and remember, the facts. Noticing patterns that emerge when observing the equations $6+4=10$, $7+3=10$, and $9+1=10$ will lead to some interesting discussions as students attempt to explain their observations.” (*Mastering the Basic Math Facts in Addition and Subtraction p. 16*). **Teacher Note:** Numeric patterns are addressed when looking at the number sequence, understanding that when counting by ones the next number in the sequence is one more each time (plus 1), and when counting backwards by ones the next number in the sequence is one less each time (minus 1). **Teacher Note:** The number of combinations a whole number has is 1 less than the whole number. Knowing and understanding that concept will help students independently discover all combinations for a given whole number. For example the whole number 2 has 1 combination ($1+1$), the whole number 3 has 2 combinations ($1+2$, $2+1$), the whole number 4 has 3 combinations ($1+3$, $2+2$, $3+1$). The number zero is to be thought of as a known partner which it’s not included within the combinations.

Potential Instructional Strategies/Lessons

Through the continuous exploration of decomposing numbers, number combinations (facts) can be found within a given number (fact families). Once students are able to decompose and compose numbers, then their understanding extends to being able to see the parts of a whole and then they are able to think about the four categories and the three numbers within to understand strategies for finding unknowns in a story problem. For example, if I know that $6=2+4$, then I can use that to solve the following problem: $6-4=[]$.

It is essential to spend time manipulating and representing numbers, and solving simple problems in which students gain a strong foundation of number sense. Having number sense and understanding numbers in the early years involves the understanding of quantity, comparing quantities, fluency and flexibility with counting, and the ability to perform simple operations with numbers. Students need to explore numbers, in a variety of ways, with a variety of materials. Learning experiences should take students from world experiences, to concrete materials, then to visual (and other) representations, before expecting abstract representations.

Students need to see and use equations in many forms. When children are comfortable representing quantities, encourage them to begin creating stories about the groupings. Encourage students to create story problems as well as solve story problems presented orally. (e.g., When decomposing numbers the teacher and students can present the task as a story problem; There were 7 children in the library. 6 children were reading. How many were writing?. Students can also tell stories about the number combinations and equations. (e.g., There are ___ apples in the basket. ___ are green and ___ are red.) Over time students will become comfortable and flexible with mathematical language and can connect concepts and terminology with meaningful referents from their own lives. Modeling the use of simple real-world story problems within instruction from the beginning will scaffold students' thinking to making connections within the operations as well.

Create a mathematical classroom which encourages collaboration and builds community. A suggestion for developing World Class Skills a South Carolina student needs to be college and career ready is to: Provide settings within the mathematical classroom that promotes the use of *Math*

Talk:

- Frequently exchange mathematical ideas and problem solving strategies.
- Children listen to understand one another. This involves thinking about what a person is saying so that you can explain it yourself or to help them explain it more clearly. It is not just being quiet when someone else is talking. Also, children need to listen so that they can ask a question or help the explainer.
- Encourages critical thinking and problem solving, collaboration and teamwork, and knowing how to learn

- Teachers can stand back or to the side of the classroom to encourage Math Talk as students interact more directly with each other.
- Most common structures:
 - Solve and Discuss: 4 to 5 students solve, explain, question, and justify at the board, while the rest of the class works the problems at their seats. 2-3 students are teacher selected to explain their methods.
 - Solve and Discuss Small Group version: *(after whole group discussion has taken place)* Students solve a problem individually within a small group. 2-3 students explain their method and solution to the rest of the group while the others are encouraged to ask questions for clarification.
 - Student Pairs and Helping Pairs: Two students work together, learning from each other, particularly in applying and practicing concepts introduced in whole-class discussion. Helping pairs-More advanced students are matched with students who are struggling.
 - Scenarios- Students act out a particular mathematical situation
 - Small Groups: Students work in groups

Introductory Lesson:

Discuss Numbers 1-10 *(adapted from Math Expressions)*

Lesson Focus: Visualize and represent numbers 1-10

Objectives:

The students will understand the 1-more and 1-less pattern by using *Stair Steps* to show 1-more and 1-less sequences for numbers 1-10. The students will visualize and represent numbers as a group of 5 and extra ones using perceptual and conceptual subitizing.

Materials:

- ‘Stair-Steps’. Each student will need a printed copy of the ‘Stair Steps’ on page 3 from the resource below:
 - https://www-k6.thinkcentral.com/content/hsp/math/hspmathmx/na/gr1/ete_9780547838717_resource.html?Unit=1&Less=1&Type=Copymaster
 - Teacher Note: When cutting out the Stair-Steps, cut along the dashed lines, and NOT the solid lines.

Whole Group:

1. Discuss Stair Steps

- Introduce Stair-Steps as a 'math tool' that will be used to help them learn about numbers 1 through 10.
- Ask for Ideas: Invite children to discuss what Stair Steps might be and how they could be used to learn about number.
- Provide each child with a set of Stair Steps. Allow a few minutes for student exploration.
- The teacher should observe how children arrange their set of steps.
- Ask children to share what they notice about the Stair Steps.

2. Sequentially Arrange the Stair Steps

- Ask children to find Stair Step 1 and place it at the top of their work space. Then have them place Stair Step 2 under it, followed by Stair Step 3 and so forth.
- Question Children as they work. (What do you notice about the Stair Steps? How are you lining them up? Are you making steps?)
- Once the sequence is complete, guide children in reciting the 1-more sequence as they touch each step
 - 1 and 1 more is _____. 2
 - 2 and 1 more is _____. 3
 - 3 and 1 more is _____, and so on. 4
- Then recite the sequence in reverse as they touch each step.
 - 1 less than 10 is _____. 9
 - 1 less than 9 is _____. 8
 - 1 less than 8 is _____, and so on. 7

3. See quantities as 5-groups: Use a 5-group and extra ones

- Ask children to find Stair Step 5 and center it on their workspace. Then have them put Stair Step 1 underneath. Help students express the total number of dots.
 - 5 and 1 more is _____. 6
- Then replace Stair Step 1 with Stair Step 2 and tell how many.
 - 5 and 2 more is _____. 7
- Then replace Stair Step 2 with Stair Step 3 and tell how many.
 - 5 and 3 more is _____. 8
- Then replace Stair Step 3 with Stair Step 4 and tell how many.
 - 5 and 4 more is _____. 9

4. Represent the 5-Group

- The teacher will use the finger rhyme, "Five Crows in a Row," to illustrate the 5-group and the extra ones in numbers 6 through 10. Children show the "crows" with their fingers and respond with the total.
 - The teacher shows 5 fingers on one hand and 2 fingers on the other hand and says the rhyme. The children each put up the same number of fingers and respond with the total. The teacher repeats the activity moving to 2 below, 3 below, and so on. Then try the numbers in random order. *For the first time the teacher should lead the activity but once students are familiar with the rhyme, you may want to invite a few children to act as Student Leaders.*
 - **Five Crows in a Row Rhyme:**
5 crows in a row.
And [1] below.
How many crows? **6 crows**
 - Teacher Note: The ability to form quick mental pictures of quantities by grouping the units is crucial to performing mathematical operations efficiently. Children benefit greatly from learning of visualize numbers without counting individual units. Visualizing numbers without counting is called perceptual subitizing. Perceptual subitizing develops into conceptual subitizing when children visualize two numbers and combine their value to find the total. Visualizing quantities in groups of fives and extra ones.

5. Informal Assessment: Student Summary

- Ask children to use Stair Steps to show 9 as a 5-group and extra ones. Then have them use words to describe their arrangement. Responses should include that 5 and 4 more is 9.
 - Teacher Note: Students can record their responses in a Math Journal.

Possible Lessons within the Unit:

Daily Activity: Number of the Day Stretch:

- https://books.google.com/books?id=vQDOAAwAAQBAJ&pg=PA39&lpg=PA39&dq=number+of+the+day+stretch&source=bl&ots=wkvMTB_eu4R&sig=YRGevESgrcZxoUPND6j74xDIS4&hl=en&sa=X&ved=0CB8Q6AEwAGoVChMI77DK_76UxglVg5WACH1nfACx#v=onepage&q=number%20of%20the%20day%20stretch&f=false

Activity: Representing Partners: Break-Aparts and Partners of a Number:

Objective: Building Number Concepts

- Children can break apart numbers using a 'Break-Apart' stick such as a pencil or a coffee stirrer. Children can use objects to represent numbers (e.g., color tiles or plastic counters). Children begin to learn about the embeddedness of numbers by breaking them into smaller components. Understanding how numbers can be broken apart (decomposed) and put back together (composed) helps build a foundation for understanding addition and subtraction.
 - Teacher Note: As students decompose a given number, the teacher can write each decomposition with an equation such as $5=4+1$, showing the total on the left and the two addends on the right. Record the equations in sequential order. Students can find patterns in all of the decompositions of a given number and eventually summarize the patterns for several numbers. (Adapted from *Progressions for the CCSS on Operations and Algebraic Thinking*)

Activity: Shake and Spill: <http://teachmath.openschoolnetwork.ca/wp-content/uploads/grade1/documents/ShakeandSpill.pdf>

- This activity has students use two sided counters to generate parts for a given whole.

Activity: Building Partners with Cubes

- Have students use the total number of cubes of one color to build a cube-train. Then have them replace one cube of the train with a cube of another color and record the partners. Continue replacing one cube at a time and recording the partners.
- Teacher Note: Tactile learners may benefit from using connecting cubes to find the sets of patterns for a given total.
- Teacher Note: Algebra: *Switching the partners* involves changing the order of the partners, or addends, within combinations embedded in a number. When the partners switch places, the total stays the same. This switching of partners demonstrate the Commutative Property of Addition.

Multisensory Part-Part-Whole Exploration: <http://teachmath.openschoolnetwork.ca/wp-content/uploads/grade1/documents/PPWMS.pdf>

- The four activities for using touch and sound instead of visual cues for PPW. The activities require the students to make mental representations of the relationships.

Lesson: Sample mini-lesson for Breaking Numbers Apart: http://www.mathsolutions.com/documents/0-941355-74-8_L.pdf

- This sample lesson offers two versions of an activity that focuses students on breaking numbers apart (decomposing), one version for kindergarten and first-grade students and another for second graders. Refer to *From a Kindergarten and First Grade Classroom*.
- The activity gives students an opportunity to reason numerically, and write addition equations. Variations and modifications should be taken into consideration based on individual student needs.

Lesson: <https://www.engageny.org/resource/grade-1-mathematics-module-1-topic-lesson-1>

- **Lesson Objective:** Analyze and describe embedded numbers (to 10) using 5-groups and number bonds.

Lesson: Grade 1 Module 1: Sums and Differences to 10: 1.ATO.1, 1.ATO.5, 1.ATO.6,

- <https://www.engageny.org/resource/grade-1-mathematics-module-1-topic-c>
- “In this first module of Grade 1, students make significant progress towards fluency with addition and subtraction of numbers to 10 as they are presented with opportunities intended to advance them from counting all to counting on which leads many students then to decomposing and composing addends and total amounts.”
- Refer to Lessons 1-8 within Module 1; Lessons 9-39 can be used in Unit 2

Lesson: More and More Buttons: <http://illuminations.nctm.org/Lesson.aspx?id=290>

- Students use buttons to create, model, and record addition sentences. They also explore commutativity in addition contexts.
- **Lesson Objective:** In this lesson students will model the addition of set, use the terms “addend” and “sum”, create addition sentences, explore the commutativity of addition, and identify addends and sums on an addition chart.

Video: Visualizing Number combinations: <https://www.teachingchannel.org/videos/visualizing-number-combinations>

- This video is of a modeled Kindergarten lesson for using quick images to identify combinations of 8.
- This lesson can be modified for first grade students as a visual way for seeing combinations and that numbers can be seen in a variety of combinations.

Lesson: Numbers in Many Ways: <http://illuminations.nctm.org/Lesson.aspx?id=293>

- Students work with subtraction at the intuitive level as they explore number families and ways to decompose numbers to 10. They will also identify members of 'fact families.' [A fact family is a set of three (or two) numbers that can be related by addition and subtraction, for example: $7 = 4 + 3$, $7 = 3 + 4$, $7 - 4 = 3$, and $7 - 3 = 4$. When the number is a double, there are only two members of the fact family. An example would be $10 - 5 = 5$, and $5 + 5 = 10$.]
- **Lesson Objective:** In this lesson students will represent numbers in flexible ways, connect numerals to the quantities they represent, and identify the addition and subtraction sentences related to a specific sum and pair of addends.

Task: Making a Ten: <https://www.illustrativemathematics.org/content-standards/1/OA/C/6/tasks/1169>

- This task is designed to help students visualize where the 10's are on a single digit addition table and explain why this is so.

Culminating Lesson:

Use Patterns to add and subtract within 10 *(adapted from Math Expressions)*

Lesson Focus: Add and subtract within 1-10

Objectives:

The students will discuss patterns with number partners.

The students will use patterns to add and subtract within 10.

The students will understand the role of 0 in addition and subtraction.

Materials:

- chart paper (1 large piece or 2 small pieces)
- markers
- Premade chart showing “Patterns with Zero”
- Premade chart showing “Patterns with Doubles”

Whole Group

1. Patterns in Partners from 2 to 10.

- As a review, create a chart showing the patterns with partners. The teacher will write the total at the student and ask for volunteers to tell you the partners for each whole number as you record the combinations on the chart. Refer to the image, Patterns with Partners, above for how the chart should look.
- Discuss the patterns observed. Include the following in the discussion:
 - The top row is the Plus 1 partners. Plus 1 partners go with the number just before the total.
 - We have doubles within 10: $1+1$, $2+2$, $3+3$, $4+4$, and $5+5$.
 - You can make the partners of 10 from the partners of 9 by adding 1 to the first partners for 9: $8+1$ becomes $9+1$, $7+2$ becomes $8+2$, $6+3$ becomes $7+3$, and $5+4$ becomes $6+4$. Then there is the new double $5+5$.
- Generalize: Then guide children to generalize that you can make the partners for a new number from the partners for the number just before by adding 1 to the first partner. Remind children they need to determine if the new number also has partners that are doubles.

2. Patterns with Zero:

- Using the chart “Patterns with Zero” discuss the patterns. Tell children to look at the first column of equations in the chart, and

tell how all the equations are the same. *(They all add zero, which is not things at all).*

- Ask children to tell a story about $10+0$ and then tell the answer.
- Encourage children to tell what the general pattern is for adding 0 to a number. *(The answer is the number that is not zero. That number does not change because you did not add anything to it.)*
- Now look at the second column and ask, “How are all these equations the same?” *(They all subtract zero, which is no things at all)*
- Ask children to tell a story about $10-0$ and then tell the answer.
- Ask, “What is the general pattern for subtracting a 0 from a number?” *(The answer is the number that is not zero. That number does not change because you did not subtract anything from it.)*
- Then look at the third column and tell how the equations are the same. *(They all subtract a number from itself, which leave zero because you took away all of the things.)*
- Ask children to tell a story about $10-10$ and then tell the answer.
- Ask, “What is the general pattern for subtracting a number from itself?” *(The answer is 0.)*

3. Partner Work:

- Encourage students to discuss the equations in the Patterns with Doubles chart with a partner. Encourage them to share their observations and any connections they make with the Patterns with Partners chart.

4. Assessment Task: 1.ATO.1, 1.ATO.3, 1.ATO.5, 1.ATO.6

- Ask students to solve the following problem:
Student A is filling a bowl with apples and oranges. Student A puts 5 apples and 5 oranges in the bowl. Is there another way to fill the bowl with apples and oranges and have exactly 10 pieces of fruit in it? (Yes or No) Explain your answer. Write number sentences to show your thinking.
- Teacher Note: Questions to think about as students problem solve. Are students able to determine all of the ways in which two number can be combined by addition to give a sum of 10? To what extent do they apply understanding of quantity, part-part-whole relationship, and compensation. To what extent do they represent sums of ten by writing number sentences or combinations.

Resources

Teacher Resources:

- **Structures of Story Problems Part-Part-Whole** and examples of problems for K-6.
 - <http://www.cbv.ns.ca/consultants/uploads/MathConsultant/Part-Part%20Whole.pdf>
- **Part-Part-Whole Cards:** <http://teachmath.openschoolnetwork.ca/wp-content/uploads/grade1/documents/ppwcards.pdf> :
 - Printable cards that can be used to practice part-part-whole relationships
- **Three Printable Tens Frames:**
 - https://illustrativemathematics.s3.amazonaws.com/attachments/000/000/433/original/three_tenframe.pdf?AWSAccessKeyId=AKIAJFC4WL6K24MWHIRQ&Expires=1434465869&Signature=xoD8R1pPj70CwAXmRjQXv6K7IzU%3D
- **Math Literature:** <http://www.the-best-childrens-books.org/math-for-kids.html>
- **Building a Math Talk Community:** <http://www.eduplace.com/math/mthexp/pdf/mathtalk.pdf>
- **Math Talk 101:** <http://www.scholastic.com/teachers/top-teaching/2014/01/math-talk-101>
- **Math Talk: The importance; Why use it?:** <http://mathsolutions.com/common-core-support/math-talk/>
- **Math Talk conversation starter posters.** <http://mason.gmu.edu/~jsuh4/teaching/resources/Buildingmathideas.pdf>
 - The last two pages include an explanation of how to use Math Talk to build mathematical ideas and discourse.
- **Explanation of a Math Mountain:** <http://mllynch.blog.greenville.k12.sc.us/files/2012/09/Family-Letter-Math-Mountains-Snip.jpg>
- **Introduction to Math Mountains:** Math Mountains show a total at the top and a set of partners for the total at the bottom. They are called 'Math Mountains' because they look like mountains.
 - **Story of the Tiny Tumblers:** "Tiny Tumblers live at the top of the Math Mountain. Every day some of them go to play on one side of the mountain, and the rest go to play on the other side." Children find the partners of the total at the top of the Math Mountain by drawing circles to show how many Tiny Tumblers they were putting on each side.
 - Tell students that the Total is at the top of the mountain (Total at the Top), and the partners roll down either side and are written there.
 - Teacher Note: Students can use Math Mountains as a math tool and then as a strategy to decompose numbers to find possible number partners and combinations. Also, Fact Families can be found using Math Mountains; $5=4+1$, $5=1+4$, $5-4=1$, $5-1=4$.

