# South Carolina College- and Career-Ready Standards for Mathematics

## Support Document

## 4th Grade

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As support for implementing the *South Carolina College- and Career-Ready Standards for Mathematics*, the standards for each grade K-5 have been grouped into possible units. In the *Table of Contents* below, the titles for those possible units are listed in a column under each grade. To see which standards are addressed in each unit for this grade and to read a brief description of the focus for each unit in this grade, click on the *Overview of Units* in the *Table of Contents*. The completed units for this grade are hyperlinked from/to the *Table of Contents* and the *Overview of Units*. The purpose of this document is to provide guidance as to how all the standards at this grade may be grouped into units and how those units might look. Since this document is merely guidance, districts should implement the standards in a manner that addresses the district curriculum and the needs of students.

## Acknowledgments

"Jean Baptiste Massieu, famous deaf educator, made a statement that is now considered a French proverb. *Gratitude is the memory of the heart.* Indeed, appreciation comes when you feel grateful from the depths of your heart. The head keeps an account of all the benefits you received and gave. But the heart records the feelings of appreciation, humility, and generosity that one feels when someone showers you with kindness." It is with sincere appreciation that we humbly acknowledge the dedication, hard work and generosity of time provided by the following individuals who are making the K-5 Mathematics Support Document possible. (http://quotations.about.com/od/ThankYou/a/Gratitude-Quotes.htm)

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| | *Table of Contents for Grades K-5* | | | | | |
|---|---|---|---|---|---|---|
| | **K** | **1st** | **2nd** | **3rd** | **4th** | **5th** |
| | **Overview of Units** | **Overview of Units** | **Overview of Units** | **Overview of Units** | **Overview of Units** | **Overview of Units** |
| **Unit 1** | Counting and Cardinality | Composing and Decomposing Numbers Through 10 | Place Value Concepts | Conceptual Understanding of Multiplication & Division | Place Value, Addition, & Subtraction with Whole Numbers | Expressions, Equations, & the Coordinate Plane |
| **Unit 2** | Understanding Relationship of Counting and Quantity | Addition and Subtraction Strategies | Developing Concepts Addition/ Subtraction | Place Value | Algebraic Thinking | Place Value |
| **Unit 3** | Count and Compare | Understanding Place Value | Fluency and Word Problems Addition/ Subtraction | Addition & Subtraction | Multiplication & Division of Whole Numbers | Operations with Whole and Decimal Numbers |
| **Unit 4** | Composing and Decomposing Numbers | Applying Place Value Concepts | Developing an Understanding of Multiplication | Application of Multiplication & Division | Fraction Equivalence | Adding and Subtracting Fractions |
| **Unit 5** | Addition and Subtraction | Comparisons and Data | Attributes Polygons and Fractional Parts | Conceptual Understanding of Fractions | Adding, Subtracting, & Multiplying with Fractions | Multiplying with Fractions |
| **Unit 6** | Patterns and Positions | Geometry and Equal Shares | Measurement Length | Data Analysis | Decimal Concepts | Dividing with Fractions |
| **Unit 7** | Two Dimensional and Three Dimensional Geometry | Measurement, Time, and Money | Measurement Time and Money | Identification and Classification of Geometric Shapes | Conversions & Problem Solving with Measurement | Classifying 2D Shapes |
| **Unit 8** | Foundations of Measurement | | Creating and Understanding Data | Problem Solving with Measurement | Geometric Classifications & Line Symmetry | Perimeter, Area, and Volume |
| **Unit 9** | Understanding Graphs and Data | | | Fluency with Multiplication & Division | Angle Measurement | Converting Measurements within a Single System |

**Grade Four Overview of Units**

*Click here to return to the K-5 Table of Contents*

| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 | Unit 7 | Unit 8 | Unit 9 |
|---|---|---|---|---|---|---|---|---|
| **Place Value, Addition, & Subtraction of Whole Numbers** | **Algebraic Thinking** | **Multiplication & Division of Whole Numbers** | **Fraction Equivalence** | **Adding, Subtracting, & Multiplying with Fractions** | **Decimal Concepts** | **Conversions & Problem Solving with Measurement** | **Geometric Classifications & Line Symmetry** | **Angle Measurement** |
| **Standards** | **Standards** | **Standards** | **Standards** | **Standards** | **Standards** | **Standards** | **Standards** | **Standards** |
| 4.NSBT.1<br>4.NSBT.2<br>4.NSBT.3<br>4.NSBT.4 | 4.ATO.1<br>4.ATO.2<br>4.ATO.3<br>4.ATO.4<br>4.ATO.5 | 4.NSBT.5<br>4.NSBT.6<br>4.ATO.3<br>4.ATO.5 | 4.NSF.1<br>4.NSF.2<br>4.NSF.5 | 4.NSF.3<br>4.NSF.4<br>4.NSF.5<br>4.MDA.4 | 4.NSF.6<br>4.NSF.7 | 4.MDA.1<br>4.MDA.2<br>4.MDA.3<br>4.MDA.8 | 4.G.1<br>4.G.2<br>4.G.3<br>4.G.4<br>4.ATO.5 | 4.MDA.5<br>4.MDA.6<br>4.MDA.7 |
| **Unit Focus** | **Unit Focus** | **Unit Focus** | **Unit Focus** | **Unit Focus** | **Unit Focus** | **Unit Focus** | **Unit Focus** | **Unit Focus** |
| Students examine the structure and patterns within the base ten system. They then use this knowledge to develop fluency with addition and subtraction of whole numbers. | To extend algebraic reasoning, students employ flexible thinking with multiplication and division to solve a variety of real-world problems. | Students employ a variety of strategies to efficiently multiply and divide multi-digit numbers. | Students strengthen their fraction sense by using a variety of models and strategies, such as the multiplicative identity element in fraction form, to generate and compare equivalent fractions. | Students use a variety of models when adding and subtracting fractions with like denominators and when multiplying a whole number and a fraction to solve real-world problems. | Students write decimals as fractions and use concrete and visual models to compare and order decimal numbers. | Students convert measurements within a single system and solve real-world problems involving a variety of measurement concepts. | Students learn specific geometric attributes, such as parallel and perpendicular lines, and use those attributes to classify shapes. The concept of line symmetry is introduced. | Students create and measure angles using a protractor. They also solve real-world problems involving unknown angle measures. |

| **Grade 4 Math Unit 1** | Return to Table of Contents | Return to Fourth Grade Overview |
|---|---|---|

Place Value, Addition, & Subtraction of Whole Numbers

**Content Standards with Clarifying Notes**

*Open Bullets Indicate Clarifying Notes*

- **4.NSBT.1** Understand that, in a multi-digit whole number, a digit represents ten times what the same digit represents in the place to its right.
  - The focus of this standard is on the relationship between the value of digits rather than the place value itself.
- **4.NSBT.2** Recognize math periods and number patterns within each period to read and write in standard form large numbers through 999,999,999.
  - When students understand math periods and the number patterns with each period, they can work with large numbers with greater ease and flexibility.
- **4.NSBT.3** Use rounding as one form of estimation and round whole numbers to any given place value.
  - Do not just start with "the rule" when rounding. The number line is a great tool to introduce rounding and it will lead to students discovering the rule on their own. Students may choose a vertical or horizontal line.
- **4.NSBT.4** Fluently add and subtract multi-digit whole numbers using strategies to include a standard algorithm.
  - Fluency means accuracy and efficiency (using a reasonable amount of steps and time), and flexibility (using a variety of strategies such as the distributive property, decomposing and recomposing numbers, etc.)

**New Academic Vocabulary for This Unit**

- math periods
- millions
- algorithm
- sum
- addend
- subtrahend
- difference
- minuend

**Prior Knowledge Required for This Unit**

In 3rd grade, students read and wrote numbers through 999,999 in standard and expanded forms (3.NSBT.4), rounded whole numbers to the nearest 10 or 100 (3.NSBT.1), and added and subtracted numbers fluently to 1,000 (3.NSBT.2).

**Subsequent Knowledge Related to This Unit**

In the Multiplication & Division of Whole Numbers Unit (Unit 3), students will use rounding to estimate products and quotients in order to check for reasonableness. Furthermore, students will use their knowledge of place value to multiply and divide using various strategies. Later, when students are introduced to decimals for the first time, they will extend their knowledge of the place value system to include tenths and hundredths.

**Relationship Among Standards in This Unit**

The primary focus of this unit is for students to develop fluency with addition and subtraction. First, students deepen their understanding of place value with an emphasis on math periods within a number and the relationship to other periods in order to read and write large numbers. Next, students use this understanding of the place value system when rounding numbers as a method for estimating, initially using number lines, with the ultimate goal of moving away from the visual model. Finally, students apply their knowledge of place value, as well as estimating, to add and subtract fluently and flexibly with any size number.