Teacher Resources for standard 1.ATO.6.b:

- **Guided Math “Number Fluency Center: Materials:** <http://mrsspruiellatschool.weebly.com/fact-fluency-center-materials-k-1-2.html>
 - This resource includes materials that can be used to differentiate number fluency centers for guided math.
- **Basic Facts:** <http://www.carrollk12.org/instruction/instruction/elementary/math/curriculum/basicfacts/default.asp>
 - This resource includes suggestions for ways to measure fact fluency.

Resources for Interactive Sites:

- **Virtual Manipulatives:** http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html
 - This resource can be used a variety of ways. Options to select: Grade, Backgrounds (i.e., Game Boards, Story Boards, Workmats), and Manipulatives (e.g., attribute blocks, attribute buttons, color tiles, connecting cubes, spinner, two-color counters)
- **Virtual Manipulative: Number Blocks Freeplay.** <http://www.mathsisfun.com/numbers/number-block-freeplay.html>
 - Suggestion: Use in whole group instruction to model decomposing and composing numbers through 10.
- **Interactive Counting Games:** <http://jmathpage.com/JIMSNumbercounting.html>
 - This Interactive site includes math activities from across the web that have been organized by topic to make math learning enjoyable and interesting.
- **Teaching Tool: Beadstring:** http://ictgames.com/brilliant_beadstring_with_colour.html
 - Choose one or two beadstrings. Practice bonds of 10 or 20, number facts to 10 or 20.
- **Interactive Whiteboard: Triangular Cards:** <http://www.topmarks.co.uk/Flash.aspx?f=triangularcardsv4>
 - Useful for demonstrating inverse operations and fact families. Select either Bonds within 10 or add to 10.
- **Interactive Addition Game:** <http://www.sheppardsoftware.com/mathgames/earlymath/bugabalooShoes.htm>
 - Five different levels to choose from to practice math facts.
- **Interactive Subtraction Games:** <http://jmathpage.com/JIMSNumbersubtraction.html>
- **Interactive Operations Games:** <http://jmathpage.com/JIMSNumberoperations.html>
- **Interactive Game:** <http://illuminations.nctm.org/Activity.aspx?id=3563>
 - Independent of partner game in which students match whole numbers, shapes, fractions, or multiplication facts to equivalent representations.”
 - Teacher Note: Use the 1-6 or 1-10 number range within Units 1 and 2.
- **Interactive Game: Making ten:** <http://illuminations.nctm.org/activitydetail.aspx?id=75>
 - Making ten using the tens frame.

Sample Formative Assessment Tasks/Questions

Assessing decomposing through 10: Ask children to write the partners for a number (4-10), switch the partners, and describe a pattern noticed.

Assessing extending the number sequence to count forward by ones to 120 starting at any number:

- Provide pencil and paper to the student. Read the problem to the student: *Mrs. Ledbetter is counting students as they enter the classroom. She has just counted the 5th student. What numbers will Mrs. Ledbetter say for the next 5 students?*

Assessing extending the number sequence to read, write and represent numbers to 100 using concrete models, and standard form; and

Assessing extending the number sequence to read and write in word form numbers zero through nineteen, and multiples of ten through ninety.

- **Number of the Day Stretch:** A way to assess students' number sense as well as their ability to represent numbers in multiple ways. Suggested questions to informally assess:
 - Why do you think it is important to be able to represent numbers in different ways? When do we usually use number words to represent numbers? When do we use numerals? When do we use pictures of diagrams? When do we use number sentences? Why do we sometimes choose one method of representation rather than another?
 - Teacher Note: This can also be assessed throughout the school year as the range of numbers increases within classroom instruction.

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Addition and Subtraction Strategies

Content Standards with Clarifying Notes*Open Bullets Indicate Clarifying Notes*

- **1.ATO.7** Understand the meaning of the equal sign as a relationship between two quantities (sameness) and determine if equations involving addition and subtraction are true.
 - Students understand that the equal sign represents a relationship where expressions on each side of the equal sign represent the same value.
- **1.ATO.8** Determine the missing number in addition and subtraction equations within 20.
 - Students should use number sense as well as concrete and pictorial models such as number lines while identifying the missing whole number within at most a four term equation.
- **1.ATO.1** Solve real-world/story problems using addition (as a joining action and as a part-part-whole action) and subtraction (as a separation action, finding parts of the whole, and as a comparison) through 20 with unknowns in all positions.
 - Students should use spoken words, concrete objects, drawings such as tape diagrams, pictorial models, length-based models (i.e., connecting cubes), number lines, and number sentences to solve story problems involving strategies of adding to, taking from, putting together, taking apart, and comparing, with the unknown as any one of the terms . In the problem such as $2 + 4 = []$; $3 + [] = 7$; and $5 = [] - 3$. Students may use a variety of basic fact strategies such as composing a 10 and decomposing a number leading to 10. Students should explain the problem-solving strategy with spoken words, concrete objects, pictorial models, and number sentences.
 - Addition and subtraction have been separated into four categories; join problems, part-part-whole problems, separate problems, and compare problems. Each category has three numbers, and any one of the three numbers can be the unknown in a story problem.
 - Teacher Note: **Joining** action-*involves* three quantities; an initial amount, a change amount (the part being added or joined), and the resulting amount (the amount after the action is over). **Part-Part-Whole** action-involves two parts that are combined into one whole. There is no meaningful distinction between the two parts within a part-part-whole situation, so there is no need to have a different problem for each parts as the unknown. **Separation action** involves three quantities; the initial amount as the whole or the largest amount, a change, and result amounts. **Compare problems** involve the comparison of two quantities, and the third amount is the difference between the two amounts. (Adapted from Van de Walle)

- **1.ATO.4** Understand subtraction as an unknown addend problem.
 - Support: Subtract 10-7 by finding the number that makes 10 when adding 7. This standard is laying the foundation for the inverse relationship between addition and subtraction. Whereas ATO.8 is finding the missing number using any strategy.
- **1.ATO.3** Apply Commutative and Associative Properties of Addition to find the sum (through 20) of two or three addends.
 - Students should use concrete, pictorial, and verbal representations of the commutative property and associative property of addition when solving. It is not important that students know the property name, but the concept the property provides.
 - Teacher note: Build on the concepts of number combinations explored in Unit 1. Students should begin to use symbols appropriately (i.e., +, -, =) within the combinations of the three quantities.
- **1.ATO.6** Demonstrate:
 - a. addition and subtraction through 20
 - b. fluency with addition and related subtraction facts through 10.
 - Students should use strategies such as counting on, making 10, decomposing a number leading to a 10, using the relationship between addition and subtraction, creating equivalent but easier or known sums, doubles plus or minus one, counting back, and the commutative property. Understand the role of 0 in addition and subtraction. Fluency is defined as efficient, accurate, and flexible. Phases of operational understanding: construct operational meaning, develop reasoning strategies, and work toward quick recall.
 - Teacher Note: The NCTM Principles and Standards for School Mathematics defines **computational fluency** as having efficient, flexible, and accurate methods for computing. Computation fluency of “mathematical fluency” with whole numbers is an essential guide for school mathematics and forms the foundation for many higher level math concepts.
- **1.NSBT.1** Extend the number sequence to:
 - a. count forward by ones to 120 starting at any number;
 - b. count by fives and tens to 100, starting at any number;
 - c. read, write and represent numbers to 100 using concrete models, standard form, and equations in expanded form;
 - d. read and write in word form numbers zero through nineteen, and multiples of ten through ninety.
 - Individually and in sequence
 - One purpose for counting by groups (“unitizing”) is to make counting more efficient.
 - Teacher Note: Extend the number sequence as the year progresses.

New Academic Vocabulary for this Unit

- unknown result
- unknown change
- unknown start
- equation
- difference
- doubles fact
- making ten
- doubles +1
- doubles -1

Prior Knowledge Required for this Unit

Before moving into Unit 2, students need to have a strong understanding of number concepts and relations begun in Kindergarten and repeated in 1st grade Unit 1. It is essential for students to know how to independently decompose and compose numbers through 10 before beginning Unit 2. Students need to have an understanding of the concepts of addition and subtraction and understand the meaning of the operations and the actions it takes to make a group larger, smaller, and on number relations.

Students will develop strategies for adding and subtracting whole numbers based on their prior work with small numbers in Kindergarten and in Unit 1. Students need to understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). Once students understand the meaning for the operations of addition and subtraction they are ready to extend their thinking and begin practicing math fact fluency.

Subsequent Knowledge Related to this Unit

In Unit 2 students will build on their understanding of number concepts and relations, extending their thinking to using strategies for adding and subtracting. They will use their understanding of the operations as a foundation to build-on as they continue to recognize connections within addition and subtraction. The focus of Unit 2 will be on taking students' understanding of numbers and applying concepts to extend their thinking and begin practicing math fact fluency. Once students recognize and understand the relationship between counting and addition and subtraction they will use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20 in Units 3 and 4. By comparing a variety of solution strategies in Unit 2, children build on their understanding of the relationship between addition and subtraction.

Unit 2 will be an introduction on addition and subtraction strategies. Unit 2 will connect new facts to previously discussed number concepts; providing opportunities for students to continually build mastery of addition and subtraction basic facts through 10 in first grade and through 20 in 2nd grade. As connections are made, and new facts/concepts are introduced students extend their understanding of strategies to use as they solve problems. Emphasis on 'making ten' is crucial because 10 is the foundation of our number system. Students should be provided with multiple experiences in exploring the different ways in which 2 addends result in a sum of 10. This knowledge becomes critical as they explore using tens as a strategy for finding unknown facts in Unit 4.

Students who understand the concepts of addition and subtraction are able to understand the connections between math facts and real

situations (i.e., the flowers in a vase with 2 lilies and 9 tulips are represented by $2+9$). These students are better equipped to effectively solve math problems by choosing the operation that makes sense (i.e., “If you put together the lilies and tulips, you get how many flowers are in the vase altogether, so you have to add $2+9$ to find that out.”). They are able to make reasonable judgments about sums and differences (i.e., “It’s a little more than 10 because $2+8=10$.”). And students who understand the concepts of addition and subtraction are better prepared to begin the task of memorizing math facts in later grades because they understand what they are being asked to memorize. It’s important to build understanding prior to focusing on fluency. (adapted from *Mastering the Basic Math Facts in Addition and Subtraction*)

Our goal is to continually reinforce the ideas related to math facts as we help students develop addition and subtraction strategies; providing a foundation for solving more complex facts in later units and grades. Continuation of providing a variety of meaningful and engaging experiences, students develop an understanding of the number combinations working towards mastery of understanding the meaning of addition and subtraction and applying operations to problem solving.

Relationship Among Standards in this Unit

The standards in this unit are clustered together with the intention of continuing to build a strong foundation of number sense, understanding numbers and the operations of addition and subtraction while developing strategies. This understanding takes time. Unit 2 is an extension from decomposing and composing numbers within 10 to relating the same concepts with applying properties of addition to add whole numbers and to creating and using strategies based on these properties to solve addition and subtraction problems of quantities within 20.

Standards are repeated with the intention of extending the range of numbers within as you progress through the year. Standards should not be taught in isolation nor are students expected to show mastery of the standard by the end of a given unit. Students need opportunities for investigations, discussions, hands-on explorations, visual models, stories problems, and time to explore concepts.

The standards in Unit 2 encourage students to begin recognizing addition and subtraction problems, write equations using symbols to represent addition and subtraction situations, use connected strategies to solve for unknowns. Discuss different types of equations, decide if they are true or false, and develop strategies for adding and subtracting within 20. The students will grasp an understanding of the procedural aspect and begin using symbols to represent different operations and facts. Strategies for adding and subtracting within 20 will be developed.

Teacher Note: Students need to attach meaning to the operations before there is any focus on fact fluency. The big Ideas about numbers that help students make sense of math facts should be at the center of teaching math facts are: our number system is a system of patterns, the order of the factors does not change the sum (commutative property), addition and subtraction are inverse operations (e.g., Fact Families), and numbers are flexible; they can be broke apart to more easily perform an operation. (adapted from *Mastering the Basic Math Facts in Addition and Subtraction*)

Potential Instructional Strategies

Learning experiences should take students from world experiences, to concrete materials, then to visual (and other) representations, before expecting abstract representations. Once students explore strategies and have a strong understanding move towards applying strategies in order to problem solve story problems.

“Provide opportunities for students to participate in shared problem solving activities to solve word problems. Use a variety of models such as drawings, words, and equations with symbols for the unknown numbers to find the solutions. Additionally students need the opportunity to explain, write and reflect on their problem-solving strategies. Students need the opportunity of writing and solving story problems involving three addends with a sum that is less than or equal to 20. Literature is a wonderful way to incorporate problem-solving in a context that young students can understand. Many literature books that include mathematical ideas and concepts have been written in recent years. For Grade 1, the incorporation of books that contain a problem situation involving addition and subtraction with numbers 0 to 20 should be included in the curriculum. As the teacher reads the story, students use a variety of manipulatives, drawings, or equations to model and find the solution to problems from the story.” (<http://www.katm.org/flipbooks/1%20FlipBook%20Final%20CCSS%202014.pdf>) **Teacher Note:** Students need to analyze word problems and avoid using keywords to solve them.

Teacher Note: 1.ATO.1: Teachers should be aware of the three types of problems; Result Unknown, Change Unknown, and Start Unknown, and provide multiple experiences for their students solving ALL three types of problems as strategies are developed. The unknown symbols should include boxes or pictures, and not letters. Use informal language (add, minus/subtract, the same as) to describe joining (putting together) and separating situations (breaking apart). (<http://www.katm.org/flipbooks/1%20FlipBook%20Final%20CCSS%202014.pdf>)

Create a mathematical classroom which encourages collaboration and builds community. A suggestion for developing World Class Skills a South Carolina student needs to be college and career ready is to:

Provide settings within the mathematical classroom that promotes the use of **Math Talk**:

- Frequently exchange mathematical ideas and problem solving strategies.
- Children listen to understand one another. This involves thinking about what a person is saying so that you can explain it yourself or to help them explain it more clearly. It is not just being quiet when someone else is talking. Also, children need to listen so that they can ask a question or help the explainer.
- Encourages critical thinking and problem solving, collaboration and teamwork, and knowing how to learn

- Teachers can stand back or to the side of the classroom to encourage Math Talk as students interact more directly with each other.
- Most common structures:
 - Solve and Discuss: 4 to 5 students solve, explain, question, and justify at the board, while the rest of the class works the problems at their seats. 2-3 students are teacher selected to explain their methods.
 - Solve and Discuss Small Group version: *(after whole group discussion has taken place)* Students solve a problem individually within a small group. 2-3 students explain their method and solution to the rest of the group while the others are encouraged to ask questions for clarification.
 - Student Pairs and Helping Pairs: Two students work together, learning from each other, particularly in applying and practicing concepts introduced in whole-class discussion. Helping pairs-More advanced students are matched with students who are struggling.
 - Scenarios- Students act out a particular mathematical situation
 - Small Groups: Students work in groups

Introductory Lesson:

1.ATO.1, 1.ATO.5, 1.ATO.6

Lesson: Ways to Make 10

In this lesson, students will count-on from embedded numbers. Students will use the strategy of counting on from the first addend. This lesson is a continuation of the concepts from Unit 1. It is important for students to have multiple experiences with finding two addends whose sum equals 10.

Objective: Represent all the number pairs of 10 as number bond diagrams from a given scenario and generate all expressions equal to 10.

Lesson Plan: (Download the Module 1, Topic B, lesson 8.) <https://www.engageny.org/resource/grade-1-mathematics-module-1-topic-b-lesson-8>

Possible Lessons within the Unit:

Lesson: Grade 1 Module 1: Sums and Differences to 10: 1.ATO.1, 1.ATO.5, 1.ATO.6,

- <https://www.engageny.org/resource/grade-1-mathematics-module-1-topic-c>
- “In this first module of Grade 1, students make significant progress towards fluency with addition and subtraction of numbers to 10 as they are presented with opportunities intended to advance them from counting all to counting on which leads many students then to

decomposing and composing addends and total amounts.”

- Refer to Lessons 9-39

Lessons: Grade 1 Module 2: Introduction to Place Value Through Addition and Subtraction Within 20

- <https://www.engageny.org/resource/grade-1-mathematics-module-2-topic>
- Module 2 serves as a bridge from students' prior work with problem solving within 10 to work within 100 as students begin to solve addition and subtraction problems involving teen numbers. Students go beyond the Level 2 strategies of counting on and counting back as they learn Level 3 strategies informally called "make ten" or "take from ten."
- Teacher Note: Module 2 Topics A-C, Lessons 1-25 can be used within this Unit.

Task: Making a ten: <https://www.illustrativemathematics.org/content-standards/1/OA/C/6/tasks/1169>

- This task is designed to help students visualize where the 10's are on a single digit addition table and explain why this is so.