**Potential Instructional Strategies/Lessons**

**Teacher Note:** 4.NSBT.1

"This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with" *(*excerpt from KATM Grade 4 flip book). Students need to understand the relationship between the base ten number system and place value so that they can explain that the value of a digit changes depending on its location in a number. Understanding how the value of a digit changes depending on where it is located in a number is essential. Multiplying or dividing a number by 10 changes the value of the digit by one whole place value position in our base ten number system.

While the focus of 4.NSBT.1 is on the multiplicative relationship between the place value of two digits in a number and will be dealt with more in the multiplication and division unit (Unit 3), this standard is included in this unit to emphasize the importance of using place value as a strategy for adding and subtracting.

**Introductory Activities:**

**Learn Zillion - Multiply by Powers of 10**

https://learnzillion.com/lessons/805-multiply-by-powers-of-10 and https://drive.google.com/file/d/0B2e1qHKMDxlgV012UTVlcEFoalE/edit

- This lesson explores the relationship between numbers on a place value chart and the activity helps students apply their new knowledge.

**Learn Zillion - Understand Relationships between Digits and their Place Value**

https://learnzillion.com/lessons/516-understand-relationships-between-digits-and-their-place-value

- This lesson explores the relationship between base ten blocks and numbers on a place value chart.

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**Teacher Note:** 4.NSBT.2

Students worked with numbers within 1,000 in 2nd and within 1,000,000 in 3rd. In 4th grade, students should be able to apply what they learned in those grades to work with larger numbers including billions. Contextual problems might include populations.

"Provide multiple opportunities in the classroom setting and use real-world context for students to read and write multi-digit whole numbers focusing on the periods within a number as the basis for reading and writing. Students also need to create numbers that meet specific criteria. For example, provide students with cards numbered 0 through 9. Ask students to select 4 to 6 cards; then, using all the cards make the largest number possible with the cards, the smallest number possible and the closest number to 5000 that is greater than 5000 or less than 5000.

There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes (see below) and vertical addition method (see below).

| MILLIONS | | | THOUSANDS | | | ONES | | |
|---|---|---|---|---|---|---|---|---|
| Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
| | | | | | | | | |

For the number one thousand, seventy-two students could write out each value vertically and then add the values.

$$\begin{array}{r} 1000 \\ 70 \\ +\quad 2 \\ \hline 1{,}072 \end{array}$$

Another misconception is that students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole" (excerpt from KATM Grade 4 Flip Book). This most likely occurs because students have been taught to line up the digits to determine which is greater rather than to focus on the place value of the digits in a number.

**Introductory Activities:**

**Georgia Department of Education - Building 1,000**
https://grade4commoncoremath.wikispaces.hcpss.org/file/view/4.NBT.1_4.NBT.2_Building1000.pdf/457296196/4.NBT.1_4.NBT.2_Building1000.pdf

- This activity could be used as a formative assessment task to determine students' level of understanding on the 3rd grade concepts of place value.

**Illuminations - Count on Math: Making Your First Million**
http://illuminations.nctm.org/Lesson.aspx?id=4018

- In this activity, students attempt to identify the concept of a million by working with smaller numerical units, such as blocks of 10 or 100, and then expanding the idea by multiplication or repeated addition until a million is reached. Additionally, they use critical thinking to analyze situations and to identify mathematical patterns that will enable them to develop the concept of very large numbers.

**Teacher Note:** 4.NSBT.3

"Rounding numbers should result in using number sense not just following a rule. Students should apply their understanding of number lines to help them round numbers. When thinking about tens, numbers with the one's digit of 1, 2, 3, and 4 are closer to the lower multiple of ten. Numbers with the one's digit of 6, 7, 8, and 9 are closer to the greater multiple of ten. Since 5 is equidistant from both multiples, it is rounded to the greater multiple of ten. This same concept applies to rounding to the nearest hundred or thousand. When teaching rounding, it is helpful for students to use a number line to visualize the process" (John SanGiovanni, Howard County Schools). For example, if asked to round 1,768 to the nearest thousand, draw a number line and label the endpoints 1000 and 2000. Have the students place the number 1,768 on the number line and point out that it is closer to 2000 so 1,768 rounds to 2000.

**Introductory Activity:**
**Illustrative Mathematics - Rounding to the Nearest Thousand**
https://www.illustrativemathematics.org/content-standards/tasks/1807

- This task introduces students to rounding larger numbers using a number line.

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**Teacher Note:** 4.NSBT.4

"Students have had exposure to many ways of adding and subtracting whole numbers. The standard algorithm starts with adding the ones and regrouping as necessary. Some students may be comfortable starting with the larger place value – this should be permitted if it makes sense to them. A mixture of problems requiring and not requiring regrouping should be given to students. Some students may use an open number line when adding and subtracting and this should be permitted" (John SanGiovanni, Howard County Schools). Students should be able to explain why their algorithm works. When explaining, students should know that it is mathematically possible to subtract a larger number from a smaller number, but that their work in 4th grade with whole numbers does not allow this as the difference would result in a negative number.

"A crucial theme in multi-digit arithmetic is encouraging students to develop strategies that they understand, can explain, and can think about, rather than merely follow a sequence of directions, rules or procedures that they don't understand. It is important for students to have seen and used a variety of strategies and materials to broaden and deepen their understanding of place value before they are required to use standard algorithms. The goal is for them to understand all the steps in the algorithm, and they should be able to explain the meaning of each digit. Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm.

This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow. Sometimes students benefit from 'being the teacher' to an imaginary student who is having difficulties applying standard algorithms in addition and subtraction situations. To promote understanding, use examples of student work that have been done incorrectly and ask students to provide feedback about the student work. It is very important for some students to talk through their understanding of connections between different strategies and standard addition and subtractions algorithms. Give students many opportunities to talk with classmates about how they could explain standard algorithms. Think-Pair-Share is a good protocol for all students.

Often students mix up when to 'carry' and when to 'borrow'. Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits. If students are having difficulty with lining up similar place values in numbers as they are adding and subtracting, it may be helpful to have them write their calculations on grid paper or lined notebook paper with the lines running vertical. This assists the student with lining up the numbers more accurately" (excerpt from KATM Grade 4 Flip Book).

**Introductory Activities:**

Illustrative Mathematics - **To Regroup or Not to Regroup**
https://www.illustrativemathematics.org/content-standards/tasks/1189

- This is an instructional task that makes students think more deeply about the regrouping process used in some subtraction problems. Since this task deals with three-digit numbers, it would be a good starting point to make sure students fully understand the subtraction concept before moving on to larger numbers.

Georgia Department of Education - **Making Sense of the Algorithm (Subtraction)**
https://grade4commoncoremath.wikispaces.hcpss.org/file/view/Making%20Sense%20of%20the%20Algorithm.pdf/402793860/Making%20Sense%20of%20the%20Algorithm.pdf

- This task allows students to make sense of the standard algorithm for subtraction. It is important you allow them to grapple with the strategies used by the student in the task. Through this grappling, students make sense of what she did to solve each problem. Through classroom discussion, student understanding will be shared and developed.

**Resources**

Engage NY Grade 4 Mathematics Module 1
https://www.engageny.org/resource/grade-4-mathematics-module-1

- This module addresses Place Value, Rounding, Addition, & Subtraction. It includes math background, teacher notes, student activities, practice pages, and a variety of assessments.

CCGPS Frameworks 4th Unit 1
www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_4_Unit1Framework.pdf

- This unit includes performance tasks and formative assessment lessons.

**Interactive Sites for Education - Place Value**
http://interactivesites.weebly.com/place-value.html

- This site has multiple interactive games that students can use to practice place value skills.

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**K-5 Math Teaching Resources - Place Value Problems**
http://www.k-5mathteachingresources.com/support-files/place-value-problems.pdf

- This provides practice for students on 4.NSBT.1.

**Howard County Schools Resources for 4.NSBT.1**
https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-1-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

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***Literature Connections***

***How Much is a Million?*** **by David Schwartz**

- This book helps children conceptualize complex numbers in a fun, humorous way.

***How Big is a Million?* by Anna Milbourne**

- This book helps children understand a million through the eyes of a penguin.

***On Beyond a Million: An Amazing Math Journey* by David M. Schwartz**

- Real-life examples provide plenty of fun facts, such as how much popcorn Americans eat in one year, or how many hairs are on a square inch of a person's head. Along with the fun comes some learning, as this counting book helps kids understand our number system.

***Count to a Million* by Jerry Pallotta**

- Although some may have their doubts, by using basic math grouping skills, readers will find themselves counting higher than they ever thought possible.

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**Howard County Schools Resources for 4.NSBT.2**

https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-1-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and web resources. Note that SC standards do not include comparing numbers.

**Bridges in Mathematics - Place Value to Millions**

http://bridges1.mathlearningcenter.org/media/Bridges_Gr4_OnlineSupplement/B4SUP-A3_NumPlVal_0409.pdf

- This provides practice for students on 4.NSBT.2.

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**National Library of Virtual Manipulatives - Place Value Number Lines**

http://nlvm.usu.edu/en/nav/frames_asid_334_g_1_t_1.html?from=topic_t_1.html

- This virtual manipulative gives students practice with placing numbers on a number line, in preparation for using number lines for rounding.

**Illustrative Mathematics - Rounding to the Nearest 100 and 1000**

https://www.illustrativemathematics.org/content-standards/tasks/1806

- This task introduces students to rounding using a number line.

**Bridges in Mathematics - Estimating to Add and Subtract**
http://bridges1.mathlearningcenter.org/media/Bridges_Gr5_OnlineSupplement/B5SUP-A1_NumEstAddSub_0509.pdf

- This provides practice for students on 4.NSBT.3 and 4.NSBT.4.

**Howard County Schools Resources for 4.NSBT.3**
https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-1-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**Matific - Rounding up to 10,000**
https://www.matific.com/us/en-us/curriculum?episode=RoundToTheNearestPowerOfTenUpTo10000G4NBT

- This provides practice for students on 4.NSBT.3.

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**Howard County Schools Resources for 4.NSBT.4**
https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-1-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**K-5 Math Teaching Resources - Adding and Subtracting Multi-digit Numbers**
http://www.k-5mathteachingresources.com/support-files/adding-and-subtracting-multi-digit-whole-numbers.pdf

- This provides practice for students on 4.NSBT.4.

**Sample Formative Assessment Tasks/Questions**

- https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-1-assessment-tasks (Assessing 4.NSBT.1)
- https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-2-assessment-tasks (Assessing 4.NSBT.2, but note that the SC standard does not include comparing numbers.)
- https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-3-assessment-tasks (Assessing 4.NSBT.3)
- https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-4-assessment-tasks (Assessing 4.NSBT.4)

Return to Table of Contents

Algebraic Thinking

**Content Standards with Clarifying Notes**

*Open Bullets Indicate Clarifying Notes*

- **4.ATO.1** Interpret a multiplication equation as a comparison (e.g. interpret 35 = 5x7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.) Represent verbal statements of multiplicative comparisons as multiplication equations.
  - "A multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., 'a is n times as much as b'). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times. Students should be given many opportunities to write and identify equations and statements for multiplicative comparisons." (Excerpt from KATM Grade 4 Flip Book)
- **4.ATO.2** Solve real-world problems using multiplication (product unknown) and division (group size unknown, number of groups unknown).
  - Students should be exposed to all three types of problems.
- **4.ATO.3** Solve multi-step, real-world problems using the four operations. Represent the problem using an equation with a variable as the unknown quantity.
- **4.ATO.4** Recognize that a whole number is a multiple of each of its factors. Find all factors for a whole number in the range 1 – 100 and determine whether the whole number is prime or composite.
- **4.ATO.5** Generate a number or shape pattern that follows a given rule and determine a term that appears later in the sequence.
  - Third grade students identified a rule for patterns. Now, students will generate the pattern from the given rule.
  - Notice that the standard does not require students to infer or guess the underlying rule for a pattern, but instead asks them to generate a pattern from a given rule.
  - In this unit, the focus will be on number patterns. Shape patterns will be addressed in Unit 8.

**New Academic Vocabulary for This Unit**

- factor pairs
- multiples
- variable
- products
- prime numbers
- rule
- multiplicative comparisons
- composite numbers
- term

**Prior Knowledge Required for This Unit**

In third grade, students learned basic multiplication and division facts of products and dividends through 100 (3.ATO.7), used equations with symbols for the unknown (3.ATO.4), solved two-step problems using multiplication and division (3.ATO.8), and identified a rule for an arithmetic pattern (3.ATO.9).

**Subsequent Knowledge Related to This Unit**

In the later Multiplication & Division Unit, students will encounter multiplicative comparisons and real world problems with larger numbers. Furthermore, they will learn algorithms for multi-digit multiplication and division. They will also generate numbers for more difficult patterns. Students' familiarity with multiplication concepts such as factors and multiples will also help them when working with division, equivalent fractions, and measurement conversions.

**Relationship Among Standards in This Unit**

The primary focus of this unit is for students to extend their algebraic reasoning. First, students will learn about multiplicative comparisons and how variables can be used within them. Next, students will gain familiarity with factors and multiples and then use this knowledge in problem solving situations. Students will also examine multiplication and division patterns and how they relate to problem solving.

Throughout the unit, students need experiences with many different types of real-world problems, presented in a variety of ways, in order to gain flexibility in problem solving. This work will continue in each of the math units throughout the school year.

**Potential Instructional Strategies/Lessons**

**Teacher Note:** 4.ATO.1 - 4.ATO.3

"Repeated addition and arrays are two ways of thinking about multiplication. Multiplication can also be thought about as multiple comparison

problems that involve a comparison of two quantities in which one is described as a multiple of the other. The relation between quantities is described in terms of how many times larger one is than the other. An example of multiplicative comparison is a 10-foot alligator is five times longer than a 2-foot alligator" (John SanGiovanni, Howard County Schools).

"Students need experiences that allow them to connect mathematical statements and number sentences or equations. This allows for an effective transition to formal algebraic concepts. They represent an unknown number in a word problem with a symbol. Word problems which require multiplication or division are solved by using drawings and equations.

Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as shown in the Common Multiplication and Division Situations Table (http://www.cpalms.org/uploads/docs/standards/mafs_table2.pdf) . They should use drawings or equations with a symbol for the unknown number to represent the problem.

Present multistep word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies" (excerpt from KATM Grade 4 Flip Book).

**Introductory Activities:**

**K-5 Math Teaching Resources - Multiplicative Comparison Problems**
http://www.k-5mathteachingresources.com/support-files/multiplication-equations-and-comparison-statements.pdf

- These practice cards help students link comparison sentences, models, and equations in multiplicative comparisons.

**Illustrative Mathematics - Comparing Money Raised**
https://www.illustrativemathematics.org/content-standards/tasks/263

- The purpose of this task is for students to solve three comparisons problems that are related by their context but are structurally different (product unknown, group size unknown, number of groups unknown).

**Teacher Note:** 4.ATO.2

When solving problems, students may use drawings and equations with a variable (a letter standing for the unknown quantity) for the unknown to represent the problem. Students should solve equal group problems and comparison problems that involve the unknown in all locations – product unknown (multiplication), group size unknown (partition division), how many in each group (division), number of groups of unknown (measurement

division), how many groups (division) and area problems that involve product unknown (multiplication) and side dimension unknown (division). Teachers should reference Common Multiplication and Division Situations Table (see Teacher Note above for link).

**Introductory Activities:**

**Thinking Blocks - choose Compare Quantities (One Step Model)**
http://www.mathplayground.com/tb_multiplication/thinking_blocks_multiplication_division.html

- This site has a tutorial on using the Thinking Blocks and then allows students to build models based on story problems.

**K-5 Math Teaching Resources - Multiplicative Comparison Problems 2**
http://www.k-5mathteachingresources.com/support-files/comparison-problems.pdf

- These practice cards take the previous cards a step farther and introduce word problems involving multiplicative comparisons.

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**Teacher Note:** 4.ATO.3

"Solving multi-step word problems requires the students to read and understand the context of the problem. Focus should be on the information given and what the problem is asking. Focus should not be on finding KEY words to indicate the four operations. When the focus is on key words, students tend to overlook the entire problem. They may misinterpret the problem because of the key word and use the incorrect operation" (John SanGiovanni, Howard County Schools).

Students need multiple experiences to solve multi-step, real-world problems, using all four operations. Students should discuss and then use various strategies. To check for reasonableness, they should also use various estimation strategies.

**Literature Connection:**
**Math Curse by Jon Scieszka**

- This book talks about the daily math problems that are encountered in everyday life and would be a great starting point for getting students thinking about real world problems.

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**Introductory Activities 4.ATO.3:**

**North Carolina Department of Education - Multistep Multiplication**
https://grade4commoncoremath.wikispaces.hcpss.org/file/view/4.OA.3_Multi-StepMultiplication.pdf/457301088/4.OA.3_Multi-StepMultiplication.pdf

- This lesson involves students estimating and solving two-step word problems using various strategies.