Task: Solving a real-world story problem: 20 Tickets: <https://www.illustrativemathematics.org/content-standards/tasks/1152>

- The purpose of the task is for students to add and subtract within 20 and represent complex addition problems with an equation to increase their understanding of and flexibility with the equals sign.

Lesson/Task: Fact Families: <https://www.illustrativemathematics.org/content-standards/1/OA/B/tasks/1214>

- “The purpose of this task is for students to identify and write sets of related addition and subtraction equations; these are often known as "fact families" because the equations are related by the same underlying relationship between the numbers. This task reinforces the commutative property of addition and using the relationship between addition and subtraction. It is best given after the students have had quite a bit of experience adding and subtracting within 10.”

Lesson/Task: Domino Addition: <https://www.illustrativemathematics.org/content-standards/tasks/1219>

- “The purpose of this task is to help students understand the commutative property of addition. Because the total number of dots is the same regardless of how a domino is oriented, the domino reinforces the idea that the addends can be written in any order.”

Task: Equality Number Sentences: <https://www.illustrativemathematics.org/content-standards/tasks/475>

- This task helps students understand the meaning of the equal sign and to use it appropriately.

Daily Activity: Number of the Day Stretch:

- https://books.google.com/books?id=vQDOAwAAQBAJ&pg=PA39&lpg=PA39&dq=number+of+the+day+stretch&source=bl&ots=wkvMTB_eu4R&sig=YRGevESgrcZxoUPND6j74xDIS4&hl=en&sa=X&ved=0CB8Q6AEwAGoVChMI77DK_76UxgIVg5WACH1nfACx#v=onepage&q=number%20of%20the%20day%20stretch&f=false

Culminating Lesson:

Lesson: Creating Story Problems: <http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathProblems>

In this activity, student will focus on reading/ listening comprehension skills as they apply to mathematics story problems, as well as on written and verbal mathematics communication skills. Using classic literature as inspiration children will apply their understanding of addition and subtraction situations and operations to create, describe, and solve story problems.

Objective:

Students will write and solve story problems involving a variety of situations, choosing strategies including- part-part- whole, comparing, grouping, doubling, counting on and counting back situations. Students will use drawings, equations, and written responses to solve single story problems.

Lesson Plan: <http://gadoe.georgiastandards.org/DMGetDocument.aspx/Lesson-plan.pdf?p=6CC6799F8C1371F69A35A3B787E603C58D358EDDDDB47CE5FDCDC021EC00DA6E8&Type=D>

Resources

Below is a suggested teaching sequence from *Mastering the Basic Math Facts in Addition and Subtraction (p.12)* suggests beginning with simpler facts and then connect each new set of facts to students' previous experiences rather than asking students to memorize 121 combinations of addition and 121 combinations of subtractions.

FOUNDATION FACTS:	
+1/+2	Students build on their understanding of counting by exploring 1 or 2 more and 1 or 2 less.
+0 facts	Using their knowledge of the concept of addition, students explore what happens when they add or subtract nothing from a quantity.
+10 facts	Adding 10 to a single-digit number results in a 2-digit sum. Students explore adding 10 in order to build understanding and automaticity that will be needed later when exploring the using-ten strategy.
Doubles	Students explore the concept of doubling and what it means to add 2 groups of equal size.
Making ten facts	Students need many opportunities to explore ways to combine numbers to form 10. It builds a strong foundation for students to build mastery of other facts.

Building on the Foundation (Facts)

Using 10s

Now that students know combinations of addends that have a sum of 10, they use their understanding of the flexibility of numbers to find way to break apart addends to create simpler facts by using tens (e.g., $9+7$ is changed to $10+6$).
I know $8+2=10$. So $8+3=11$. 3 is 1 more than 2 and $8+2=10$, so $8+3=11$

Using doubles

Students' knowledge of doubles facts is now put to use to find unknown facts that are near doubles (e.g., $4+5$ might be thought of as $4+4+1$).

Teacher Resources:

- **KATM Grade 1 Flipbook:** <http://www.katm.org/flipbooks/1%20FlipBook%20Final%20CCSS%202014.pdf>
 - Common Core Standards for Mathematics Flip Book Grade 1
- **Part-Part-Whole Cards:** <http://teachmath.openschoolnetwork.ca/wp-content/uploads/grade1/documents/ppwcards.pdf> :
 - Printable cards that can be used to practice part-part-whole relationships
- **Math Literature:** <http://www.the-best-childrens-books.org/math-for-kids.html>
- **Building a Math Talk Community:** <http://www.eduplace.com/math/mthexp/pdf/mathtalk.pdf>
- **Math Talk 101:** <http://www.scholastic.com/teachers/top-teaching/2014/01/math-talk-101>
- **Math Talk: The importance; Why use it?:** <http://mathsolutions.com/common-core-support/math-talk/>
- **Math Talk conversation starter posters:** <http://mason.gmu.edu/~jsuh4/teaching/resources/Buildingmathideas.pdf>
 - The last two pages include an explanation of how to use Math Talk to build mathematical ideas and discourse.
- **Structures of Story Problems Part-Part-Whole:** <http://www.cbv.ns.ca/consultants/uploads/MathConsultant/Part-Part%20Whole.pdf>
- <http://www.k-5mathteachingresources.com/support-files/add-and-subtract-to-20.pdf>

Teacher Resources for standard 1.ATO.6.b:

- **Guided Math “Number Fluency Center: Materials:** <http://mrsspruiellatschool.weebly.com/fact-fluency-center-materials-k-1-2.html>
 - This resource includes materials that can be used to differentiate number fluency centers for guided math.
- **Basic Facts:** <http://www.carrollk12.org/instruction/instruction/elementary/math/curriculum/basicfacts/default.asp>
 - This resource includes suggestions for ways to measure fact fluency.

Resources for Interactive Sites:

- **Virtual Manipulatives:** http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html
 - This resource can be used a variety of ways. Options to select: Grade, Backgrounds (i.e., Game Boards, Story Boards, Workmats), and Manipulatives (e.g., attribute blocks, attribute buttons, color tiles, connecting cubes, spinner, two-color counters)
- **Virtual Manipulative:** Number Blocks Freeplay. <http://www.mathsisfun.com/numbers/number-block-freeplay.html>
 - Suggestion: Use in whole group instruction to model decomposing and composing numbers through 10.
- **Teaching Tool:** Beadstring:http://ictgames.com/brilliant_beadstring_with_colour.html
 - Choose one or two beadstrings. Practice bonds of 10 or 20, number facts to 10 or 20.
- **Interactive Modeling Tool:** Thinking Blocks Jr.:http://www.mathplayground.com/tb_addition_jr/thinking_blocks_junior.html
 - Model and Solve Word Problems
 - Addition and Subtraction Practice with small numbers (options for biggest number being 10 or 20)
- **Interactive Modeling Tool:** Thinking Blocks: http://www.mathplayground.com/ThinkingBlocks/thinking_blocks_modeling%20tool.html
- **Interactive Game:** Sum Sense: <http://resources.oswego.org/games/SumSense/sumadd.html>
 - Single digit addition. Drop and drag cards to make 'sum' sense.
- **Interactive Game:** Sum Sense: Single digit subtraction: <http://resources.oswego.org/games/SumSense/sumsub.html>
 - Drop and drag cards to make 'sum' sense.
- **Interactive Operations Games:** <http://jmathpage.com/JIMSNumberoperations.html>
- **Interactive Game:** <http://illuminations.nctm.org/Activity.aspx?id=3563>
 - Independent of partner game in which students match whole numbers, shapes, fractions, or multiplication facts to equivalent representations.”
 - Teacher Note: Use the 1-6 or 1-10 number range within Units 1 and 2.
- **Interactive Game:** Number Bonds 10: http://www.mathplayground.com/number_bonds_10.html

Sample Formative Assessment Tasks/Questions

Assessing 1.ATO.1: The following links include a task for students to solve. <https://www.illustrativemathematics.org/content-standards/1/OA/A/1>

Assessing 1.ATO.2: <https://www.illustrativemathematics.org/1.OA.A.2>

Assessing Sums and Differences to 10: <https://www.engageny.org/resource/grade-1-mathematics-module-1>

- Click on Mid-Module Assessment or End-of-Module Assessment under Downloadable Resources for samples

Assessing extending the number sequence to read, write and represent numbers to 100 using concrete models, and standard form; and

Assessing extending the number sequence to read and write in word form numbers zero through nineteen, and multiples of ten through ninety.

- **Number of the Day Stretch:** A way to assess students' number sense as well as their ability to represent numbers in multiple ways. Suggested questions to informally assess:
 - Why do you think it is important to be able to represent numbers in different ways? When do we usually use number words to represent numbers? When do we use numerals? When do we use pictures of diagrams? When do we use number sentences? Why do we sometimes choose one method of representation rather than another?
 - Teacher Note: This can also be assessed throughout the school year as the range of numbers increases within classroom instruction.

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Place Value Concepts

Content Standards with Clarifying Notes

Open Bullets Indicate Clarifying Notes

- **2.NSBT.1** Understand place value through 999 by demonstrating that:
 - a. 100 can be thought of as a bundle (group) of 10 tens called a “hundred”; b. the hundreds digit in a three-digit number represents the number of hundreds, the tens digit represents the number of tens, and the ones digit represents the number of ones; c. three-digit numbers can be decomposed in multiple ways (e.g., 524 can be decomposed as 5 hundreds, 2 tens and 4 ones or 4 hundreds, 12 tens, and 4 ones, etc.)
 - *Second graders should come to see a set/group of 10 tens as a new unit called 100 (hundred).*
 - *It is important to note that 3 digit numbers can be decomposed in multiple ways (as a basis for later concepts of addition/subtraction regrouping).*
- **2.NSBT.2** Count by tens and hundreds to 1,000 starting with any number
 - *example: “Count by 10’s starting at 350” (350, 360, 370, 380, etc)*
 - *example: “Count by 100’s starting at 350” (350, 450, 550, 650, etc)*
- **2.NSBT.3** Read, write and represent numbers through 999 using concrete models, standard form, and equations in expanded form
 - *concrete models could be diagrams/pictures or actual manipulatives*
 - *standard form is numerical form (e.g. 387)*
 - *equations in expanded form (e.g. $300 + 80 + 7 = 387$) [note: expanded form does NOT have to occur in any sequential order (e.g. $80 + 7 + 300 = 387$)]*
- **2.NSBT.4** Compare two numbers with up to three digits using words and symbols (i.e., >, =, or <)
 - *Students should be required to compare numbers with words (greater than, less than, equal to) as well as symbols.*
- **2.ATO.3** Determine whether a number through 20 is odd or even using pairings of objects, counting by twos, or finding two equal addends to represent the number (e.g., $3 + 3 = 6$).
 - *The focus of this standard is based on the conceptual understanding of even and odd numbers. An even number is an amount that can be made of two equal parts with no leftovers. An odd number is one that is not even or cannot be made of two equal parts. **The number endings of 0, 2, 4, 6, and 8 are only an interesting and useful pattern or observation and should not be used as the definition of an even number.** (Van de Walle & Lovin, 2006, p. 292)*

New Academic Vocabulary for this Unit

- number value
- = (equal to)
- < (less than)
- > (greater than)
- even number
- number place
- expanded form

Prior Knowledge Required for this Unit

In first grade, students used concrete models to represent numbers up to 100 in expanded form. First graders also demonstrated that ten ones are thought of as a bundle. In second grade, students should use the correct terminology that 100 ones or 10 tens = one hundred (100).

First graders also learned to count forward by ones to 120; starting at any number as well as by 5's and 10's to 100, starting at any number. In this unit, students are required to count by 10's or 100's starting at any number to 1,000.

Subsequent Knowledge Related to this Unit

In this Unit, students will develop conceptual understandings of place value through 999; this understanding will lead to students developing strategies for addition and subtraction based on place value in the next unit-- Unit 2.

In 3rd grade, students will then use knowledge of place value to round numbers to nearest 10's and 100's. They will also learn to multiply one-digit numbers by multiples of 10 using knowledge of place value.

It is important that students develop place value concepts to use in rounding as well as for addition/subtraction because students are not expected to use addition/subtraction standard algorithms until 4th grade.

Relationship Among Standards in this Unit

The various standards in the unit were combined to develop place value concepts especially in decomposing numbers to represent place values. These conceptual understandings will help students develop addition and subtraction strategies especially when regrouping is needed.

Expanded form (125 can be written as $100 + 20 + 5$) is a valuable skill when students use place value strategies to add and subtract large numbers in **2.NSBT.7**.

Students will also use place value knowledge to develop definitions for odd and even numbers in standard **2.ATO.3**. When students pull numbers apart, separating the 10's and 1's place, they will see that the "tens" is always divisible by two (can be halved), therefore the one's place number must determine if the overall number is even or odd.

If students are able to write out numbers in expanded form (**2.NSBT.3**), this could assist them with skip counting by 100's and by 10's in 2.NSBT 2 as they should be able to see which number is increasing (ex: count by 100's starting at 482; $(400 + 80 + 2)$. $(500 + 80 + 2)$, $(600 + 80 + 2)$)

Potential Instructional Strategies/ Lessons

2. ATO. 3 Odd and Even Numbers

(Adapted from <https://www.teachervision.com/math/lesson-plan/48936.html>)

Objective

In this lesson, the concept of odd and even numbers is explored using manipulatives. A hundreds chart is used to show the alternating pattern of odd and even numbers, and students are asked to extend the pattern to identify additional odd and even numbers. Students will identify odd and even numbers and patterns in number sequences.

Materials

Provide each student with:

- 2 copies of a hundreds chart
 - (link to printable hundred's chart) http://www.mcps.org/FBE_Files/100.pdf
- 50 counters
- 2 green crayons

Procedure

Show students how to tell whether a number from 1 to 6 is even or odd using counters. Explain that if the number of counters can be arranged so each counter is paired with another, the number is even. Further explain that if there is an unpaired counter, or a counter is left over, then the number is odd. Model this

process for students, and then have them use counters to find which numbers are odd or even. Have students use the green crayon to color the squares with the numbers they identify as even on their hundreds charts. Ask students to continue to use this method to find all the even numbers up to 20, recording them on the charts using a green crayon.

Ask students to describe patterns they see on their charts, and elicit that every other number is green, or even. Have students use their green crayons to continue the pattern on the charts up to the number 50. Ask students to circle one even number and one odd number between 20 and 50 on their charts. Then ask students to have their partner use counters to verify their choices. Encourage students to arrange the counters in groups of 10. Discuss their findings.

Extension: Discuss how the chart has alternating stripes, or columns, and ask students what is the same in each green-colored column (the digit in the ones place in each number) and what is different (the digit in the tens place). Elicit that the digit in the ones place in each even number is 0, 2, 4, 6, or 8. Write several numbers between 50 and 100 on the chalkboard, and ask students to identify whether each is odd or even.

2.NSBT.3 Introduction to Writing Expanded Form (adapted from <http://illuminations.nctm.org/Lesson.aspx?id=3691>)

- Printable Expanded Place Value Form Cards
 - <http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/3-5/PlaceValueCards%20AS.pdf>
- Printable Spinner
 - <http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/3-5/Spinner.pdf>

Objectives:

Determine the value of each digit in a three-digit number.

Compose and decompose numbers using standard and expanded form.

Use place value to compare the values of numbers

Materials:

Base ten blocks

Cardstock

Place Value Playing Cards Activity Sheet

Spinners

Paper clip and pencil for spinner

Before class, copy the Place Value Playing Cards Activity Sheet and Spinner Activity Sheet on cardstock. Students will work in pairs, so make a copy for each pair of students.

Procedure:

To introduce standard form, write the words "three hundred forty-five" on the board. Write just the words, not the numeral. Ask a volunteer to come to the board and write the words using digits. State that this number, 345, is written in standard form. Explain that standard form is simply the numerical form of a number. To get students to think about what this number means, have them write everything they know about the number in standard form. To focus students' thoughts, encourage them to make observations about each digit and what that digit represents in the number. Discuss their answers.

Students may make these observations about 345:

- It has three digits.
- It includes ones, tens, hundreds.
- It has a 3 in the hundreds place.
- It has a 4 in the tens place.
- It has a 5 in the ones place.
- It is a number written in standard form.
- It is greater than 340 but less than 350.

Organize students into pairs. Hand out the base ten blocks. Ask groups to model the number 345 using the fewest base ten blocks possible. Remind students that a flat represents 100, a rod represents 10, and a unit represents 1. Groups should end up with 3 flats, 4 rods, and 5 units.

With the entire class, count the value of the blocks. Ask, "How many flats do you have?" [3.] "What is the total value?" [300.] Explain that 3 hundreds is worth 300 while you count the flats aloud, "100, 200, 300." Write "300" on the board. Do the same with the rods and the units, writing "+ 40" and "+ 5" on the same line as 300. That is, when you've finished counting, the following should appear on the board:

$$300 + 40 + 5$$

Tell students that what you have written on the board is the *expanded form* of 345. Provide a few more examples of numbers in standard form, and have students work together to create the numbers using base ten blocks before they write down the expanded form.

Group students into pairs and pass out the sets of place value cards, one set to each pair. Explain that pairs will play a game. The first student uses the place value cards to compose a number in expanded form, and shows it to the second student. The second student writes the standard form of the number on a slip of paper and shows it to the first student so he or she can confirm or reject. Have students reverse roles.