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**Teacher Note:** 4.ATO.4
"Students need to develop an understanding of the concepts of number theory such as prime numbers and composite numbers. This includes the relationship of factors and multiples. Multiplication and division are used to develop concepts of factors and multiples. Division problems resulting in remainders are used as counterexamples of factors. Review vocabulary so that students have an understanding of terms such as factor, product, multiples, and odd and even numbers.

Multiples: Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20). To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:
all even numbers are multiples of 2
all even numbers that can be halved twice (with a whole number result) are multiples of 4
all numbers ending in 0 or 5 are multiples of 5

Factors: Provide students with counters to find the factors of numbers. Have them find ways to separate the counters into equal subsets. For example, have them find several factors of 10, 14, 25 or 32, and write multiplication expressions for the numbers. Another way to find the factor of a number is to use arrays from square tiles or drawn on grid papers. Have students build rectangles that have the given number of squares. Knowing how to find factors for a number assists in developing reasonableness when dividing, problem solving, and simplifying fractions.

Prime and Composite: **Definitions of prime and composite numbers should not be provided, but determined after many strategies have been used in finding all possible factors of a number.** A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number. Students investigate whether numbers are prime or composite by
building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2

rectangles, 1 x 7 and 7 x 1, therefore it is a prime number)
finding factors of the number
Students should develop a process for finding factor pairs so they can do this for any number 1 – 100 with efficiency.
Example: Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

Starting with a number chart of 1 to 20, use multiples of prime numbers to eliminate later numbers in the chart. For example, 2 is prime but 4, 6, 8, 10, 12 . . . are composite. After working with the numbers 1 to 20, consider using a hundreds chart and have the students color code multiples of numbers. The color will help students see emerging patterns which they can discuss.
Encourage the development of rules that can be used to aid in the determination of composite numbers. For example, other than 2, if a number ends in an even number (0, 2, 4, 6 and 8), it is a composite number" (excerpt from KATM Grade 4 Flip Book).

**Introductory Activities 4.ATO.4:**
**Sieve of Eratosthenes**
https://grade4commoncoremath.wikispaces.hcpss.org/file/view/The+Sieve+of+Eratosthenes.pdf
http://www.visnos.com/demos/sieve-of-eratosthenes
The first link is a lesson in which students use color coding on a hundreds chart to identify the prime numbers between 1 and 100. The second link is a virtual sieve that is also color coded.

**Understanding Factors and Multiples (and Determining Prime and Composite Numbers)**
https://learnzillion.com/lessonsets/123-find-and-understand-factors-and-determine-if-a-number-is-a-multiple-of-a-given-number-for-whole-numbers-0-100
11 LearnZillion lessons (similar to PowerPoint) that provide strategies for students working with factors and multiples, prime and composite numbers.

**Factor Game**
http://mathsolutions.com/wp-content/uploads/978-1-935099-02-4_NL36_L1.pdf
- This partner game from Math Solutions is good for practice in identifying factors of given numbers.

**Illustrative Mathematics - The Locker Game**

https://www.illustrativemathematics.org/content-standards/tasks/938

- The purpose of this instructional task is for students to deepen their understanding of factors and multiples of whole numbers.

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**Teacher Note:** 4.ATO.5

"Students need to recognize patterns, extend patterns, describe patterns, and create patterns. Repeated patterns have a part or a core that repeats over and over. Examples of repeated patterns can include using shapes and numbers. You can describe a pattern by stating the rule it is following. Number Changing patterns are examples of number sequences. A sequence is a set of numbers that are arranged in a certain order. Each number in the sequence is called a term.

Examples:

Repeating pattern: 1,4,7, 1,4,7, 1,4,7, 1,4,7, 147, 1,4,7, 1,4,7

Changing pattern: 1,4,7,10,13,16,19,22,25,28,31,34,37" (John SanGiovanni, Howard County Schools).

"In order for students to be successful later in the formal study of algebra, their algebraic thinking needs to be developed. Understanding patterns is fundamental to algebraic thinking. Students have experience in identifying arithmetic patterns, especially those included in addition and multiplication tables. Contexts familiar to students are helpful in developing students' algebraic thinking.

Students should generate numerical or geometric patterns that follow a given rule. They should look for relationships in the patterns and be able to describe and make generalizations.

As students generate numeric patterns for rules, they should be able to "undo" the pattern to determine if the rule works with all of the numbers generated. For example, given the rule, "Add 4" starting with the number 1, the pattern 1, 5, 9, 13, 17, … is generated. In analyzing the pattern, students need to determine how to get from one term to the next term. Teachers can ask students, "How is a number in the sequence related to the one that came before it?", and "If they started at the end of the pattern, will this relationship be the same?" Students can use this type of questioning in analyzing numbers patterns to determine the rule.

Often, students think that results are random. There is no pattern. Another common misconception when students are working with repeating patterns is that they will often repeat what is given rather than looking at what "chunks" or part of the pattern is actually being repeated. Example:

Given the pattern 6,9,12,6,9,12,6,9,… If the student is asked “what is the next number in the pattern”, they may respond with “6” because they are returning to the beginning of the given pattern and repeat it from there. Students should be encouraged to look for the repeating set” (excerpt from KATM Grade 4 Flip Book).

**Introductory Activities 4.ATO.5:**

**Number and Shape Patterns**

https://learnzillion.com/lessonsets/195-generate-number-or-shape-patterns-that-follow-a-given-rule-and-identifying-pattern-features

- 9 LearnZillion lessons (similar to PowerPoint) that provide a variety of strategies and activities involving patterns.

**Developing Algebraic Thinking Using Manipulatives**

http://www.mathsolutions.com/documents/Developing_Algebraic_Thinking_i35.pdf

- Students use pattern blocks as a tool to explore patterns that grow and solve problems. Using the contexts of trees and fish that grow in consistent and predictable ways, students build, extend, describe, and represent patterns.

**Resources**

**CCGPS Frameworks 4th Unit 2**

https://www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_4_Unit2Framework.pdf

- Parts of this unit include performance tasks and formative assessment lessons.

**Howard County Schools Resources for 4.ATO.1**

https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-1-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**Howard County Schools Resources for 4.ATO.2**

https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-2-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**Howard County Schools Resources for 4.ATO.3**
https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-3-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**Math Playground - Solve it Math!**
http://www.mathplayground.com/wp_videos.html

- This site offers numerous multi-step problems for students to solve and then gives step by step video solutions. This site is best suited for struggling students.

**Math Playground - Word Problems with Katie**
http://www.mathplayground.com/WordProblemsWithKatie2.html

- This game gives the student multiplication and division single and multistep word problems to solve. This site is best suited for struggling students.

**Math Playground - Algebra Puzzles**
http://www.mathplayground.com/algebra_puzzle.html

- Students use multiplication and division skills to determine the value of the objects. This is a great way to practice multi-step problems solving and writing equations. Students can choose two different levels (beginner or advanced).

**North Carolina State Department of Education - Problem Solving Decks**
http://mathlearnnc.sharpschool.com/UserFiles/Servers/Server_4507209/File/Problem%20Solving%20Decks%20%28K-8%29/Problem%20Solving%20Deck%20B%20Student%20Sheets.pdf (student copy for showing work)
http://mathlearnnc.sharpschool.com/UserFiles/Servers/Server_4507209/File/Problem%20Solving%20Decks%20%28K-8%29/Problem%20Solving%20Deck%20B%20Cards.pdf

- These cards offer wonderful opportunities for thinking and solving problems.

**Port Angeles School District - Practice Problems and Strategies**
http://www.portangelesschools.org/students/grade-4-St.html#PSstrat

- This site has numerous problem-solving strategies and sample problems for each strategy.

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**Howard County Schools Resources for 4.ATO.4**
https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-4-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**BBC Skillswise - Factors and Multiples**
http://www.bbc.co.uk/skillswise/topic/multiples-and-factors

- This site has a video, games and practice sheets that can be printed out, and interactive quizzes.

**Bridges in Mathematics - Primes, Composites, and Common Factors**
http://catalog.mathlearningcenter.org/files/pdfs/SecB5SUP-A2_NumPrimes-201304.pdf

- This is two lessons, with student practice sheets, on identifying prime and composite numbers, as well as finding factors of numbers between 1 and 100.

**Illuminations - Factorize**
http://illuminations.nctm.org/Activity.aspx?id=3511

- This interactive tool allows students to build arrays in order to identify all the factors of a number.

**BrainPop - Sortify with Multiples and Factors**
https://www.brainpop.com/games/sortify-multiplication/

- This interactive game gives students number tiles and asks the student to sort them based on categories that the students choose.

**Math Live from Learn Alberta - Multiples, Factors, Primes, and Composites**
http://www.learnalberta.ca/content/me5l/html/math5.html?goLesson=2

- This is an interactive tutorial that also includes an activity sheet and assessment.

**Crickweb - Venn Diagram Comparing Multiples**
http://www.crickweb.co.uk/ks2numeracy-properties-and-ordering.html

- This interactive tool allows students to sort multiples of different numbers using a Venn Diagram.

**Calculation Nation - Factor Dazzle**

http://calculationnation.nctm.org/Games/Game.aspx?GameId=A0537FC6-3B08-4AFC-9AD6-0CC5E3BC9B86

- This game gives students practice with identifying factors.

***You Can Count on Monsters* by Richard Evan Schwartz**

- This book presents the concepts of prime numbers and factoring in a novel and colorful way.

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**Howard County Schools Resources for 4.ATO.5**

https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-5-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**Sample Formative Assessment Tasks/Questions**

- https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-1-assessment-tasks (assessing 4.ATO.1)
- https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-2-assessment-tasks (assessing 4.ATO.2)
- https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-3-assessment-tasks (assessing 4.ATO.3)
- https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-4-assessment-tasks (assessing 4.ATO.4)
- https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-5-assessment-tasks (Assessing 4.ATO.5 - Use Quarter 4 Tasks for numbers)

Return to Table of Contents

**Grade 4 Math Unit 3** Return to Table of Contents Return to Fourth Grade Overview

Multiplication & Division with Whole Numbers

**Content Standards with Clarifying Notes**

*Open bullets indicate clarifying notes.*

- 4.NSBT.5 Multiply up to a four-digit number by a one-digit number and multiply a two-digit number by a two-digit number using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using rectangular arrays, area models and/or equations.
  - This standard calls for students to multiply numbers using a variety of strategies.
- 4.NSBT.6 Divide up to a four-digit dividend by a one-digit divisor using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.
  - This standard calls for students to explore division through various strategies.
- 4.ATO.3 Solve multi-step, real-world problems using the four operations. Represent the problem using an equation with a variable as the unknown quantity.
  - Although this standard does not directly mention interpreting remainders, this is a critical skill that students must practice in a variety of problem-solving situations. "Students should not just think of remainders as "R3" or "left over." Remainders should be put in context and dealt with accordingly" (*Teaching Student-Centered Mathematics: Grades 3-5* by John A. Van de Walle, pg. 62). These would include dropping off the remainder, rounding up to the next whole number, sharing it as a fraction, or leaving it as leftover.
- 4.ATO.5 Generate a number or shape pattern that follows a given rule and determine a term that appears later in the sequence.
  - Third grade students identified a rule for patterns. Now, students will generate the pattern from the given rule.
  - Notice that the standard does not require students to infer or guess the underlying rule for a pattern, but instead asks them to generate a pattern from a given rule.
  - In this unit, the focus will be on number patterns. Shape patterns will be addressed in Unit 8.

**New Academic Vocabulary for This Unit**

- **area model for multiplication**
- **partial products algorithm**
- **partial quotients algorithm**

**Prior Knowledge Required for This Unit**

In third grade, students learned basic multiplication and division facts of products and dividends through 100 (3.ATO.7), multiplied one-digit whole numbers by multiples of 10 in the range 10-90 (3.NSBT.3), used equations with symbols for the unknown (3.ATO.4), solved two-step problems using multiplication and division (3.ATO.8), and identified a rule for an arithmetic pattern (3.ATO.9).

**Subsequent Knowledge Related to This Unit**

Use of the standard algorithm for multiplication, and understanding why it works, is an expectation in the 5th grade. Students will also apply the area model of multiplication when multiplying decimals and fractions in 5th grade.

In 5th grade, students will also divide up to a four-digit dividend by a two-digit divisor, using strategies based on place value, the properties of operations, and the relationship between multiplication and division.

**Relationship Among Standards in This Unit**

The primary focus of this unit is for students to utilize a variety of strategies to efficiently multiply and divide multi-digit numbers. First, students use their place value understanding to employ new strategies for multiplying multi-digit numbers. Next, students explore division as the inverse operation of multiplication, again relying on strategies that highlight their knowledge of place value. Finally, students apply their knowledge of multi-digit multiplication and division to patterns and real-world problems.

**Potential Instructional Strategies/Lessons**

**Teacher Note:** 4.NSBT.5

Students who develop flexibility in breaking numbers apart (decomposing numbers) have a better understanding of the importance of place value and the Distributive Property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication, work flexibly to solve word problems, and transfer this understanding when dividing whole numbers.

When asking students to gain understanding about multiplying larger numbers be sure to provide frequent opportunities to engage in mental math exercises. When doing mental math, it is difficult to even attempt to use a strategy that one does not fully understand. Also, it is a natural tendency to use numbers that are 'friendly' (multiples of 10) when doing mental math, and this promotes its understanding.

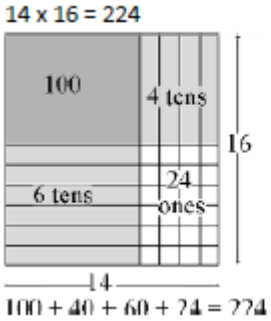
Example:

There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

| Student 1 | Student 2 | Student 3 |
|---|---|---|
| 25 x12 I broke 12 up into 10 and 2 and 25 x 10 = 250<br>25 x 2 = 50<br>250 +50 = 300 | 25 x 12 I broke 25 up into 5 groups of 5<br>5 x 12 = 60 I have 5 groups of 5 in 25<br>60 x 5 = 300 | 25 x 12<br>I doubled 25 and cut 12 in half to get 50<br>50 x 6 = 300 |

Use of place value and the Distributive Property are applied in the scaffold examples below.

- To illustrate 154 x 6 students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the Distributive Property, 154 X 6 = (100 + 50 + 4) x 6 = (100 x 6) + (50 X 6) + (4 X 6) = 600 + 300 + 24 = 924.
- The area model shows the partial products.




Using the area model, students first verbalize their understanding:

- 10 x 10 is 100
- 4 x 10 is 40
- 10 x 6 is 60, and
- 4 x 6 is 24.

They use different strategies to record this type of thinking.

14 x 16 = 224 Students explain this strategy with base 10 blocks, drawings, or numbers.

- Matrix Model: This model should be introduced after students have facility with the strategies shown above.

| | 20 | 5 | |
|---|---|---|---|
| 20 | 400 | 100 | 500 |
| 4 | 80 | 20 | 100 |
| | 480 + | 120 | 600 |

Example:

What would an array area model of 74 X 38 look like?

| | 70 | 4 |
|---|---|---|
| 30 | 70 × 30 = 2,100 | 4 × 30 = 120 |
| 8 | 70 × 8 = 560 | 4 × 8 = 32 |

2100 + 560 + 120 + 32 = 2,812

(Excerpt from KATM Grade 4 Flip Book)

**Introductory Activities:**

**Using Arrays to Multiply Bigger Numbers, Georgia Department of Education**

http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathArrays#task29

**Illuminations - Multiply and Conquer**

http://illuminations.nctm.org/Lesson.aspx?id=3210

- Students decompose 2-digit numbers, model area representations using the distributive property and partial product arrays, and align paper-and-pencil calculations with the arrays. The lessons provide conceptual understanding of what occurs in a 2-digit multiplication problem. Partial product models serve as transitions to understanding the standard multiplication algorithm.

**Learn Zillion - Use an Array to Multiply a Two Digit Number by a One Digit Number**

https://learnzillion.com/lesson_plans/6737-use-an-array-to-multiply-a-two-digit-number-by-a-one-digit-number#fndtn-lesson

- This lesson introduces multiplying a one digit number by a two digit by breaking apart their arrays into easily manageable multipliers.

**Learn Zillion - Use Place Value Understanding to Multiply Three and Four Digit Numbers**

https://learnzillion.com/lesson_plans/6751-use-place-value-understanding-to-multiply-three-and-four-digit-numbers#fndtn-lesson

- This lesson organize the steps to solve a multi-digit multiplication problem by setting up an area model of multiplication.

**Learn Zillion - Use an Area Model for Multiplication of Two-digit Numbers by Two-digit Numbers**

https://learnzillion.com/lesson_plans/6075-use-an-area-model-for-multiplication-of-two-digit-numbers-by-two-digit-numbers#fndtn-lesson

- This lesson demonstrates how to multiply a 2 digit number by another 2 digit number by applying understanding of the area model for multiplication.

**Teacher Note:** 4.NSBT.6

Two types of division problems are sharing (also call partitioning division) and repeated subtraction (also called measurement division.) In partition problems the whole is shared or distributed among a known number of sets/groups to determine the size of each. When the number of sets/groups is unknown, but the size of the equal sets/groups is known, the problems are measurement division.

Partition Division: Jake has 24 oranges. He wants to share them equally among his 6 friends. How many oranges does each friend get?