In the following period, collect the place value cards and redistribute the number cards (excluding the "+" symbols), passing out the cards as evenly as possible

so that each student has only 1 or 2 cards in his or her hands. For example, if you have 28 students, each student would get 1 card. If you have fewer students, some students may get 2 cards. If you have more than 28 students, pair students so that all may participate. Have a volunteer use their paper clip, pencil, and spinner to generate a 3-digit number—for example, 6, 3, and 5. Ask students to use each of the numbers exactly once to create the greatest number possible in their notebooks.

Ask, "If we put each number in the hundreds place, what would be the value of each number?" For the example roll, students would hold up the 600, 300, and 500 cards, and walk to the front of the room. Discuss which of the numbers is the greatest. [600] Students with the 300 and 500 cards may then sit down. Do the same with the 30 and 50 cards to pick the tens. [50] That will leave 3 in the ones place.

Take the three cards (600, 50, 3) and stack them together (so the 5 overlaps the 0 in the tens place of 600, and the 3 overlaps the 0 in the ones place of 50) to show that standard form is 653. Ask, "How do we know this is the largest number?" Try to elicit that the digit with the greatest value, 6, is in the hundreds place.



As time allows, continue this activity with other numbers. Have students overlap the cards to solidify the composition of numbers. Because math is not a spectator sport, it would be good if each student's numbers are used at least once during this activity. To keep kids interested, you can alternatively ask for the least number that could be formed, or the largest odd number, or the least even number, or other types of numbers.

Assessment Options

1. As students work with the base ten blocks, observe if they are correctly identifying the value of the blocks. For example, if they have 3 tens rods, are they counting by tens to identify the value as 30?
2. Give each student an index card and a spinner. Have each student spin a number 3 times and write all 3 digits on the top of his or her card. Have students write the greatest number they can make with those 3 digits in both standard and expanded form. Ask them to write an explanation on the index card of how they know this is the biggest number they can make with those digits.

2.NSBT.3 Building Base-Ten Numbers

(Adapted from https://www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_2_Unit1Framework.pdf)

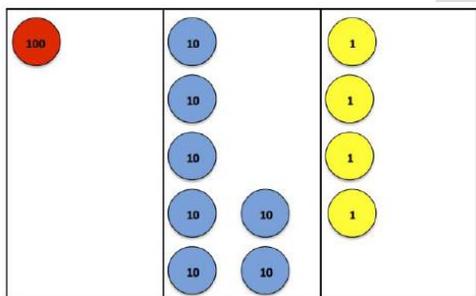
Your task is to build numbers and identify them. Use dice and base ten blocks or models to complete this exercise.

1. Roll all 3 dice at once.
2. Create the smallest number possible, using all three dice.
3. Create the largest number possible, using all three dice.
4. Using the smallest number of base ten pieces possible, draw (or build) a model of each number you recorded. Have the flats represent hundreds, the rods represent tens, and the unit cubes represent ones. Use words to write how each of the two numbers is spoken. Write the numbers in expanded notation.
5. Repeat the same exercise two more times and record.
6. When you have completed all your rolls, put a star beside the greatest number you rolled. Then put a check beside the smallest number you rolled. Explain how you know what the largest and smallest numbers are possible when using three dice.

Roll #1

Smallest Number	Model with Drawings	Expanded Notation (ex. $200 + 30 + 6$)
Largest Number	Model with Drawings	Expanded Notation

2.NSBT.4 Comparing 3 digit numbers. (Downloadable PDF file (save to your desktop) https://www.engageny.org/file/92856/download/math-g2-m3-topic-f-lesson-16.pdf?token=4V10IbATvJHPuoDgsDpR17T5Vlcw8ql_OTWFyrHcDKs)



In this 60 minute lesson, students use Place Value Disks (created from colored construction paper) to represent 3 digit numbers.

- o printable place value chart: http://lrt.ednet.ns.ca/PD/BLM/pdf_files/place_value_charts/02_to_hundreds.pdf

Resources

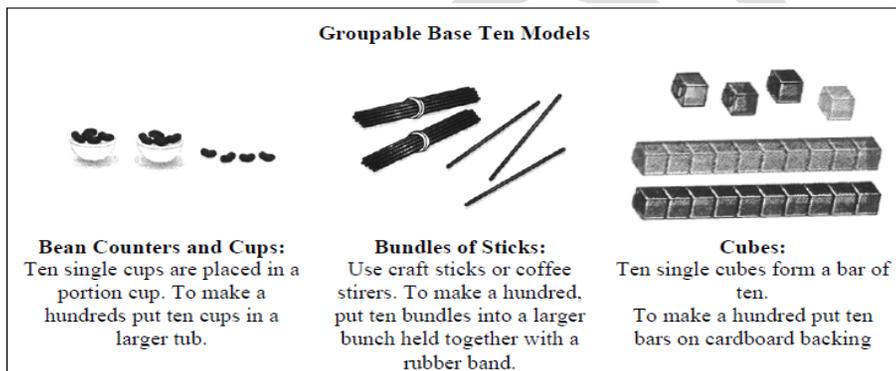
Videos:

- **2.NSBT.1** Place Value Video
 - <https://youtu.be/omkDLmfvetk>
- **2.NSBT.1** How to decompose numbers video
 - <https://learnzillion.com/lessons/2465-add-by-decomposing-numbers>
- **2.NSBT.3** How to write Expanded Form Video
 - <https://learnzillion.com/lessons/3685-read-and-write-numbers-in-expanded-form-using-arrow-cards>

Websites:

- **2.NSBT.1** interactive game (penguins and place value)
 - http://www.bbc.co.uk/schools/starship/maths/games/place_the_penguin/big_sound/full.shtml
- **2.ATO.3** printable 100's chart
 - http://www.mcps.org/FBE_Files/100.pdf
- **2.NSBT.3** printable Flat/Rod/Unit sheet
 - http://www.mathatube.com/files/base_ten_blocks-flats-rods-units-2.pdf
- **2.NSBT.3** interactive game where students find numbers in expanded form
 - <http://www.topmarks.co.uk/place-value/place-value-charts>

Images:



- image from https://www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_2_Unit1Framework.pdf

Sample Formative Assessment Tasks/Questions

Problem:

Mr. Robert asked Anna to help another second grader learn about even and odd numbers. He handed Anna 16 red counters and 19 blue counters to help her show and explain ways to identify even and odd numbers to her classmate. What could Anna show and say to her classmate?

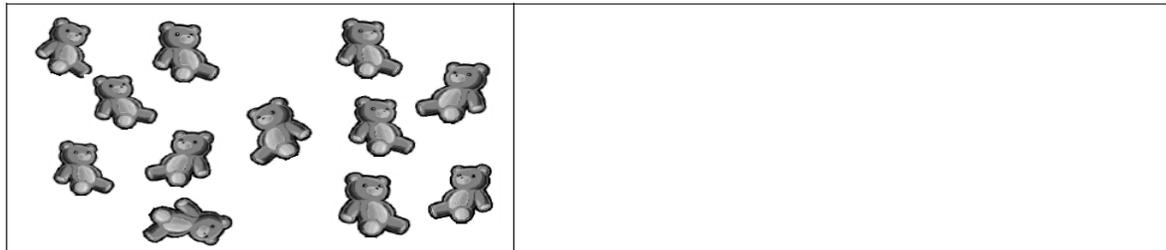
Is 16 an even or odd number?	Is 19 an even or odd number?
Draw and explain.	Draw and explain.
16 is an _____ number because _____ <small>(even or odd)</small>	13 is an _____ number because _____ <small>(even or odd)</small>
_____	_____
_____	_____
_____	_____

Possible ways to formatively assess:

2.ATO.3: (Adapted <https://www.georgiastandards.org/Common-Core/Common>)

Even or Odd: (adapted from <https://www.engageny.org/file/118071/download/math-g2-m6-end-of-module-assessment.pdf?token=TFaZc-yoTKtp6HQiQods-R1xg7xcLclcFdgGP8yOMM>)

Does the picture below show an even or an odd number of teddy bears? Explain your thinking using pictures, numbers, or words in the box on the right.



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Developing Concepts Addition/Subtraction

Content Standards with Clarifying Notes
Open Bullets Indicate Clarifying Notes

- **2.NSBT.5** Add and subtract **fluently** through 99 using knowledge of place value and properties of operations
 - Fluently and fluency describe a student’s ability to compute with accuracy, flexibility, and efficiency (Kilpatrick, Swafford, & Findell, 2001).
 - The standard algorithm for carrying and borrowing is not a focus or expectation for second grade; students should be encouraged first to develop strategies based on place value and properties of operations first, then the standard algorithm will be introduced later in 4th grade.
 - Students are expected to fluently add and subtract two digit numbers; therefore students should be given ample opportunities to independently solve 2 digit equations.

- **2.NSBT.6** Add up to four two-digit numbers using strategies based on knowledge of place value and properties of operations

Example: $67 + 25 = \underline{\quad}$

Place Value Strategy:
I broke both 67 and 25 into tens and ones. 6 tens plus 2 tens equals 8 tens. Then I added the ones. 7 ones plus 5 ones equals 12 ones. I then combined my tens and ones. 8 tens plus 12 ones equals 92.

Decomposing into Tens:
I decided to start with 67 and break 25 apart. I knew I needed 3 more to get to 70, so I broke off a 3 from the 25. I then added my 20 from the 22 left and got to 90. I had 2 left. 90 plus 2 is 92. So, $67 + 25 = 92$

Commutative Property:
I broke 67 and 25 into tens and ones so I had to add $60+7+20+5$. I added 60 and 20 first to get 80. Then I added 7 to get 87. Then I added 5 more. My answer is 92.

- **2.NSBT.7** Add and subtract through 999 using concrete models, drawings, and symbols which convey strategies connected to place value understanding.
- **2.NSBT.8** Determine the number that is 10 or 100 more or less than a given number through 1,000 and explain the reasoning verbally and in writing

- This standard focuses ONLY on adding and subtracting 10 or 100 from a given number. Students are not expected to explain reasoning for adding or subtracting numbers that are multiples of 10 or 100 (e.g. 40 or 300).
- **2.MDA.5** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences through 99 on a number line diagram.
- Students will use number lines with whole numbers as both a means of solving addition and subtraction, but also as an introduction to using rulers

New Academic Vocabulary for this Unit

- number line
- commutative property
- associative property

Prior Knowledge Required for this Unit

In Unit 1 (Place Value) students decompose numbers into expanded form; they will use this knowledge to construct their own addition and subtraction strategies. *(Teacher Note: Standard algorithms for addition and subtraction are not introduced until 4th grade).*

In 1st grade, students applied commutative and associative properties of addition to find sums of two or three addends (through 20). In this unit, 2nd graders will use these same properties of addition (as well as subtraction) to construct methods for adding multi-digit numbers and up to four 2-digit numbers. (ex. $35 + 26 + 18 + 89$).

Subsequent Knowledge Related to this Unit

In the next unit, Unit 3-- Fluency and Word Problems Addition/ Subtraction, students will use the conceptual strategies they develop in this Unit 2 for addition and subtraction to build fluency (computations performed flexibly and efficiently) and skill application (using addition and subtraction within real world problems and scenarios).

In third grade, students will apply properties of operations (commutative, associative, and distributive) to multiply and divide.

Relationship Among Standards in this Unit

2.NBTS.6 and 2.NBTS.7 both require students to build conceptual number sense with addition and subtraction using strategies they invent themselves. Teachers are not expected to give students algorithms to use which may by-pass students' ability to construct number sense based on place value and properties of operations.

2.MDA.5 introduces a number line as an operational tool (for addition and subtraction) for whole numbers 0 - 99. Students also use a hundred's chart in this unit to grasp adding or subtracting 10 from a number to find patterns in the resulting answer in standard 2.NSTB.8.

Potential Instructional Strategies

Students should be given ample opportunities to use manipulatives (either concrete or virtual) to generate strategies for addition and subtraction. Virtual manipulatives can be used for whole group lessons, or for independent work if students have access to technology

- virtual manipulatives website:
 - http://nlvm.usu.edu/en/nav/frames_asid_154_g_1_t_1.html?from=category_g_1_t_1.html

After allowing students chances to manipulate models, move students to pictorial stage. Initially, provide students with copies of flats/rods/units they can use to add or subtract (see Flats/Rods/Units under Resources)

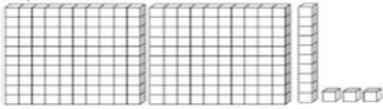
(adapted from N.C. Instructional Support Tools)

2.NSBT.7 Allow students to solve equations with manipulatives (flats/rods/units) (adapted from N.C. Instructional Support Tools)

Example: $213 - 124 = \underline{\quad}$

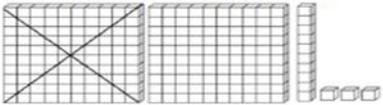
Student A

I used place value blocks. I made a pile of 213.

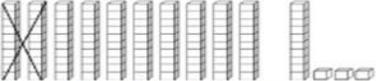


I then started taking away blocks.

First, I took away a hundred which left me with 1 hundred and thirteen.



Now, I only need to take away 24.
I need to take away 2 tens but I only had 1 ten so I traded in my last hundred for 10 tens. Then I took two tens away leaving me with no hundreds and 9 tens and 3 ones.



I then had to take 4 ones away but I only have 3 ones. I traded in a ten for 10 ones. I then took away 4 ones.



This left me with no hundreds, 8 tens and 9 ones. My answer is 89. $213 - 124 = 89$

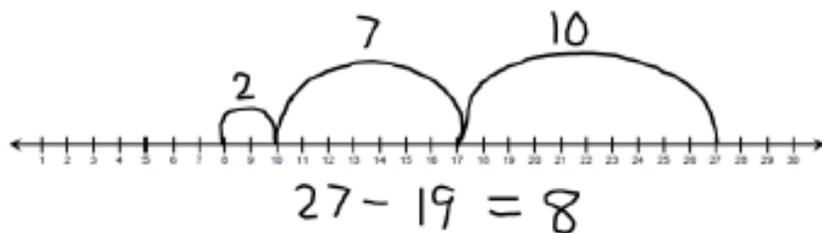


2.MDA.5 Have students solve subtraction equations (or word problems) using a number line

- printable number line:
 - http://lrt.ednet.ns.ca/PD/BLM/pdf_files/number_lines/number_lines_number_2.pdf

Example: There were 27 students on the bus. 19 got off the bus. How many students are on the bus?

Student A: I used a number line. I started at 27. I broke up 19 into 10 and 9. That way, I could take a jump of 10. I landed on 17. Then I broke the 9 up into 7 and 2. I took a jump of 7. That got me to 10. Then I took a jump of 2. That's 8. So, there are 8 students now on the bus.



(adapted from N.C. Instructional Support Tools)

2.NSTB.8 Determining 10 more or 10 less than a given number in Number Talk Discussion

(adapted from https://www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_2_Unit2Framework.pdf)

MATERIALS

- 200 chart per student (*printable 200's chart in Word format:* http://lrt.ednet.ns.ca/PD/BLM/word_files/number_charts/1%20-200_chart.doc)
- Class 200 Chart
- Transparent counters or highlighters

Procedure:

Gather students in the meeting area. Display the class the 200 chart. Give each student a 200 chart. Select a starting number. Have students place a transparent counter on it or highlight it. Give students directions one at a time using the terms add 10, subtract 10, add 1, subtract 1, 10 more, 10 less, 1 more, and 1 less. After each clue, give students the opportunity to count up using their chart, if they need to and then have students move their transparent counter to the new number.

Model this with the class, using only 3 or 4 directions. When the last direction has been given, ask students what number their transparent counter is on.

Sample direction set:

- *Place your counter on 16.*
- *Add 10. (students should move their counter to 26)*
- *Subtract 1. (students should move their counter to 25)*
- *Move ahead 10 more. (students should move their counter to 35)*
- *What number is the counter covering? (35)*

Resources

Videos:

- 2. NSBT.7 video adding numbers using decomposing (place value)
 - <https://learnzillion.com/lessons/2465-add-by-decomposing-numbers>
- 2.MDA.5: video explaining number lines with addition
 - <https://learnzillion.com/lessons/2687-add-by-using-a-number-line>
- 2.NSBT.6: video demonstrating adding 4 digit numbers
 - <https://learnzillion.com/lessons/2804-add-up-to-four-2-digit-numbers-using-place-value>

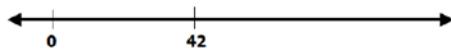
Worksheets/Printables

- 2.NSBT. 7 printable sheet with Flat/Rod/Units
 - http://www.mathatube.com/files/base_ten_blocks-flats-rods-units-2.pdf

Sample Formative Assessment Tasks/Questions

2.MDA.5 Sample Task Adapted from http://schools.nyc.gov/NR/rdonlyres/CAC1375E-6DF9-475D-97EE-E94BAB0BEFAB/0/NYCDOEG2MathCarolsNumbers_Final.pdf

Carol's teacher drew a number line on the board.



3. About where would 85 be? Place 85 on the number line where it belongs.
4. About where would 21 be? Place 21 on the number line where it belongs.
5. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

Sample Answer:

Carol's teacher drew a number line on the board.



1. About where would 85 be? Place 85 on the number line where it belongs.
2. About where would 21 be? Place 21 on the number line where it belongs.
3. About where would 31 be? Place 31 on the number line where it belongs.

Tell Carol how you knew where to place 31 and why.

You place the 31 in the middle and a little smaller because 31 is ten more than 21 and is 11 less than 42.

2.NSTB.7 and 2.NSTB.8 Addition Strategies / 10 more/less than a number

(adapted from NYS Common Core mathematics curriculum)

Susan and James solved $125 + 32$ in different ways. Explain why both ways are correct.

<p><i>Susan's Way:</i></p> $125 + 32$ $125 \xrightarrow{+10} 135 \xrightarrow{+10} 145 \xrightarrow{+10} 155 \xrightarrow{+2} 157$	<p><i>James' Way:</i></p> $125 + 32$ $125 + 30 + 2 = 157$
<p><i>Explanation:</i></p>	<p><i>Explanation:</i></p>

2.NSTB.7 Comparing different strategies for adding (adapted from NYS Common Core mathematics curriculum)

Linda and Keith added $127 + 59$ differently. Explain why Linda's work and Keith's work are both correct.

<p><i>Linda's work:</i></p> $\begin{array}{r} 127 \\ + 59 \\ \hline 16 \\ 70 \\ + 100 \\ \hline 186 \end{array}$	<p><i>Keith's work:</i></p> $\begin{array}{r} 127 \\ + 59 \\ \hline 186 \end{array}$
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Conceptual Understanding of Multiplication and Division

Content Standards with Clarifying Notes*Open bullets indicate clarifying notes*

- **3.ATO.1** Use concrete objects, drawings, and symbols to represent multiplication facts of two single-digit whole numbers and explain the relationship between the factors (i.e., 0-10) and the product.
 - Develop conceptual understanding which means using concrete/hands-on methods to explore multiplication
 - Develop vocabulary for the terms in a multiplication fact.
- **3.ATO.2** Use concrete objects, drawings and symbols to represent division without remainders and explain the relationship among the whole number quotient (i.e., 0 – 10), divisor (i.e., 0 – 10), and dividend.
 - Develop conceptual understanding which means using concrete/hands-on methods to explore division
 - Develop vocabulary for the terms in a division fact.
- **3.ATO.3** Solve real-world problems involving equal groups, area/array, and number line models using basic multiplication and related division facts. Represent the problem situation using an equation with a symbol for the unknown.
 - Develop conceptual understanding of how models and equations represent the problem situations.

New Academic Vocabulary for This Unit

- | | | |
|------------|-----------|------------------|
| ● factor | ● product | ● multiplication |
| ● dividend | ● divisor | ● quotient |
| ● division | | |

Prior Knowledge Required for this Unit

In 2nd grade, students used repeated addition to find the total number of objects arranged in rectangular arrays and wrote expressions to express the total as a sum of equal addends. (2.ATO.4) In this unit rectangular arrays are the basis for introduction to multiplication and division.

Subsequent Knowledge Related to this Unit

Students will use conceptual understanding of multiplication and division from this unit to determine unknown whole numbers in multiplication and division fact equations, to understand the relationship between multiplication and division, and to solve problems related to multiplication and division. In Unit 4 students apply conceptual understanding of multiplication and division by using properties of operations and other strategies to solve problems and explain their reasoning. In Unit 8 students will use multiplication and division when solving problems with measurement. In Unit 9, students extend their understanding when multiplying a single digit number by a multiple of 10. By the end of 3rd grade, students are expected to demonstrate fluency with basic multiplication and division facts (3.ATO.7)

Relationship Among Standards in this Unit

Students develop conceptual understanding of what multiplication and division are, and explore the relationships between the two operations.

Potential Instructional Strategies/Lessons

Teacher Note:

- Students can more readily develop an understanding of multiplication concepts if they see visual representations of the computation process. For example, they can picture students in a marching band arranged in equal rows or chairs set up in rows in an auditorium. Using models can help students develop meaning for operations and lessen the abstraction of operations. Students can use various objects and materials to make models that will help them make sense of operations, including the following: movable objects such as counters, craft sticks, tiles, and containers to put them in; multi-cubes or other interlocking cubes; visual materials such as pictures; arrays (an arrangement of rows and columns); base ten blocks; money (pennies, nickels, dimes, quarters, dollars); grid paper; number lines; hundreds charts; and tape diagrams. Models and representations can be used to help students understand what the symbols in operations mean. Students need to work with representations that help them see the relationships between multiplication and repeated addition, and between multiplication and division.

Introducing Arrays (3.ATO.1, 3.ATO.2, 3.ATO.3)

<https://www.engageny.org/resource/grade-3-mathematics-module-1-topic-lesson-1>

(Download the Module 1 lesson, topic A.)

- This lesson focuses on understanding equal groups as multiplication. The Look especially at the activity sheets. The emphasis there is on arrays as repeated addition which extends the 2nd grade introduction to arrays.

www.youtube.com/watch?v=DDoFS9CNFs0

- This video is designed for teachers not students. The video demonstrates an introduction to multiplication with arrays, and provides suggestions as to how students might build models. Use the strategies introduced in the first 4 minutes 12 seconds of the video as background teacher information to introduce multiplication with arrays.

https://learnzillion.com/lesson_plans/141-farmer-john-s-seeds-relate-multiplication-and-division-to-the-array-model-using-equal-groups

(must set up a free account)

- This student activity supports conceptual understanding of how to organize equal groups into an array and how multiplication and division can be represented with arrays.

Relationship Between Multiplication and Division (3.ATO.3)

<http://blog.aimsedu.org/2013/10/01/addressing-mathematical-practice-standards-through-multiplication-and-division-word-problems/>

- This lesson addresses the relationship between multiplication and division through the use of real-life problems.

Resources

Operations with Rectangular Arrays (3.ATO.1, 3.ATO.2, 3.ATO.3)

<http://www.softschools.com/counting/games/multiplication.jsp>

- game with visual models

www.youtube.com/watch?v=M_DQHbBKq00

- basic introduction to multiplication with rectangular arrays

https://www.youtube.com/watch?v=UKlz2aWa_Mg

- math talk using arrays for multiplication

www.youtube.com/watch?v=c7UwNboK0I8

- multiplication using an array

<https://www.youtube.com/watch?v=yMrVfwupaGw>

- multiplication with arrays to find products and factors

www.emis.de/proceedings/PME28/RR/RR018_Outhred.pdf

- research findings on student drawings of rectangular arrays

www.youtube.com/watch?v=erpHiUHK-3A

- basic introduction to division with rectangular arrays

Operations with a Variety of Concrete Models (3.ATO.1, 3.ATO.2, 3.ATO.3)

Examples of Models

http://www.eworkshop.on.ca/edu/resources/guides/Guide_Math_K_6_Volume_5.pdf

- *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6 - Volume 5*, pp 28-29 (*Using Models to Represent Facts of Multiplication and Division*) gives examples of a variety of models to represent multiplication and division facts.

Hopping on a Number Line

<http://illuminations.nctm.org/Lesson.aspx?id=1251>

- In this interactive lesson, students generate products using the number line model. This model highlights the measurement aspect of multiplication and is a distinctly different representation of the operation. The order (commutative) property of multiplication is also introduced. Students are encouraged to predict products and to answer puzzles involving multiplication.

www.scooe.org/depts/ci/math/.../Bar%20Diagrams.do

- Using bar diagrams

Children's Literature

Count on Pablo, by Barbara deRubertis (1999)

- Pablo and his grandmother prepare and sell vegetables at an outdoor market. The story provides an exploration of $\times 5$ facts as Pablo and his grandmother sell onions tied in pairs ($\times 2$ facts), tomatoes in boxes of 10 ($\times 10$ facts), and peppers in bags of 5 ($\times 5$ facts). Pablo skip-counts to determine the number of vegetables being prepared for market, but through classroom explorations, the story allows for an easy transition from skip-counting to multiplication.

Amanda Bean's Amazing Dream by Cindy Neuschwander

- This book makes a convincing case to children about why they should learn to multiply. The story helps children see what multiplication is, how it relates to the world around them, and how learning to multiply can help them. At the end of the story, the section titled "For Parents, Teachers, and Other Adults" explains what's important for children to learn about multiplication and suggests three activities to extend children's learning.

The Hershey's Milk Chocolate Multiplication Book by Jerry Pallotta

- This book uses the arrays in a chocolate bar to teach multiplication.
-

Culminating Activity

<https://learnzillion.com/lessons/2999-solve-word-problems-using-the-idea-of-equal-groups>

- sample explanation of multiplication

<http://www.commoncoresheets.com/Interactive.php?Worksheet=Math/Interactive/3oa1>

- Go to Multiplication and Division links and select topics such as Creating Equations from Arrays, Rectangular Arrays, Writing Multiplication Equations, Rewriting Addition to Multiplication, Multiplying with Arrays, Dividing with Number Lines, and No Remainder. This site now has some interactive practice for use on the Promethean board.

Sample Formative Assessment Tasks/Questions

http://www.achieve.org/files/NYCDOEG3MathCookieDough_Final.SW_.pdf

- performance tasks

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Place Value

Content Standards with Clarifying Notes

Open bullets indicate clarifying notes

- **3.NSBT.1** Use place value understanding to round whole numbers to the nearest 10 or 100.
 - Teacher Note: This is the first time students have been introduced to rounding.
- **3.NSBT.4** Read and write numbers through 999,999 in standard form and equations in expanded form.
- **3.NSBT.5** Compare and order numbers through 999,999 and represent the comparison using the symbols $>$, $=$, or $<$.

New Academic Vocabulary for This Unit

- rounding
- benchmark numbers
- order numbers (greatest to least and least to greatest)
- equation

Prior Knowledge Required for this Unit

In 2nd grade, students learned to read, write and represent numbers through 999 using concrete models, standard form, and equations in expanded form (2.NSBT.3), compared three digit numbers (2.NSBT.4), learned place value through 999 (2.NSBT.1), and added 10 or 100 more or less to a given number up to 1000 (2.NSBT.8). This is the students' first introduction to rounding.

Subsequent Knowledge Related to this Unit

In fourth grade, students will use rounding as one form of estimation and round whole numbers to any given place value.

Relationship Among Standards in this Unit

Understanding place value and having the ability to demonstrate that understanding are vital in the development of mathematical thinking. In this unit students will use place value concepts to round, read, write, order and compare numbers through 999,999.

Potential Instructional Strategies/Lessons

Rounding: 3.NSBT.1

Teacher Note:

In *Teaching Student-Centered Mathematics Grades 3-5 (2006)*, John Van de Walle explains that in our number system, some numbers are “nice” or easy to think about and work with. “What makes a number nice is sort of fuzzy. However, numbers such as 100, 500, and 750 are easier to use than 96, 517, and 762. Multiples of 100 are very nice, and multiples of 10 are not bad either... Flexible thought with numbers and many estimation skills are related to the ability to substitute a nice number for one that is not so nice. The substitution may be to make a mental computation easier, to compare it to a familiar reference, or simply to store the number in memory more easily. You might say, ‘Last night it took me 57 minutes to do my homework’ or ‘Last night it took me about an hour to do my homework.’ The first expression is more precise; the second substitutes a rounded number for better communication.”

In the past, students were taught rules for rounding numbers. Unfortunately, the emphasis was placed on applying the rule correctly (If the next digit is 5 or greater, round up; otherwise, leave the number alone.) Knowing when rounding is appropriate/useful and understanding how the thinking process works are important in making rounding a skill students can and will use.

A common roadblock in rounding numbers is that students cannot identify the number that is halfway between two consecutive tens or hundreds (e.g., 35 is halfway between 30 and 40; 750 is halfway between 700 and 800.) A useful tool to address this concept is the number line.

A number line with nice numbers highlighted is the focus of the rounding strategies and activities in this unit.

Just Hanging A-Round

<http://www.cpalms.org/Public/PreviewResourceLesson/Preview/46508>

In this lesson, students will demonstrate knowledge of rounding in problem solving with or without the aid of a number line.

Reading and Writing Numbers: 3.NSBT.4

Teacher Note:

A common misconception in reading and writing larger numbers is that the student recognizes simple multi-digit numbers, such as 30 or 400, but does not understand that the position of a digit determines its value. The student mistakes the numeral 306 for thirty-six or writes 4008 when asked to record four hundred eight. Such errors are often due to a misunderstanding of zero's use as a place holder. Close monitoring of student responses is necessary to identify and correct place value misconceptions. Another common error is the use of "and" when reading whole numbers. A student might read 306 (three hundred six) as three hundred and six, or 2,478 (two thousand, four hundred seventy-eight) as two thousand, four hundred and seventy-eight. The word "and" is used when reading decimal or mixed numbers only.

Place Value to 999,999

http://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=48&ved=0CEkQFjAHOChqFQoTCLfGpK7IIMYCFQPQgAodqDwAyw&url=http%3A%2F%2Fwww.esc11.net%2Fcms%2Flib3%2FTX21000259%2FCentricity%2FDomain%2F91%2F10_M030101.pdf&ei= mOAVbfPGoOggwSo-YDYDA&usq=AFQjCNEADGwh74QI81Mu-Nd8B5OI0tQTLA&sig2=1_0SqWywIw_9DiYs5zuY5Q

- In this five-day unit, students use base-ten blocks and place value charts to describe place value in numbers up to 999,999. Standard form, expanded notation, word form, and the value of each digit are investigated. Blackline masters of all support materials are included.

Expanded Form

<http://studyjams.scholastic.com/studyjams/jams/math/numbers/expanded-notation.htm>

- changing numbers to and from expanded form.

Place Value to 100,000

<http://www.toonuniversity.com/flash.asp?err=503&engine=15>

- reading and writing numbers

Place Value and Expanded Form

<http://mathlessons.about.com/od/fourthgradelessons/a/Lesson-Plan-Expanded-Notation.htm>

- variety of practice

Number Boards

<http://www.primaryresources.co.uk/online/numberboard2.swf>

<http://www.ictgames.com/arrowcards.html>

- Both of these websites provide visuals to show place value and expanded form for given numbers

Order and Compare Numbers: 3.NSBT.5

http://www.eduplace.com/math/mw/background/3/01/te_3_01_place_ideas.html

- unit on ordering greater numbers, including place value concepts

http://www.teach-nology.com/teachers/lesson_plans/math/35orderingnum.html

- group activity to order large numbers

<http://www.crickweb.co.uk/ks2numeracy-calculation.html>

- ordering numbers to 10,000s

<http://www.learnnc.org/lp/pages/2961?style=print>

- small group activity for comparing numbers

<https://www.pinterest.com/pin/268034615296174514/>

- visual of whole class activity

Resources

EngageNY Grade 3 Module 2: Place Value and Problem Solving with Units of Measure

Lesson plans, activities, assessment, and resources for place value (you will need to select the ones that address the standards).

<https://www.engageny.org/resource/grade-3-mathematics-module-2>

Learn Zillion - Round Whole Numbers to the Nearest 10 or 100

<https://learnzillion.com/lessonsets/370-round-whole-numbers-to-the-nearest-10-or-100>

- 3 LearnZillion lessons (similar to powerpoint) to help students round numbers

<https://wordpress.com/2013/11/17/tricks-are-not-for-kids/>

- Strategy for rounding using a number line vs teaching tricks

Rounding Games

<http://www.mathnook.com/math/skill/roundinggames.php>

Teachers Pay Teachers Freebies - Center Games

<https://www.teacherspayteachers.com/Product/Rounding-With-a-Number-Line-155326>

<https://www.teacherspayteachers.com/Product/Place-Value-737311>

Guess My Number

This activity allows practice with place value skills in a game-like challenge. Can be adapted to any level of learning or ability.

http://www.mathsolutions.com/documents/0-941355-75-6_L3.pdf

Children's Literature

Coyotes All Around by Stuart J. Murphy

- Clever Coyote thinks it's time for lunch — and also time to show her friends how, with some simple rounding, she can add up numbers in her head. The story is also filled with lots of coyote factoids. Suggested classroom activities are listed at the end of the book.
- <http://reckoningnreading.weebly.com/lesson-plans-pre-made.html>

This lesson plan to from Tarleton State University supports *Coyotes All Around* and is written for 3rd Grade. An added plus, the handouts are included at the end of the document.

Sir Cumference and All the King's Tens by Cindy Neuschwander

- Understanding how numbers move in value from one place to the other is foundational for all other mathematical operations. *Sir Cumference and All the King's Tens* helps students to visualize the value of each digit in the place value chart. Visitors to the Royal Palace are grouped by tens and then by hundreds and ultimately by thousands. The illustrations and storyline help students to create a picture of how grouping numbers by ten easily lends itself to counting large numbers and combining them in other mathematical operations.

A Place for Zero: A Math Adventure by Angeline Sparagna Lopresti

- Zero is all alone in the land of Digitaria. He can't play addemup with the other numbers, because he has nothing to add. Join Zero as he goes on a journey to discover his place. His quest takes him from the mysterious workshop of Count infinity to the palace of King Multiplus, where Zero meets a stranger who looks surprisingly familiar.

How Much is a Million? by David Schwartz

- A popular book for teaching very large numbers, filled with examples that kids will relate to. Schwartz uses a series of examples to demonstrate how big a million is, then a billion, then a trillion.

Sample Formative Assessment Tasks/Questions

Rounding to the Nearest Ten and Hundred

<https://www.illustrativemathematics.org/content-standards/3/NBT/A/1/tasks/1805>

<https://grade3commoncoremath.wikispaces.hcpss.org/Assessing+3.NBT.1> (assessing for 3.NSBT.1)

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Place Value, Addition, & Subtraction of Whole Numbers

Content Standards with Clarifying Notes*Open Bullets Indicate Clarifying Notes*

- **4.NSBT.1** Understand that, in a multi-digit whole number, a digit represents ten times what the same digit represents in the place to its right.
 - The focus of this standard is on the relationship between the value of digits rather than the place value itself.
- **4.NSBT.2** Recognize math periods and number patterns within each period to read and write in standard form large numbers through 999,999,999.
 - When students understand math periods and the number patterns with each period, they can work with large numbers with greater ease and flexibility.
- **4.NSBT.3** Use rounding as one form of estimation and round whole numbers to any given place value.
 - Do not just start with "the rule" when rounding. The number line is a great tool to introduce rounding and it will lead to students discovering the rule on their own. Students may choose a vertical or horizontal line.
- **4.NSBT.4** Fluently add and subtract multi-digit whole numbers using strategies to include a standard algorithm.
 - Fluency means accuracy and efficiency (using a reasonable amount of steps and time), and flexibility (using a variety of strategies such as the distributive property, decomposing and recomposing numbers, etc.)