Measurement Division: Jake has 24 oranges. He puts them into baskets containing 4 oranges each. How many baskets did he use?

Not all division situations come out evenly. Students need to understand the meaning of remainders. Initial instruction should include using counters and arrays to show how sometimes counters are "left over". Remainders can have different effects on answers depending on the context of the problem. Students need to interpret the meaning of the remainder. The remainder may be discarded because it has no effect on the answer. The remainder can force the answer to go to the next whole number (John SanGiovanni, Howard County Schools).

In fourth grade, students build on their third grade work with division within 100. Students need many varied opportunities to develop their understandings by using problems in and out of context.

Examples:

A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils that she wants to put in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some

students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.

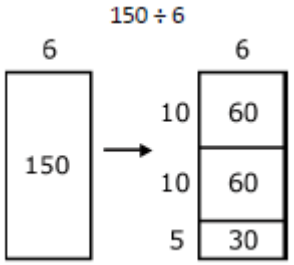
Using Place Value: 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4)

Using Multiplication: 4 x 50 = 200, 4 x 10 = 40, 4 x 5 = 20; 50 + 10 + 5 = 65; so 260 ÷ 4 = 65

| Student 1 | Student 2 | | | Student 3 |
|---|---|---|---|---|
| 592 divided by 8<br>There are 70 8's in 560<br>592 - 560 = 32<br>There are 4 8's in 32<br>70 + 4 = 74 | 592 divided by 8<br>I know that 10 8's is 80<br>If I take out 50 8's that is 400<br>592 - 400 = 192<br>I can take out 20 more 8's which is 160<br>192 - 160 = 32<br>8 goes into 32 4 times<br>I have none left<br>I took out 50, then 20 more,<br>then 4 more<br>That's 74 | 592<br>-400<br>192<br>- 160<br>32<br>-32<br>0 | <br>50<br><br>20<br><br>4<br> | I want to get to 592<br>8 x 25 = 200<br>8 x 25 = 200<br>8 x 25 = 200<br>200 + 200 + 200 = 600<br>600 - 8 = 592<br>I had 75 groups of 8 and<br>took one away, so there are<br>74 teams |

Using an Open Array or Area Model

After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.




150 ÷ 6 Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, 6 times what number is a number close to 150? They recognize that 6 x 10 is 60 so they record 10 as a factor and partition

the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.

2. Recognizing that there is another 60 in what is left they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6 x 5 is 30. They write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Students express their calculations in various ways:

a.

$$\begin{array}{r} 150 \\ -60(6 \times 10) \\ \hline 90 \\ -60(6 \times 10) \\ \hline 30 \\ -30(6 \times 5) \\ \hline 0 \end{array} \qquad 150 \div 6 = 10 + 10 + 5 = 25$$

b. $150 \div 6 = (60 \div 6) + (60 \div 6) + (30 \div 6) = 10 + 10 + 5 = 25$

(Excerpt from KATM Grade 4 Flip Book)

**Introductory Activities:**

**Math Steps - Division**

https://www.eduplace.com/math/mathsteps/4/d/4.division.ideas.html

- This lesson uses base-ten blocks to introduce, and then further develop, the concept of division beyond basic facts.

**Learn Alberta - Division of Whole Numbers**

http://www.learnalberta.ca/content/me5l/html/math5.html?goLesson=9

- This interactive lesson looks at estimating quotients, finding quotients using base-ten blocks, and then transferring this knowledge to the traditional algorithm.

**Learn Zillion - Divide Two-digit Dividends using Friendly Multiples**

https://learnzillion.com/lesson_plans/8961-divide-two-digit-dividends-using-friendly-multiples#fndtn-lesson

- This lesson teaches students to divide 2 digit dividends by using friendly multiples of the divisor (partial quotients).

**Teacher Notes:** 4.ATO.3 and 4.ATO.5

These standards were introduced in the previous unit. See Unit 2 for explanations and introductory activities. In this unit, these standards should be addressed with larger numbers and more complex tasks.

**Resources**

**Engage NY Grade 4 Mathematics Module 3**
https://www.engageny.org/resource/grade-4-mathematics-module-3

- This module stresses place value understanding and visual representations to solve multiplication and division problems with multi-digit numbers. It includes math background, teacher notes, student activities, practice pages, and a variety of assessments.

**Grade 4, Mission 3: Multiplying and Dividing Big Numbers**
https://www.zearn.org/curriculum/9

- On the Zearn website, there are projectables, practice and more that support EngageNY (see unit above).

**HCPS Frameworks 4th Unit 2**
https://www.georgiastandards.org/Georgia-Standards/Frameworks/4th-Math-Unit-2.pdf

- This unit includes performance tasks and formative assessment lessons.

**Howard County Schools Resources for 4.NSBT.5**
https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-5-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**Bridges in Mathematics - Multi-digit Multiplication**
http://bridges1.mathlearningcenter.org/media/Bridges_Gr4_OnlineSupplement/B4SUP-A5_NumOpMDigitMulti_0409.pdf

**Bridges in Mathematics - Estimating to Multiply & Divide**
http://bridges1.mathlearningcenter.org/media/Bridges_Gr4_OnlineSupplement/B4SUP-A4_NumEstMultDiv_0409.pdf

**Matific Expanded Algorithm (free for teachers)**
https://www.matific.com/us/en-us/grades/4G?episode=MultiplicationGridExpandedExpanded

- This interactive lesson relates the area model of multiplication to the partial products algorithm. There are additional lessons at this site that could follow this one.

**BBC Skillswise - The Amoeba Multiplication Game**
http://downloads.bbc.co.uk/skillswise/maths/ma12pape/game/ma12pape-game-written-multiplication/multiplication.swf

- This game provides practice with the area model of multiplication in a step-by-step format.

**K-5 Math Teaching Resources - Doubling and Halving**
http://www.k-5mathteachingresources.com/support-files/multiplication-strategy-doubling-and-halving.pdf

- This resource introduces the strategy of doubling one factor and halving the other to help simplify and solve problems.

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**Howard County Schools Resources for 4.NSBT.6**
https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-6-about-the-math-learning-targets-and-rigor

https://grade4commoncoremath.wikispaces.hcpss.org/4.NBT.6

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**Illuminations The Quotient Cafe**
http://illuminations.nctm.org/activity.aspx?id=4197

- This interactive tool gives students practice with the partial quotients algorithm.

**Illustrative Mathematics Mental Division Strategy**

https://www.illustrativemathematics.org/content-standards/tasks/1774

- This task would be ideal to help students develop mental strategies to think about division during instruction. This strategy is often referred to as using "compatible numbers."

**Sample Formative Assessment Tasks/Questions**

- https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-5-assessment-tasks (assessing 4.NSBT.5)
- http://www.jennyray.net/uploads/1/2/9/7/12975776/multi-digit_multiplication_strategies_grade_4_revised_may_2012.pdf (assessing 4.NSBT.5)
- https://hcpss.instructure.com/courses/107/pages/4-dot-nbt-dot-6-assessment-tasks (assessing 4.NSBT.6)
- https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-3-assessment-tasks (assessing 4.ATO.3)
- https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-5-assessment-tasks (assessing 4.ATO.5)

Return to Table of Contents

Fraction Equivalence

**Content Standards with Clarifying Notes**

Open bullets indicate clarifying notes.

- 4.NSF.1 Explain why a fraction (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100), a/b, is equivalent to a fraction, n×a/nxb, by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
  - This standard refers to visual fraction models. This includes area models, linear models (number lines) or it could be a collection/set model.
- 4.NSF.2 Compare two given fractions (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100) by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2 and represent the comparison using the symbols >, =, or <.
  - This standard should focus on comparing fractions using number sense, instead of using strategies (methods) such as the butterfly method which does not contribute to students' fraction understanding.
- 4.NSF.5 Express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100 and use this technique to add two fractions with respective denominators of 10 and 100.
  - In this unit, the equivalent fraction section will be addressed. Addition and subtraction of the fractions will occur in the next unit.

**New Academic Vocabulary for This Unit**

**All the following vocabulary was introduced in previous grade levels.**

- unit fraction
- set model
- area model
- linear model
- numerator
- denominator
- whole number
- mixed number
- fraction equivalence
- partition (verb)

**Prior Knowledge Required for This Unit**

In third grade, students learned about fraction equivalence (with the denominators 2, 3, 4, 6, 8, 10) by demonstrating an understanding that:
a. two fractions are equal if they are the same size, based on the same whole, or at the same point on a number line;
b. fraction equivalence can be represented using set, area, and linear models;
c. whole numbers can be written as fractions (e.g., 4 = 4 /1 and 1 = 4/ 4 );
d. fractions with the same numerator or same denominator can be compared by reasoning about their size based on the same whole (3.NSF.2).
Students will also need their knowledge of multiplication and division when moving from recognizing concrete models of equivalent fractions to generating equivalent fractions mathematically.