New Academic Vocabulary for This Unit

- | | | | |
|------------|-------------|--------------|--------------|
| ● math | ● algorithm | ● addend | ● difference |
| ● periods | ● sum | ● subtrahend | ● minuend |
| ● millions | | | |

Prior Knowledge Required for this Unit

In 3rd grade, students read and wrote numbers through 999,999 in standard and expanded forms (3.NSBT.4), rounded whole numbers to the nearest 10 or 100 (3.NSBT.1), and added and subtracted numbers fluently to 1,000 (3.NSBT.2).

Subsequent Knowledge Related to this Unit

In the Multiplication & Division of Whole Numbers Unit (Unit 3), students will use rounding to estimate products and quotients in order to check for reasonableness. Furthermore, students will use their knowledge of place value to multiply and divide using various strategies. Later, when students are introduced to decimals for the first time, they will extend their knowledge of the place value system to include tenths and hundredths.

Relationship Among Standards in this Unit

The primary focus of this unit is for students to develop fluency with addition and subtraction. First, students deepen their understanding of place value with an emphasis on math periods within a number and the relationship to other periods in order to read and write large numbers. Next, students use this understanding of the place value system when rounding numbers as a method for estimating, initially using number lines, with the ultimate goal of moving away from the visual model. Finally, students apply their knowledge of place value, as well as estimating, to add and subtract fluently and flexibly with any size number.

Potential Instructional Strategies/Lessons

Teacher Note: 4.NSBT.1

“This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with” (excerpt from KATM Grade 4 flip book). Students need to understand the relationship between the base ten number system and place value so that they can explain that the value of a digit changes depending on its location in a number. Understanding how the value of a digit changes depending on where it is located in a number is essential. Multiplying or dividing a number by 10 changes the value of the digit by one whole place value position in our base ten number system.

While the focus of 4.NSBT.1 is on the multiplicative relationship between the place value of two digits in a number and will be dealt with more in the multiplication and division unit (Unit 3), this standard is included in this unit to emphasize the importance of using place value as a strategy for adding and subtracting.

Introductory Activities:

Learn Zillion - Multiply by Powers of 10

<https://learnzillion.com/lessons/805-multiply-by-powers-of-10> and <https://drive.google.com/file/d/0B2e1qHKMDxlgV012UTVlcEFoalE/edit>

- This lesson explores the relationship between numbers on a place value chart and the activity helps students apply their new knowledge.

Learn Zillion - Understand Relationships between Digits and their Place Value

<https://learnzillion.com/lessons/516-understand-relationships-between-digits-and-their-place-value>

- This lesson explores the relationship between base ten blocks and numbers on a place value chart.

Teacher Note: 4.NSBT.2

Students worked with numbers within 1,000 in 2nd and within 1,000,000 in 3rd. In 4th grade, students should be able to apply what they learned in those grades to work with larger numbers including billions. Contextual problems might include populations.

“Provide multiple opportunities in the classroom setting and use real-world context for students to read and write multi-digit whole numbers focusing on the periods within a number as the basis for reading and writing. Students also need to create numbers that meet specific criteria. For example, provide students with cards numbered 0 through 9. Ask students to select 4 to 6 cards; then, using all the cards make the largest number possible with the cards, the smallest number possible and the closest number to 5000 that is greater than 5000 or less than 5000.

There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes (see below) and vertical addition method (see below).

MILLIONS			THOUSANDS			ONES		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

For the number one thousand, seventy-two students could write out each value vertically and then add the values.

$$\begin{array}{r}
 1000 \\
 70 \\
 + \underline{2} \\
 1,072
 \end{array}$$

Another misconception is that students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole" (excerpt from KATM Grade 4 Flip Book). This most likely occurs because students have been taught to line up the digits to determine which is greater rather than to focus on the place value of the digits in a number.

Introductory Activities:

Georgia Department of Education - Building 1,000

https://grade4commoncoremath.wikispaces.hcps.org/file/view/4.NBT.1_4.NBT.2_Building1000.pdf/457296196/4.NBT.1_4.NBT.2_Building1000.pdf

- This activity could be used as a formative assessment task to determine students' level of understanding on the 3rd grade concepts of place value.

Illustrations - Count on Math: Making Your First Million

<http://illuminations.nctm.org/Lesson.aspx?id=4018>

- In this activity, students attempt to identify the concept of a million by working with smaller numerical units, such as blocks of 10 or 100, and then expanding the idea by multiplication or repeated addition until a million is reached. Additionally, they use critical thinking to analyze situations and to identify mathematical patterns that will enable them to develop the concept of very large numbers.

Teacher Note: 4.NSBT.3

“Rounding numbers should result in using number sense not just following a rule. Students should apply their understanding of number lines to help them round numbers. When thinking about tens, numbers with the one’s digit of 1,2,3, and 4 are closer to the lower multiple of ten. Numbers with the one’s digit of 6, 7, 8, and 9 are closer to the greater multiple of ten. Since 5 is equidistant from both multiples, it is rounded to the greater multiple of ten. This same concept applies to rounding to the nearest hundred or thousand. When teaching rounding, it is helpful for students to use a number line to visualize the process” (John SanGiovanni, Howard County Schools). For example, if asked to round 1,768 to the nearest thousand, draw a number line and label the endpoints 1000 and 2000. Have the students place the number 1,768 on the number line and point out that it is closer to 2000 so 1,768 rounds to 2000.

Introductory Activity:**Illustrative Mathematics - Rounding to the Nearest Thousand**

<https://www.illustrativemathematics.org/content-standards/tasks/1807>

- This task introduces students to rounding larger numbers using a number line.

Teacher Note: 4.NSBT.4

“Students have had exposure to many ways of adding and subtracting whole numbers. The standard algorithm starts with adding the ones and regrouping as necessary. Some students may be comfortable starting with the larger place value – this should be permitted if it makes sense to them. A mixture of problems requiring and not requiring regrouping should be given to students. Some students may use an open number line when adding and subtracting and this should be permitted” (John SanGiovanni, Howard County Schools). Students should be able to explain why their algorithm works. When explaining, students should know that it is mathematically possible to subtract a larger number from a smaller number, but that their work in 4th grade with whole numbers does not allow this as the difference would result in a negative number.

“A crucial theme in multi-digit arithmetic is encouraging students to develop strategies that they understand, can explain, and can think about, rather than merely follow a sequence of directions, rules or procedures that they don't understand. It is important for students to have seen and used a variety of strategies and materials to broaden and deepen their understanding of place value before they are required to use standard algorithms. The goal is for them to understand all the steps in the algorithm, and they should be able to explain the meaning of each digit. Start with a student’s understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm.

This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow. Sometimes students benefit from 'being the teacher' to an imaginary student who is having difficulties applying standard algorithms in addition and subtraction situations. To promote understanding, use examples of student work that have been done incorrectly and ask students to provide feedback about the student work. It is very important for some students to talk through their understanding of connections between different strategies and standard addition and subtraction algorithms. Give students many opportunities to talk with classmates about how they could explain standard algorithms. Think-Pair-Share is a good protocol for all students.

Often students mix up when to 'carry' and when to 'borrow'. Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits. If students are having difficulty with lining up similar place values in numbers as they are adding and subtracting, it may be helpful to have them write their calculations on grid paper or lined notebook paper with the lines running vertical. This assists the student with lining up the numbers more accurately" (excerpt from KATM Grade 4 Flip Book).

Introductory Activities:

Illustrative Mathematics - To Regroup or Not to Regroup

<https://www.illustrativemathematics.org/content-standards/tasks/1189>

- This is an instructional task that makes students think more deeply about the regrouping process used in some subtraction problems. Since this task deals with three-digit numbers, it would be a good starting point to make sure students fully understand the subtraction concept before moving on to larger numbers.

Georgia Department of Education - Making Sense of the Algorithm (Subtraction)

<https://grade4commoncoremath.wikispaces.hcps.org/file/view/Making%20Sense%20of%20the%20Algorithm.pdf/402793860/Making%20Sense%20of%20the%20Algorithm.pdf>

- This task allows students to make sense of the standard algorithm for subtraction. It is important you allow them to grapple with the strategies used by the student in the task. Through this grappling, students make sense of what she did to solve each problem. Through classroom discussion, student understanding will be shared and developed.

Resources

Engage NY Grade 4 Mathematics Module 1

<https://www.engageny.org/resource/grade-4-mathematics-module-1>

- This module addresses Place Value, Rounding, Addition, & Subtraction. It includes math background, teacher notes, student activities, practice pages, and a variety of assessments.

CCGPS Frameworks 4th Unit 1

www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_4_Unit1Framework.pdf

- This unit includes performance tasks and formative assessment lessons.

Interactive Sites for Education - Place Value

<http://interactivesites.weebly.com/place-value.html>

- This site has multiple interactive games that students can use to practice place value skills.

K-5 Math Teaching Resources - Place Value Problems

<http://www.k-5mathteachingresources.com/support-files/place-value-problems.pdf>

- This provides practice for students on 4.NSBT.1.

Howard County Schools Resources for 4.NSBT.1

<https://grade4commoncoremath.wikispaces.hcpss.org/4.NBT.1>

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

Literature

How Much is a Million? by David Schwartz

- This book helps children conceptualize complex numbers in a fun, humorous way.

***How Big is a Million?* by Anna Milbourne**

- This book helps children understand a million through the eyes of a penguin.

***On Beyond a Million: An Amazing Math Journey* by David M. Schwartz**

- Real-life examples provide plenty of fun facts, such as how much popcorn Americans eat in one year, or how many hairs are on a square inch of a person's head. Along with the fun comes some learning, as this counting book helps kids understand our number system.

***Count to a Million* by Jerry Pallotta**

- Although some may have their doubts, by using basic math grouping skills, readers will find themselves counting higher than they ever thought possible.

Howard County Schools Resources for 4.NSBT.2

<https://grade4commoncoremath.wikispaces.hcps.org/4.NBT.2>

- This page includes lesson plans, print resources, LearnZillion video links, and web resources. Note that SC standards do not include comparing numbers.

Bridges in Mathematics - Place Value to Millions

http://bridges1.mathlearningcenter.org/media/Bridges_Gr4_OnlineSupplement/B4SUP-A3_NumPIVal_0409.pdf

- This provides practice for students on 4.NSBT.2.

National Library of Virtual Manipulatives - Place Value Number Lines

http://nlvm.usu.edu/en/nav/frames_asid_334_g_1_t_1.html?from=topic_t_1.html

- This virtual manipulative gives students practice with placing numbers on a number line, in preparation for using number lines for rounding.

Illustrative Mathematics - Rounding to the Nearest 100 and 1000

<https://www.illustrativemathematics.org/content-standards/tasks/1806>

- This task introduces students to rounding using a number line.

Bridges in Mathematics - Estimating to Add and Subtract

http://bridges1.mathlearningcenter.org/media/Bridges_Gr5_OnlineSupplement/B5SUP-A1_NumEstAddSub_0509.pdf

- This provides practice for students on 4.NSBT.3 and 4.NSBT.4.

Howard County Schools Resources for 4.NSBT.3

<https://grade4commoncoremath.wikispaces.hcpss.org/4.NBT.3>

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

Matific - Rounding up to 10,000

<https://www.matific.com/us/en-us/curriculum?episode=RoundToTheNearestPowerOfTenUpTo10000G4NBT>

- This provides practice for students on 4.NSBT.3.

Howard County Schools Resources for 4.NSBT.4

<https://grade4commoncoremath.wikispaces.hcpss.org/4.NBT.4>

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

K-5 Math Teaching Resources - Adding and Subtracting Multi-digit Numbers

<http://www.k-5mathteachingresources.com/support-files/adding-and-subtracting-multi-digit-whole-numbers.pdf>

- This provides practice for students on 4.NSBT.4.

Sample Formative Assessment Tasks/Questions

- <https://grade4commoncoremath.wikispaces.hcpss.org/Assessing+4.NBT.1> (Assessing 4.NSBT.1)
- <https://grade4commoncoremath.wikispaces.hcpss.org/Assessing+4.NBT.2> (Assessing 4.NSBT.2, but note that the SC standard does not include comparing numbers.)
- <https://grade4commoncoremath.wikispaces.hcpss.org/Assessing+4.NBT.3> (Assessing 4.NSBT.3)
- <https://grade4commoncoremath.wikispaces.hcpss.org/Assessing+4.NBT.4> (Assessing 4.NSBT.4)

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Algebraic Thinking

Content Standards with Clarifying Notes*Open Bullets Indicate Clarifying Notes*

- **4.ATO.1** Interpret a multiplication equation as a comparison (e.g. interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.) Represent verbal statements of multiplicative comparisons as multiplication equations.
 - “A multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., ‘a is n times as much as b’). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times. Students should be given many opportunities to write and identify equations and statements for multiplicative comparisons.” (Excerpt from KATM Grade 4 Flip Book)
- **4.ATO.2** Solve real-world problems using multiplication (product unknown) and division (group size unknown, number of groups unknown).
 - Students should be exposed to all three types of problems.
- **4.ATO.3** Solve multi-step, real-world problems using the four operations. Represent the problem using an equation with a variable as the unknown quantity.
- **4.ATO.4** Recognize that a whole number is a multiple of each of its factors. Find all factors for a whole number in the range 1 – 100 and determine whether the whole number is prime or composite.
- **4.ATO.5** Generate a number or shape pattern that follows a given rule and determine a term that appears later in the sequence.
 - Third grade students identified a rule for patterns. Now, students will generate the pattern from the given rule.
 - Notice that the standard does not require students to infer or guess the underlying rule for a pattern, but instead asks them to generate a pattern from a given rule.
 - In this unit, the focus will be on number patterns. Shape patterns will be addressed in Unit 8.

New Academic Vocabulary for This Unit

- factor pairs
- multiples
- variable
- products
- prime numbers
- rule
- multiplicative comparisons
- composite numbers
- term

Prior Knowledge Required for this Unit

In third grade, students learned basic multiplication and division facts of products and dividends through 100 (3.ATO.7), used equations with symbols for the unknown (3.ATO.4), solved two-step problems using multiplication and division (3.ATO.8), and identified a rule for an arithmetic pattern (3.ATO.9).

Subsequent Knowledge Related to this Unit

In the later Multiplication & Division Unit, students will encounter multiplicative comparisons and real world problems with larger numbers. Furthermore, they will learn algorithms for multi-digit multiplication and division. They will also generate numbers for more difficult patterns. Students' familiarity with multiplication concepts such as factors and multiples will also help them when working with division, equivalent fractions, and measurement conversions.

Relationship Among Standards in this Unit

The primary focus of this unit is for students to extend their algebraic reasoning. First, students will learn about multiplicative comparisons and how variables can be used within them. Next, students will gain familiarity with factors and multiples and then use this knowledge in problem solving situations. Students will also examine multiplication and division patterns and how they relate to problem solving.

Throughout the unit, students need experiences with many different types of real-world problems, presented in a variety of ways, in order to gain flexibility in problem solving. This work will continue in each of the math units throughout the school year.

Potential Instructional Strategies/Lessons

Teacher Note: 4.ATO.1 - 4.ATO.3

“Repeated addition and arrays are two ways of thinking about multiplication. Multiplication can also be thought about as multiple comparison problems that involve a comparison of two quantities in which one is described as a multiple of the other. The relation between quantities is described in terms of how many times larger one is than the other. An example of multiplicative comparison is a 10-foot alligator is five times longer than a 2-foot alligator” (John SanGiovanni, Howard County Schools).

“Students need experiences that allow them to connect mathematical statements and number sentences or equations. This allows for an effective transition to formal algebraic concepts. They represent an unknown number in a word problem with a symbol. Word problems which require multiplication or division are solved by using drawings and equations.

Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as shown in the Common Multiplication and Division Situations Table (http://www.cpalms.org/uploads/docs/standards/mafs_table2.pdf). They should use drawings or equations with a symbol for the unknown number to represent the problem.

Present multistep word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies” (excerpt from KATM Grade 4 Flip Book).

Introductory Activities:

K-5 Math Teaching Resources - Multiplicative Comparison Problems

<http://www.k-5mathteachingresources.com/support-files/multiplication-equations-and-comparison-statements.pdf>

- These practice cards help students link comparison sentences, models, and equations in multiplicative comparisons.

Illustrative Mathematics - Comparing Money Raised

<https://www.illustrativemathematics.org/content-standards/tasks/263>

- The purpose of this task is for students to solve three comparisons problems that are related by their context but are structurally different (product unknown, group size unknown, number of groups unknown).

Teacher Note: 4.ATO.2

When solving problems, students may use drawings and equations with a variable (a letter standing for the unknown quantity) for the unknown to represent the problem. Students should solve equal group problems and comparison problems that involve the unknown in all locations – product unknown (multiplication), group size unknown (partition division), how many in each group (division), number of groups of unknown (measurement division), how many groups (division) and area problems that involve product unknown (multiplication) and side dimension unknown (division). Teachers should reference Common Multiplication and Division Situations Table (see Teacher Note above for link).

Introductory Activities:**Thinking Blocks - choose Compare Quantities (One Step Model)**

http://www.mathplayground.com/tb_multiplication/thinking_blocks_multiplication_division.html

- This site has a tutorial on using the Thinking Blocks and then allows students to build models based on story problems.

K-5 Math Teaching Resources - Multiplicative Comparison Problems 2

<http://www.k-5mathteachingresources.com/support-files/comparison-problems.pdf>

- These practice cards take the previous cards a step farther and introduce word problems involving multiplicative comparisons.

Teacher Note: 4.ATO.3

“Solving multi-step word problems requires the students to read and understand the context of the problem. Focus should be on the information given and what the problem is asking. Focus should not be on finding KEY words to indicate the four operations. When the focus is on key words, students tend to overlook the entire problem. They may misinterpret the problem because of the key word and use the incorrect operation” (John SanGiovanni, Howard County Schools).

Students need multiple experiences to solve multi-step, real-world problems, using all four operations. Students should discuss and then use various strategies. To check for reasonableness, they should also use various estimation strategies.

Literature Connection:**Math Course by Jon Scieszka**

- This book talks about the daily math problems that are encountered in everyday life and would be a great starting point for getting students thinking about real world problems.

Introductory Activities 4.ATO.3:**North Carolina Department of Education - Multistep Multiplication**

https://grade4commoncoremath.wikispaces.hcpss.org/file/view/4.OA.3_Multi-StepMultiplication.pdf/457301088/4.OA.3_Multi-StepMultiplication.pdf

- This lesson involves students estimating and solving two-step word problems using various strategies.

Teacher Note: 4.ATO.4

“Students need to develop an understanding of the concepts of number theory such as prime numbers and composite numbers. This includes the relationship of factors and multiples. Multiplication and division are used to develop concepts of factors and multiples. Division problems resulting in remainders are used as counterexamples of factors. Review vocabulary so that students have an understanding of terms such as factor, product, multiples, and odd and even numbers.

Multiples: Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20). To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- all even numbers are multiples of 2
- all even numbers that can be halved twice (with a whole number result) are multiples of 4
- all numbers ending in 0 or 5 are multiples of 5

Factors: Provide students with counters to find the factors of numbers. Have them find ways to separate the counters into equal subsets. For example, have them find several factors of 10, 14, 25 or 32, and write multiplication expressions for the numbers. Another way to find the factor of a number is to use arrays from square tiles or drawn on grid papers. Have students build rectangles that have the given number of squares.

Knowing how to find factors for a number assists in developing reasonableness when dividing, problem solving, and simplifying fractions.

Prime and Composite: **Definitions of prime and composite numbers should not be provided, but determined after many strategies have been used in finding all possible factors of a number.** A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number. Students investigate whether numbers are prime or composite by

- building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1×7 and 7×1 , therefore it is a prime number)
- finding factors of the number

Students should develop a process for finding factor pairs so they can do this for any number 1 – 100 with efficiency.

Example: Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

Starting with a number chart of 1 to 20, use multiples of prime numbers to eliminate later numbers in the chart. For example, 2 is prime but 4, 6, 8, 10, 12 . . . are composite. After working with the numbers 1 to 20, consider using a hundreds chart and have the students color code multiples of numbers. The color will help students see emerging patterns which they can discuss.

Encourage the development of rules that can be used to aid in the determination of composite numbers. For example, other than 2, if a number ends in an even number (0, 2, 4, 6 and 8), it is a composite number” (excerpt from KATM Grade 4 Flip Book).

Introductory Activities 4.ATO.4:

Sieve of Eratosthenes

<https://grade4commoncoremath.wikispaces.hcps.org/file/view/The+Sieve+of+Eratosthenes.pdf>

<http://www.visnos.com/demos/sieve-of-eratosthenes>

- The first link is a lesson in which students use color coding on a hundreds chart to identify the prime numbers between 1 and 100. The

second link is a virtual sieve that is also color coded.

Understanding Factors and Multiples (and Determining Prime and Composite Numbers)

<https://learnzillion.com/lessonsets/123-find-and-understand-factors-and-determine-if-a-number-is-a-multiple-of-a-given-number-for-whole-numbers-0-100>

- 11 LearnZillion lessons (similar to powerpoint) that provide strategies for students working with factors and multiples, prime and composite numbers.

Factor Game

http://mathsolutions.com/wp-content/uploads/978-1-935099-02-4_NL36_L1.pdf

- This partner game from Math Solutions is good for practice in identifying factors of given numbers.

Illustrative Mathematics - The Locker Game

<https://www.illustrativemathematics.org/content-standards/tasks/938>

- The purpose of this instructional task is for students to deepen their understanding of factors and multiples of whole numbers.

Teacher Note: 4.ATO.5

“Students need to recognize patterns, extend patterns, describe patterns, and create patterns. Repeated patterns have a part or a core that repeats over and over. Examples of repeated patterns can include using shapes and numbers. You can describe a pattern by stating the rule it is following. Number Changing patterns are examples of number sequences. A sequence is a set of numbers that are arranged in a certain order. Each number in the sequence is called a term.

Examples:

Repeating pattern: 1,4,7, 1,4,7, 1,4,7, 1,4,7, 147, 1,4,7, 1,4,7

Changing pattern: 1,4,7,10,13,16,19,22,25,28,31,34,37” (John SanGiovanni, Howard County Schools).

“In order for students to be successful later in the formal study of algebra, their algebraic thinking needs to be developed. Understanding patterns is fundamental to algebraic thinking. Students have experience in identifying arithmetic patterns, especially those included in addition and multiplication tables. Contexts familiar to students are helpful in developing students’ algebraic thinking.

Students should generate numerical or geometric patterns that follow a given rule. They should look for relationships in the patterns and be able to describe and make generalizations.

As students generate numeric patterns for rules, they should be able to “undo” the pattern to determine if the rule works with all of the numbers generated. For example, given the rule, “Add 4” starting with the number 1, the pattern 1, 5, 9, 13, 17, ... is generated. In analyzing the pattern, students need to determine how to get from one term to the next term. Teachers can ask students, “How is a number in the sequence related to the one that came before it?”, and “If they started at the end of the pattern, will this relationship be the same?” Students can use this type of questioning in analyzing numbers patterns to determine the rule.

Often, students think that results are random. There is no pattern. Another common misconception when students are working with repeating patterns is that they will often repeat what is given rather than looking at what “chunks” or part of the pattern is actually being repeated. Example: Given the pattern 6,9,12,6,9,12,6,9,... If the student is asked “what is the next number in the pattern”, they may respond with “6” because they are returning to the beginning of the given pattern and repeat it from there. Students should be encouraged to look for the repeating set” (excerpt from KATM Grade 4 Flip Book).

Introductory Activities 4.ATO.5:

Number and Shape Patterns

<https://learnzillion.com/lessonsets/195-generate-number-or-shape-patterns-that-follow-a-given-rule-and-identifying-pattern-features>

- 9 LearnZillion lessons (similar to powerpoint) that provide a variety of strategies and activities involving patterns.

Developing Algebraic Thinking Using Manipulatives

http://www.mathsolutions.com/documents/Developing_Algebraic_Thinking_i35.pdf

- Students use pattern blocks as a tool to explore patterns that grow and solve problems. Using the contexts of trees and fish that grow in consistent and predictable ways, students build, extend, describe, and represent patterns.

Resources

CCGPS Frameworks 4th Unit 2

https://www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_4_Unit2Framework.pdf

- Parts of this unit include performance tasks and formative assessment lessons.

Howard County Schools Resources for 4.ATO.1

<https://grade4commoncoremath.wikispaces.hcpss.org/4.OA.1>

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

Howard County Schools Resources for 4.ATO.2

<https://grade4commoncoremath.wikispaces.hcpss.org/4.OA.2>

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

Howard County Schools Resources for 4.ATO.3

<https://grade4commoncoremath.wikispaces.hcpss.org/4.OA.3>

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

Math Playground - Solve it Math!

http://www.mathplayground.com/wp_videos.html

- This site offers numerous multi-step problems for students to solve and then gives step by step video solutions. This site is best suited for struggling students.

Math Playground - Word Problems with Katie

<http://www.mathplayground.com/WordProblemsWithKatie2.html>

- This game gives the student multiplication and division single and multistep word problems to solve. This site is best suited for struggling students.

Math Playground - Algebra Puzzles

http://www.mathplayground.com/algebra_puzzle.html

- Students use multiplication and division skills to determine the value of the objects. This is a great way to practice multi-step problems solving and writing equations. Students can choose two different levels (beginner or advanced).

North Carolina State Department of Education - Problem Solving Decks

http://mathlearnnc.sharpschool.com/UserFiles/Servers/Server_4507209/File/Problem%20Solving%20Decks%20%28K-8%29/Problem%20Solving%20Deck%20B%20Student%20Sheets.pdf (student copy for showing work)

http://mathlearnnc.sharpschool.com/UserFiles/Servers/Server_4507209/File/Problem%20Solving%20Decks%20%28K-8%29/Problem%20Solving%20Deck%20B%20Cards.pdf

- These cards offer wonderful opportunities for thinking and solving problems.

Port Angeles School District - Practice Problems and Strategies

<http://www.portangelesschools.org/students/grade-4-St.html#PSstrat>

- This site has numerous problem-solving strategies and sample problems for each strategy.

Howard County Schools Resources for 4.ATO.4

<https://grade4commoncoremath.wikispaces.hcpss.org/4.OA.4>

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

BBC Skillswise - Factors and Multiples

<http://www.bbc.co.uk/skillswise/topic/multiples-and-factors>

- This site has a video, games and practice sheets that can be printed out, and interactive quizzes.

Bridges in Mathematics - Primes, Composites, and Common Factors

http://catalog.mathlearningcenter.org/files/pdfs/SecB5SUP-A2_NumPrimes-201304.pdf

- This is two lessons, with student practice sheets, on identifying prime and composite numbers, as well as finding factors of numbers between 1 and 100.

Illuminations - Factorize

<http://illuminations.nctm.org/Activity.aspx?id=3511>

- This interactive tool allows students to build arrays in order to identify all the factors of a number.

BrainPop - Sortify with Multiples and Factors

<https://www.brainpop.com/games/sortify-multiplication/>

- This interactive game gives students number tiles and asks the student to sort them based on categories that the students choose.

Math Live from Learn Alberta - Multiples, Factors, Primes, and Composites

<http://www.learnalberta.ca/content/me5l/html/math5.html?goLesson=2>

- This is an interactive tutorial that also includes an activity sheet and assessment.

Crickweb - Venn Diagram Comparing Multiples

<http://www.crickweb.co.uk/ks2numeracy-properties-and-ordering.html>

- This interactive tool allows students to sort multiples of different numbers using a Venn Diagram.

Calculation Nation - Factor Dazzle

<http://calculationnation.nctm.org/Games/Game.aspx?GameId=A0537FC6-3B08-4AFC-9AD6-0CC5E3BC9B86>

- This game gives students practice with identifying factors.

***You Can Count on Monsters* by Richard Evan Schwartz**

- This book presents the concepts of prime numbers and factoring in a novel and colorful way.

Howard County Schools Resources for 4.ATO.5

<https://grade4commoncoremath.wikispaces.hcpss.org/4.OA.5>

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

Sample Formative Assessment Tasks/Questions

<https://grade4commoncoremath.wikispaces.hcps.org/Assessing+4.OA.1> (assessing 4.ATO.1)

<https://grade4commoncoremath.wikispaces.hcps.org/Assessing+4.OA.2> (assessing 4.ATO.2)

<https://grade4commoncoremath.wikispaces.hcps.org/Assessing+4.OA.3> (assessing 4.ATO.3)

<https://grade4commoncoremath.wikispaces.hcps.org/Assessing+4.OA.4> (assessing 4.ATO.4)

<https://grade4commoncoremath.wikispaces.hcps.org/Assessing+4.OA.5> (assessing 4.ATO.5)

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DRAFT

Expressions, Equations, and the Coordinate Plane

Content Standards with Clarifying Notes

Open bullets indicate clarifying notes.

- **5.ATO.1** Evaluate numerical expressions involving grouping symbols (i.e., parentheses, brackets, braces).
 - Include the use of all four operations
- **5.ATO.2** Translate verbal phrases into numerical expressions and interpret numerical expressions as verbal phrases.
 - This standard does not include the use of variables
 - Include the use of all four operations
- **5.ATO.3** Investigate the relationship between two numerical patterns.
 - a. Generate two numerical patterns given two rules and organize in tables;
 - b. Translate the two numerical patterns into two sets of ordered pairs;
 - c. Graph the two sets of ordered pairs on the same coordinate plane;
 - d. Identify the relationship between the two numerical patterns.
- **5.G.1** Define a coordinate system.
 - a. The x- and y- axes are perpendicular number lines that intersect at 0 (the origin);
 - b. Any point on the coordinate plane can be represented by its coordinates;
 - c. The first number in an ordered pair is the x-coordinate and represents the horizontal distance from the origin;
 - d. The second number in an ordered pair is the y-coordinate and represents the vertical distance from the origin.
- **5.G.2** Plot and interpret points in the first quadrant of the coordinate plane to represent real-world and mathematical situations.

New Academic Vocabulary for This Unit

- | | | |
|------------------|---------------------|----------|
| ● brackets | ● coordinate system | ● origin |
| ● braces | ● coordinate plane | ● x-axis |
| ● first quadrant | ● ordered pair | ● y-axis |
| ● x-coordinate | ● y-coordinate | |

Prior Knowledge Required for this Unit

In this unit, students extend their work from grade 3 (3.ATO.5) and grade 4 (4.ATO.3) where students start learning the conventional order when simplifying numerical expressions. In addition, students apply their reasoning of the four operations as well as place value while describing the relationship between numbers.

This unit extends the work from grade four, where students generate numerical patterns when they are given one rule. In grade five, students are given two rules and generate two numerical patterns. Additionally, this is the first time students are introduced to the coordinate plane. Students will define a coordinate system, plot, and interpret points in the first quadrant.

Subsequent Knowledge Related to this Unit

Students need multiple experiences with expressions that use grouping symbols throughout the year to develop understanding of when and how to use parentheses, brackets, and braces. First, students use these symbols with whole numbers (extending beyond basic facts.) Then, the symbols can be used as students add, subtract, multiply and divide decimals and fractions throughout the year. In grade 6, students will be formally introduced to the Order of Operations and (6.EE.1). Additionally, students translate between verbal phrases and numerical expressions. These standards are the foundation for writing and evaluating numerical and algebraic expressions that will include whole-number exponents in Grade 6 (6.EE.1, 6.EE.2, 6.EE.3 and 6.EE.4).

In science units (Force and Motion), students will apply their understanding of the coordinate plane. Students will extend their understanding to graphing in all four quadrants in grade 6 as well as (6.NS.6 and 6.NS.8).

Relationship Among Standards in this Unit

Once students have had rich learning experiences with numerical expressions and the coordinate plane, students apply these skills to model numerical expressions within the coordinate plane. (5.ATO.3)

Potential Instructional Strategies/Lessons

Teacher Note: 5.ATO.1 and 5.ATO.2

“Students may believe the order in which a problem with mixed operations is written is the order to solve the problem. The use of the mnemonic

phrase “Please Excuse My Dear Aunt Sally” to remember the order of operations (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction) can also mislead students to always perform multiplication before division and addition before subtraction. To correct this thinking, students need to understand that addition and subtraction are inverse operations and multiplication and division are inverse operations, as in they have the same “impact”. At this level, students need opportunities to explore the “impact” of the various operations on numbers and solve equations starting with the operation of greatest “impact”. This standard also calls for students to verbally describe the relationship between expressions without actually calculating them. 5.ATO.2 calls for students to apply their reasoning of the four operations as well as place value while describing the relationship between numbers. The standard does not include the use of variables, only numbers and signs for operations.” (Excerpt from KATM Grade 5 flip book)

Possible Strategies:

Why Do We Need an Order of Operations? (5.ATO.1)

<https://www.illustrativemathematics.org/content-standards/5/OA/A/tasks/1606>

- The purpose of this task is to help students think about the reason for the mathematical convention known as the "order of operations." However, formal instruction of “Order of Operations” should not be taught in grade 5.

Video Game Scores (5.ATO.2)

<https://www.illustrativemathematics.org/content-standards/tasks/590>

- This task requires students to write and interpret numerical expressions. The focus of this problem is not on numerical answers, but instead on building and interpreting expressions that could be entered in a calculator or communicated to another student unfamiliar with the context.

The next three activities are similar. All challenge students to reach a target number with a given set of numbers (rolled dice or other random method) by using any operation and including parentheses as needed (5.ATO.1)

Bowling for Numbers <https://www.illustrativemathematics.org/content-standards/tasks/969>.

Clear the Board http://mathsolutions.com/wp-content/uploads/0-941355-75-6_L.pdf

Target Number Dash <http://www.k-5mathteachingresources.com/support-files/target-number-dash.pdf>

Teacher Note: 5.G.1 and 5.G.2

“Students need to understand the underlying structure of the coordinate system and see how axes make it possible to locate points anywhere on a coordinate plane. This is the first time students are working with coordinate planes, and only in the first quadrant. It is important that students create the coordinate grid themselves. This can be related to two number lines, perpendicular lines, and reliance on previous experiences with moving along a number line.