**Subsequent Knowledge Related to This Unit**

Students will use their understanding of fractions and fraction equivalence in the next unit (Adding, Subtracting, and Multiplying with Fractions) and when working with Decimal Concepts in Unit 6.

**Relationship Among Standards in This Unit**

The focus of this unit is fraction equivalence - focusing on visual models and number sense to build a strong foundation. Students work with a variety of models to create equivalent fractions (including using benchmark fractions) and to compare fractions (including mixed numbers).

**Potential Instructional Strategies/Lessons**

**Teacher Notes for 4.NSF.1:**
Equivalent fractions are fractions that represent equal value. They are numerals that name the same fractional number. When we say that fractions are equivalent there is an underlying assumption that the wholes are the same size. Students need to understand this concept. A focus question should be "Are the wholes the same size?" Fraction manipulatives should be used when first introducing the concept of equivalency. Students should explore, using fraction strips or pattern blocks, which fractions are equivalent before moving to a procedure to find equivalent fractions (John SanGiovanni, Howard County Schools).

Common Misconceptions: Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing ½ to sixths. They would multiply the denominator by 3 to get ⅙, instead of multiplying the numerator by 3 also.

Their focus is only on the multiple of the denominator, not the "whole fraction". It is important that students use a fraction in the form of one such as 3/3 so that the numerator and denominator do not contain the original numerator or denominator.
(Excerpt from KATM Grade 4 Flip Book)

**Teaching Channel introduction and model lesson on equivalent fractions**
https://www.teachingchannel.org/videos/understanding-modeling-and-creating-equivalent-fractions-core-challenge

**Fraction Progression from Illustrative Mathematics**
https://www.illustrativemathematics.org/progressions

- Fractions Progression Module consisting of videos paired with illustrative tasks to help teachers better understand the development of concepts and skills around fractions from grades 3-5.

**Introductory Activities:**
**Illustrative Mathematics - Explaining Fraction Equivalence with Pictures**
https://www.illustrativemathematics.org/content-standards/tasks/743

- The purpose of this task is to provide students with an opportunity to explain fraction equivalence through visual models in a particular example.

**Illustrative Mathematics - Fractions and Rectangles**
https://www.illustrativemathematics.org/content-standards/tasks/881

- The primary goal of this task is for students to use pictures to explain the equivalence between 3/12 and 1/4. This task helps set students on the path to understanding that if you divide the numerator and denominator by the same whole number, you get an equivalent fraction. It is as if we are "putting smaller pieces together to form bigger pieces," as opposed to subdividing the pieces as seen in *Explaining Fraction Equivalence with Pictures* (task above).

**Illuminations - Equivalent Fractions**
http://illuminations.nctm.org/Activity.aspx?id=3510

- This technology connection is an activity to create equivalent fractions by dividing shapes and matching them to number line locations.

**Learn Alberta - Equivalent Fractions**
http://www.learnalberta.ca/content/me5l/html/math5.html?goLesson=4

- This lesson on equivalent fractions includes an interactive video, an activity sheet, and an assessment task.

**PBS - Thirteen Ways at Looking at One-Half**

http://pbskids.org/cyberchase/math-games/thirteen-ways-looking-half/

- This interactive tool shows students how a simple fraction such as ½ can be represented in many ways using the same shape.

**Teacher Notes for 4.NSF.2:**

There are several ways to compare fractions. Before moving to the procedure of finding a common denominator, fraction sense, and reasoning should be the focus.

- One way of comparing fractions is to look at benchmark fractions. Benchmark fractions are fractions that are easy to visualize such as ¼, ½, ¾. So you look at fractions in relation to ½. When comparing the two fractions 7/9 and 3/8, students should think 7/9 is greater than ½ and 3/8 is less than ½ so 7/9 is greater than 3/8.
- Students can also draw pictures to represent the fractions making certain that the wholes are the same.
- When comparing fractions with the same numerators, such as 3/5 and 3/10, it should be obvious to the students because the numerator names the same number of parts so they need to look to the denominators and see that 3 out of 10 parts is less than 3 out of 5 parts.
- You can also compare each fraction to one. When comparing 5/6 and 7/8. Think that it takes 1/6 to get to one whole and it takes 1/8 to get to one whole. Since 1/8 is smaller than 1/6, 7/8 is larger than 5/6.
- You can compare fractions using a common denominator procedure. When comparing ¾ and 2/3, find the common denominator of 12 and calculate that ¾ = 9/12 and 2/3 is equal to 8/12 so ¾ > 2/3 (John SanGiovanni, Howard County Schools).

**Introductory Activities:**

**Illustrative Mathematics - Using Benchmarks to Compare Fractions**

https://www.illustrativemathematics.org/content-standards/tasks/812

- This task is intended primarily for instruction. The goal is to provide examples for comparing two fractions, 15 and 27 in this case, by finding a benchmark fraction which lies in between the two.

**North Carolina Department of Education - Fraction Buckets**

https://grade4commoncoremath.wikispaces.hcpss.org/file/view/4.NF.1_4.NF.2_FractionBuckets.pdf

- This lesson asks students to compare two fractions with different numerators and different denominators by comparing them to benchmarks of ½ and 1.

**Learn Alberta - Comparing and Ordering Fractions**

http://www.learnalberta.ca/content/me5l/html/math5.html?goLesson=5

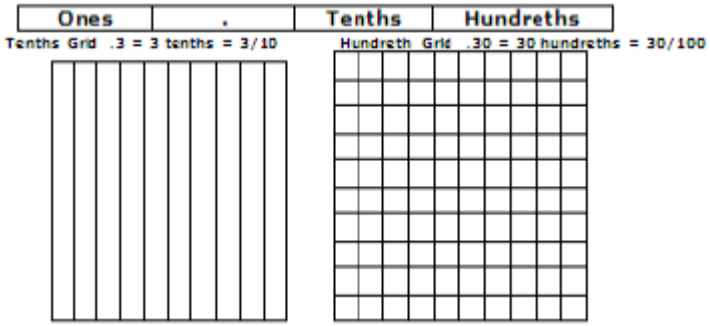
- This lesson on comparing and ordering fractions includes an interactive video, an activity sheet, and an assessment task.

**Teacher Notes for 4.NSF.5:**

This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (4.NF.6 and 4.NF.7), experiences that allow students to shade decimal grids (10x10 grids) can support this work.

Student experiences should focus on working with grids rather than algorithms. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

This work in fourth grade lays the foundation for performing operations with decimal numbers in fifth grade.




Students can use base ten blocks, graph paper, and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

Base Ten Blocks: students may represent 3/10 with 3 longs and may also write the fraction as 30/100 with the whole in this case being the flat (the flat represents one hundred units with each unit equal to one hundredth).

Common Misconception: Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that .30 is greater than 0.3.

(Excerpt from KATM Grade 4 Flip Book)

**Introductory Activities:**

**Illustrative Mathematics - Fraction Equivalence**

https://www.illustrativemathematics.org/content-standards/tasks/154

- The primary goal of this task is for students to use illustrations to explain why tenths and hundredths can make equivalent fractions.

**Resources**

**Engage NY Grade 4 Mathematics Module 5**

https://www.engageny.org/resource/grade-4-mathematics-module-5

- This module addresses Fraction Equivalence, Ordering, and Operations. It includes math background, teacher notes, student activities, practice pages, and a variety of assessments.

**Grade 4, Mission 5: Equivalent Fractions**

https://www.zearn.org/curriculum/19

- On the Zearn website, there are projectables, practice and more that support EngageNY (see unit above).

**CCGPS Frameworks 4th Unit 3**

https://www.georgiastandards.org/Georgia-Standards/Frameworks/4th-Math-Unit-3.pdf

- This unit includes performance tasks and formative assessment lessons.

**Learn Zillion - Grade 4, Unit 5 Understanding Fraction Equivalence and Comparison**

https://hcpss.learnzillion.com/resources/64183-grade-4-unit-5

- This unit includes video lessons, practice resources for students, and a summative assessment. Students begin to integrate their understanding of fractions with their understanding of comparing numbers. They use and build on their knowledge of fractions to discover different strategies for comparing and building equivalent fractions. By partitioning proportionally and exploring the idea of transitivity, students deepen their understanding of equivalence and comparison.

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**Howard County Schools Resources for 4.NF.1**

https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-1-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Matific Equivalent Fractions Episodes (free for teachers)**

https://www.matific.com/us/en-us/search?q=equivalent%20fractions

- This page offers multiple episodes and interactive worksheets on equivalent fractions.