Possible Strategies:

- Multiple experiences with plotting points are needed. Provide points plotted on a grid and have students name and write the ordered pair. Have students describe how to get from the origin to the location of the plotted points. Encourage students to articulate directions, attending to precision as they move and/or plot additional points.
- Present real-world and mathematical problems and have students graph points in the first quadrant of the coordinate plane. Gathering and graphing data is a valuable experience for students. It helps them to develop an understanding of coordinates and what the overall graph represents. Students also need to analyze the graph by interpreting the coordinate values in the context of the situation. For example, students may gather and analyze data comparing speed of a student made vehicle to the height of a ramp. The ordered pair (5, 20) represents a in 5 seconds the vehicle traveled 20 feet.
- Students may think the order in plotting a coordinate point is not important. Have students plot points so that the position of the positive coordinates is switched. For example, have students plot (3, 4) and (4, 3) and discuss the order used to plot the points. Have students create directions for others to follow so that they become aware of the importance of direction and distance.” (Excerpt from KATM Grade 5 flip book)

Meerkat Coordinate Plane Task (5.G.1 and 5.G.2)

<https://www.illustrativemathematics.org/content-standards/tasks/1516>

- This task presents a real-world mathematical problem that requires students to answer questions by drawing and interpreting the meaning of points in the first quadrant of the coordinate plane.

Culminating Activity for 5.G.1 and 5.G.2:**Graph Points on a Coordinate Grid Using Ordered Pairs (5.G.1 and 5.G.2)**

https://learnzillion.com/lesson_plans/1003-graph-points-on-a-coordinate-grid-using-ordered-pairs

- This lesson LearnZillion lesson builds on students' understanding of coordinate grids and locations of points on a plane by introducing ordered pairs as a more specific way of identifying points.

Teacher Note: 5.ATO.3

“Students have experienced generating and analyzing numerical patterns using a given rule in Grade 4. Now in 5th grade, given two rules with an apparent relationship, students should be able to identify the relationship between the resulting sequences of the terms in one sequence to the corresponding terms in the other sequence. For example, starting with 0, multiply the next digit by 4 and starting with 0, multiply the next digit by 8 which generates each sequence of numbers (0, 4, 8, 12, 16, ...) and (0, 8, 16, 24, 32,...). Students should see that the terms in the second sequence are double the terms in the first sequence, or that the terms in the first sequence are half the terms in the second sequence.

Based on data generated in the above bullet, have students form ordered pairs and graph them on a coordinate plane. Patterns can be also observed from the graphs. The graph of both sequences of numbers is a visual representation that will show the relationship between the two sequences of numbers. Encourage students to represent the sequences in T-Charts so they can see a connection between the graph and the sequences.” (Excerpt from KATM Grade 5 flip book)

Possible Strategies:**Generate numerical patterns by examining the context of real-world scenarios (5.ATO.3)**

https://learnzillion.com/lesson_plans/1006-generate-numerical-patterns-by-examining-the-context-of-real-world-scenarios

- The goal of this LearnZillion lesson is to build understanding of numerical patterns in the context of real-world problems.

Compare Corresponding Terms in Numerical Patterns by Using Line Graphs on the Coordinate Grid (5.ATO.3)

https://learnzillion.com/lesson_plans/1001-compare-corresponding-terms-in-numerical-patterns-by-using-line-graphs-on-the-coordinate-grid

- This LearnZillion lesson uses science data to show the math skills. Includes student practice, too.

Resources

Espresso

<http://gregtangmath.com/espresso>

- This game involves filling in missing operators so each expression equals its target number. (5.ATO.1)

Bracket Basics

<http://www.bracketbasics.co.uk/activity/>

- This game involves filling in given numbers so that the expression equals a target number. (5.ATO.1)

The Fly on the Ceiling by Dr. Julie Glass

<https://www.youtube.com/watch?v=HfecU1nqKFc>

- Literature connection; introduces students to Rene Descartes and the history of the coordinate plane

EngageNY 5th grade Module 6 - Problem Solving with the Coordinate Plane

<https://www.engageny.org/resource/grade-5-mathematics-module-6>

- In this unit, students define a coordinate system for the first quadrant of the coordinate plane and use it to solve problems. This unit includes math background, teacher notes, student activities, practice pages, and a variety of assessments.

Dino Dig

<http://www.counton.org/games/virtualmathfest/dinosaur.html>

- This game involves selecting points to identify locations of dinosaur bones. (5.G.1, 5.G.2)

Billy Bug

<http://www.oswego.org/ocsd-web/games/BillyBug/bugcoord.html>

- Interactive game for finding points in the first quadrant of the coordinate plane. (5.G.1, 5.G.2)

Sample Formative Assessment Tasks/Questions

Performance Task for 5.G.2

Granny's Balloon Trip

<http://www.insidemathematics.org/assets/common-core-math-tasks/granny%27s%20balloon%20trip.pdf>

- This task challenges a student to use knowledge of scale to organize and represent data from a table on a graph.

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.OA.1> (assessing 5.ATO.1)

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.OA.2> (assessing 5.ATO.2)

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.OA.3> (assessing 5.ATO.3)

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.G.1> (assessing 5.G.1)

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.G.2> (assessing 5.G.2)

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5th Grade Unit 2

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Place Value

Content Standards with Clarifying Notes

Open bullets indicate clarifying notes

- **5.NSBT.1** Understand that, in a multi-digit whole number, a digit in one place represents 10 times what the same digit represents in the place to its right, and represents $\frac{1}{10}$ times what the same digit represents in the place to its left.
- **5.NSBT.2** Use whole number exponents to explain:
 - a. patterns in the number of zeroes of the product when multiplying a number by powers of 10;
 - b. patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.
- **5.NSBT.3** Read and write decimals in standard and expanded form. Compare two decimal numbers to the thousandths using the symbols $>$, $=$, or $<$.
- **5.NSBT.4** Round decimals to any given place value within thousandths

New Academic Vocabulary for This Unit

- exponents
- power(s) of 10
- thousandths

Prior Knowledge Required for this Unit

In 4th grade, place value of whole numbers focused on understanding math periods and the patterns found in those periods. (4.NSBT.1). In 5th grade, students apply the same pattern (math periods) while working with decimal numbers, using whole number exponents to when multiplying and dividing by a power of 10. This is the first time students have been introduced to the use of whole number exponents to denote powers of 10.

Students extend their understanding of the base-ten system to how numbers compare and how numbers round for decimal numbers to thousandths.

Subsequent Knowledge Related to this Unit

In Unit 9, students will use these skills when converting units within the metric system. As the year progresses, having a solid understanding of decimal numbers will strengthen their flexibility when working with fractions, decimals, and percents in problem-solving situations.

Students need to have a firm grasp of place value (whole numbers and decimal numbers) for future work with computing with numbers, exponents and scientific notation.

Relationship Among Standards in this Unit

The standards included in this unit provide students with a foundation to work flexibly within the base ten place value system.

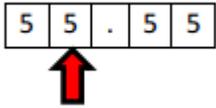
Potential Instructional Strategies/Lessons

Teacher Note: 5.NSBT.1 and 5.NSBT.2

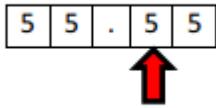
Standard 5.NSBT.1 calls for students to reason about the magnitude of numbers. “Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is $\frac{1}{10}$ th the size of the tens place. In fourth grade, students examined the relationships of the digits in numbers for whole numbers only. This standard extends this understanding to the relationship of decimal fractions. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons. Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.” (Excerpt from KATM Grade 5 flip book)

Example: (from KATM Grade 5 flip book)

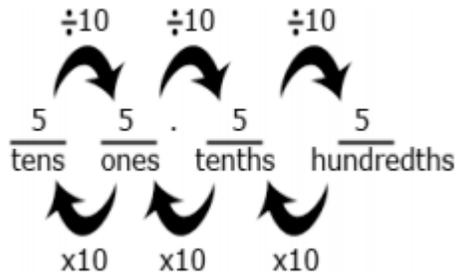
To extend this understanding of place value to their work with decimals, students use a model of one unit; they cut it into 10 equal pieces, shade in, or describe $\frac{1}{10}$ of that model using fractional language (“This is 1 out of 10 equal parts. So it is $\frac{1}{10}$ ”. I can write this using $\frac{1}{10}$ or 0.1”). They repeat the process by finding $\frac{1}{10}$ of a $\frac{1}{10}$ (e.g., dividing $\frac{1}{10}$ into 10 equal parts to arrive at $\frac{1}{100}$ or 0.01) and can explain their reasoning, “0.01 is $\frac{1}{10}$ of $\frac{1}{10}$ thus is $\frac{1}{100}$ of the whole unit.” In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.



The 5 that the arrow points to is $\frac{1}{10}$ of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is $\frac{1}{10}$ of 50 and 10 times five tenths.



The 5 that the arrow points to is $\frac{1}{10}$ of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is 10 times five hundredths.



Standard 5.NS.B.2 calls for students to have multiple experiences working with connecting the patterns in the number of zeroes of the product when multiplying a number by powers of 10. In addition, students should have multiple experiences exploring patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

Example:

$2.5 \times 10^3 = 2.5(10 \times 10 \times 10) = 2.5 \times 1,000 = 2,500$. Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right.

$$350 \div 10^3 = 350 \div 1,000 = 0.350 = 0.35$$

$$350 \div 10 = 35, 35 \div 10 = 3.5$$

$$3.5 \div 10 = 0.35, \text{ or } 350 \times \frac{1}{10}, 35 \times \frac{1}{10}, 3.5 \times \frac{1}{10}$$

“This will relate well to subsequent work with operating with fractions. This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10, the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left.”

Students might write:

- $36 \times 10 = 36 \times 10^1 = 360$
- $36 \times 10 \times 10 = 36 \times 10^2 = 3,600$
- $36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36,000$
- $36 \times 10 \times 10 \times 10 \times 10 = 36 \times 10^4 = 360,000$

Students might think and/or say:

**I noticed that every time, I multiplied by 10 I added a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left.*

**When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So I had to add a zero at the end to have the 3 represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones).*

Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense.

- $523 \times 10^3 = 523,000$ The place value of 523 is increased by 3 places.
- $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places.
- $52.3 \times 10^1 = 523$ The place value of 52.3 is decreased by one place.

(Examples from KATM Grade 5 flip book)

Possible Strategies:

Use a place value chart to show how place value increases to the left in a multi-digit number (5.NSBT.1)

https://learnzillion.com/lesson_plans/599-use-a-place-value-chart-to-show-how-place-value-increases-to-the-left-in-a-multi-digit-number

- In this LearnZillion lesson, students work with the idea that the tens place is ten times as much as the ones place, and the ones place is $\frac{1}{10}$ th the size of the tens place.

Understand how place value decreases with each shift to the right in a multi-digit number by using a place value chart (5.NSBT.1)

https://learnzillion.com/lesson_plans/602-understand-how-place-value-decreases-with-each-shift-to-the-right-in-a-multi-digit-number-by-using-a-place-value-chart

- In this LearnZillion lesson, students will work with the idea that the ones place is $\frac{1}{10}$ of the place to its left.

Kipton's Scale (5.NSBT.1)

<https://www.illustrativemathematics.org/content-standards/tasks/1562>

- This task is set in the context of weighing objects and bundles of 10, 100, and 1,000 objects, it helps students visualize that bundling 10 units of a given place value will create 1 unit of the next highest place value.

Multiply whole numbers by powers of 10 (5.NSBT.2)

https://learnzillion.com/lesson_plans/601-multiply-whole-numbers-by-powers-of-10-using-knowledge-of-place-value

- In this LearnZillion lesson, students learn to use whole number exponents to denote powers of ten.

Use place value to explain the pattern when a decimal is divided by a power of 10 (5.NSBT.2)

https://learnzillion.com/lesson_plans/603-use-place-value-to-explain-the-pattern-when-a-decimal-is-divided-by-a-power-of-10

- In this LearnZillion lesson, students divide powers of ten with decimal numbers. Students need to be provided with opportunities to explore this concept and come to this understanding; this should not just be taught as a procedure.

Multiplying Decimals by 10 (5.NSBT.2)

<https://www.illustrativemathematics.org/content-standards/tasks/1620>

- The purpose of this task is to help students understand and explain why multiplying a decimal number by 10 shifts all the digits one place to the left.

Marta's Multiplication Error (5.NSBT.2)

<https://www.illustrativemathematics.org/content-standards/tasks/1524>

- This task highlights a common misconception among students deriving the rules for multiplying a number by a power of 10.

Teacher Note: 5.NSBT.3 and 5.NSBT.4

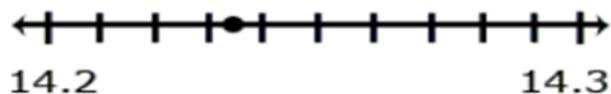
Standard 5.NSBT.3 requires students to read and write decimals in standard and expanded form including fractions. “Students should build on their work from Fourth Grade, where they worked with both decimals and fractions interchangeably. Expanded form is included to build upon work in 5.NSBT.1 and 5.NSBT.2 and deepen students’ understanding of place value. Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation. This investigation leads them to understanding equivalence of decimals ($0.8 = 0.80 = 0.800$).” (Excerpt from KATM Grade 5 flip book)

Example: Some equivalent forms of 0.72 are:

$\frac{72}{100}$	$\frac{7}{10} + \frac{2}{100}$
$7 \times \left(\frac{1}{10}\right) + 2 \times \left(\frac{1}{100}\right)$	$0.70 + 0.02$
$\frac{70}{100} + \frac{2}{100}$	0.720
$7 \times \frac{1}{10} + 2 \times \frac{1}{100} + 0 \times \frac{1}{1000}$	$\frac{720}{1000}$

Standard 5.NSBT.4 requires students to round decimals to any given place value within thousandths. “Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding. When rounding a decimal to a given place, students may identify the two possible answers, and use their understanding of place value to compare the given number to the possible answers.” (Excerpt from KATM Grade 5 flip book)

Example: Round 14.235 to the nearest tenth.



Students recognize that the possible answer must be in tenths thus, it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).

Possible Strategies:

Compare two decimals using a number line and comparison symbols (5.NSBT.3)

https://learnzillion.com/lesson_plans/314-compare-two-decimals-using-a-number-line-and-comparison-symbols#fndtn-lesson

- In this LearnZillion lesson, students will use their understanding of place value to order and compare decimal numbers using a number line.

Drawing Pictures to Illustrate Decimal Comparisons (5.NSBT.3)

<https://www.illustrativemathematics.org/content-standards/tasks/1801>

- The purpose of this task is for students to compare decimal numbers using pictures or diagrams.

Comparing Decimals on the Number Line (5.NSBT.3)

<https://www.illustrativemathematics.org/content-standards/tasks/1802>

- This task involves using number lines to compare decimal numbers. The numbers selected in this task are purposefully chosen to target student misconceptions.

Placing Thousandths on the Number Line (5.NSBT.3)

<https://www.illustrativemathematics.org/content-standards/tasks/1803>

- This task primarily deals with comparing decimal numbers on a number line. It also requires students to draw upon what they know about the base ten system.

Place Value Game (Greg Tang Math) (5.NSBT.3)

<http://gregtangmath.com/placevalue>

- In this online game, students match numbers to their place value. You can choose two different difficulty levels with whole numbers or decimals

Write decimals in expanded form (5.NSBT.3)

<https://learnzillion.com/lessons/3285-write-decimals-in-expanded-form>

- In this LearnZillion lesson students will learn how to read and write numbers to the thousandths in expanded notation using base ten blocks.

Are these equivalent to 9.52? (5.NSBT.3)

<https://www.illustrativemathematics.org/content-standards/tasks/1813>

- The purpose of this Illustrative Math task is to help students develop the understanding that a single base-ten number can be represented in many different ways.

Round Decimals to Any Place Using Number Lines (5.NSBT.4)

<https://learnzillion.com/lessonsets/212-round-decimals-to-any-place-using-number-lines>

- 3 LearnZillion lessons (similar to powerpoint) focusing on rounding decimals to the nearest tenth and hundredth using number lines.

Rounding to Tenths and Hundredths (5.NSBT.4)

<https://www.illustrativemathematics.org/content-standards/tasks/1804>

- The purpose of this task is for students to use the position of a number on the number line to round the number without knowing its exact value.

Resources

EngageNY Grade 5 Module 1: Place Value and Decimal Fractions

<https://www.engageny.org/resource/grade-5-mathematics-module-1>

- In this unit, students' understanding of the patterns in the base ten system are extended from Grade 4's work with place value of multi-digit whole numbers and decimals to hundredths to the thousandths place. This unit includes math background, teacher notes, student activities, practice pages, and a variety of assessments.

CCGPS (Georgia) 5th Grade, Unit 2 Decimals

https://www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_5_Unit2Framework.pdf

- Georgia's Decimal unit. Includes many activities, math background, and assessments.

Rounding Decimals (5.NSBT.5)

<http://studyjams.scholastic.com/studyjams/jams/math/decimals-percents/rounding-decimals.htm>

- Colorful, interactive, step by step directions for rounding decimals from Scholastic Study Jams. Includes a brief assessment after the lesson.

Sample Formative Assessment Tasks/Questions

Decimals Performance Assessment Task (5.NSBT.3 and 5.NSBT.4)

<http://www.insidemathematics.org/assets/common-core-math-tasks/decimals.pdf>

- This task challenges a student to use knowledge of place value system to represent and compare rational numbers. A student must use place value understanding to justify or explain how to order a set of decimal values. A student must be able to make sense of rational values to generate numbers falling between two values.

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.NBT.1> (assessing 5.NSBT.1)

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.NBT.2> (assessing 5.NSBT.2)

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.NBT.3> (assessing 5.NSBT.3)

<https://grade5commoncoremath.wikispaces.hcpss.org/Assessing+5.NBT.4> (assessing 5.NSBT.4)

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