**Crickweb- Fraction Machine**

http://www.crickweb.co.uk/ks2numeracy-tools.html (scroll down)

- This is an interactive tool that allows students to explore fraction equivalence.

**Matching Math - Equivalent Fractions**

http://www.sheppardsoftware.com/mathgames/fractions/memory_equivalent1.htm

- This game gives students practice in identifying equivalent fractions with models.

**Math Playground Online Game- Triplets**

http://www.mathplayground.com/Triplets/Triplets.html

- In this online game, students make groups of equivalent fractions.

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**Howard County Schools Resources for 4.NF.2**

https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-2-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Matific Comparing Fractions Episodes (free for teachers)**

https://www.matific.com/us/en-us/search?q=comparing%20fractions

- This page offers multiple episodes and interactive worksheets on equivalent fractions.

**Greg Tang Math - Satisfraction**

http://gregtangmath.com/satisfraction

- Simplify and Compare Options give students the opportunity to practice their fraction skills.

**Math Coach's Corner - Multiple Strategies for Comparing Fractions**

http://www.mathcoachscorner.com/2013/12/multiple-strategies-for-comparing-fractions/

- This blogpost discusses using number sense to compare fractions. It includes links to other posts on building number sense with fractions, as well as a free comparing fractions resource to use with students.

**Howard County Schools Resources for 4.NF.5**

https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-5-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Sample Formative Assessment Tasks/Questions**

https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-1-assessment-tasks (Assessing 4.NSF.1)

https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-2-assessment-tasks (Assessing 4.NSF.2)

https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-5-assessment-tasks (Assessing 4.NSF.5)

Return to Table of Contents

**Grade 4 Unit 5** Return to Table of Contents Return to Fourth Grade Overview

Adding, Subtracting, & Multiplying with Fractions

**Content Standards with Clarifying Notes**
*(Open bullets indicate clarifying notes)*

- **4.NSF.3** Develop an understanding of addition and subtraction of fractions (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100) based on unit fractions.
  a. Compose and decompose a fraction in more than one way, recording each composition and decomposition as an addition or subtraction equation;
  b. Add and subtract mixed numbers with like denominators;
  c. Solve real-world problems involving addition and subtraction of fractions referring to the same whole and having like denominators.
- **4.NSF.4** Apply and extend an understanding of multiplication by multiplying a whole number and a fraction (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100).
  a. Understand a fraction a/b as a multiple of 1/b;
  b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number;
  c. Solve real-world problems involving multiplication of a fraction by a whole number (i.e., use visual fraction models and equations to represent the problem).
- **4.NSF.5** Express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100 and use this technique to add two fractions with respective denominators of 10 and 100.
  - In the previous unit, students made equivalent fractions. Now, they will apply that skill to add and subtract.
- **4.MDA.4** Create a line plot to display a data set (i.e., generated by measuring length to the nearest quarter-inch and eighth-inch) and interpret the line plot.

**New Academic Vocabulary for This Unit**

All vocabulary introduced in prior grades

**Prior Knowledge Required for This Unit**

In third grade, students were introduced to fractions (3.NSF.1), fraction equivalence (3.NSF.2) and mixed numbers (3.NSF.3). In the previous unit of fourth grade (Unit 4), students continued to develop their understanding of fractions by creating equivalent fractions and comparing fractions.

**Subsequent Knowledge Related to This Unit**

Students will extend their understanding of adding, subtracting, and multiplying fractions in 5th grade (5th grade, Units 4 and 5) and apply these fraction skills in fifth grade when solving problems involving perimeter and area (5th grade, unit 8). Students will also divide fractions beginning in 5th grade.

**Relationship Among Standards in This Unit**

The focus of this unit is to build an understanding of addition and subtraction of fractions and multiplication of a fraction by a whole number. This understanding is formed through hands on activities that allow students to form a deep understanding rather than tricks and strategies that simply get an answer.

**Potential Instructional Strategies/Lessons**

**Teacher Notes for 4.NSF.3:**

Addition and subtraction of fractions with like denominators can easily be solved using an algorithm of adding or subtracting the numerators and keeping the same denominator. However, prior to this, students need instruction on the conceptual understanding of adding and subtracting fractions and what a reasonable answer looks like. Questions such as "will the answer when you add 7/8 + 3/8 be more or less than a whole and why?" should be part of instruction. Concrete materials should be used to introduce addition and subtraction prior to moving to the algorithm.

- Students need to see that a fraction can be decomposed just like whole numbers. There are many different ways to decompose a fraction. Understanding this helps students see the value of fractions and enhances their fraction sense. 7/10 = 1/10 + 1/10 +1/10 + 1/10 +1/10 + 1/10 + 1/10 or 3/10 + 3/10 + 1/10 or 5/10 + 2/10 or 4/10 + 2/10 + 1/10.
- A mixed number is a whole number and a fraction. Students need to see that 3 ¼ is the same as 3 + ¼. This can be connected to the previous standard of decomposing fractions.
- Drawing a picture and writing an equation are two effective strategies when solving word problems with fractions. Writing an equation helps students translate word phrases into numbers. Encourage students to use these two strategies instead of looking for keywords, which should be avoided because keywords can indicate different operations depending on the context in the problem (John SanGiovanni, Howard County Schools).

A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

Example:
Susan and Avery need 8 ⅜ feet of ribbon to package gift baskets. Susan has 3 ⅛ feet of ribbon and Avery has 5 ⅜ feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.
The student thinks: I can add the ribbon Susan has to the ribbon Avery has to find out how much ribbon they have altogether. Susan has 3 ⅛ feet of ribbon and Avery has 5 ⅜ feet of ribbon. I can write this as 3 ⅛ + 5 ⅜. I know they have 8 feet of ribbon by adding the 3 and 5. They also have ⅛ and ⅜ which makes a total of 4/8 more. Altogether they have 8 4/8 feet of ribbon. 8 4/8 is larger than 8 ⅜ so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, ⅛ foot.

Mixed numbers were introduced in 3rd grade, but this is the first time operations have been used with mixed numbers. Previously, students have explored mixed numbers on a number line or when naming a fractional part. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers into improper fractions. Keep in mind the Concrete-Representational-Abstract (CRA) approach to teaching fractions. Students need to be able to "show" their thinking using concrete and/or representations BEFORE they move to abstract thinking.
Example: While solving the problem 3 ¾ + 2 ¼ students could do the following:

| Student 1 | Student 2 | Student 3 |
|---|---|---|
| $3 + 2 = 5$ *and* $\frac{3}{4} + \frac{1}{4} = 1$ | $3\frac{3}{4} + 2 = 5\frac{3}{4} + \frac{1}{4} = 6$ | $3\frac{3}{4} = \frac{15}{4}$ *and* $2\frac{1}{4} = \frac{9}{4}$ *so* $\frac{15}{4} + \frac{9}{4} + \frac{24}{4} = 6$ |

Common Misconception: Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole. (Excerpt from KATM Grade 4 Flip Book)

**Fraction Progression from Illustrative Mathematics**
https://www.illustrativemathematics.org/progressions

- Fractions Progression Module consisting of videos paired with illustrative tasks to help teachers better understand the development of concepts and skills around fractions from grades 3-5.

**Introductory Activities:**
**Illustrative Mathematics - Making 22 Seventeenths in Different Ways**
https://www.illustrativemathematics.org/content-standards/tasks/837
This task is related to adding fractions with the same denominator. The main purpose is to emphasize that there are many ways to decompose a

fraction as a sum of fractions, similar to decompositions of whole numbers that students should have seen in earlier grades. With opportunity for classroom discussion, it could be useful in instruction.

**Illustrative Mathematics - Peaches**

https://www.illustrativemathematics.org/content-standards/tasks/968

This task provides a context where it is appropriate for students to subtract fractions with a common denominator; it could be used for either assessment or instructional purposes. For this particular task, teachers should anticipate two types of solution approaches: one where students subtract the whole numbers and the fractions separately and one where students convert the mixed numbers to improper fractions and then proceed to subtract. Both methods are legitimate approaches to solving this problem and the comparison of the two opens the opportunity for a great classroom discussion on being strategic about choosing an approach to solving a problem.

**Teacher Notes for 4.NSF.4:**

**A unit fraction is a fraction that describes one part of the whole.**

Unit fractions always have a numerator of one. Students need to see that fraction parts can be counted, just like we count whole numbers. So if we count 1 orange, 2 oranges, 3 oranges, etc., we can also count 1 fourth, 2 fourths and 3 fourths. So if 3 oranges can be thought of as 3 groups of one orange, then ¾ can be represented as 3 groups of ¼ or 3 x ¼.

**Prior to teaching the procedure for multiplying a whole number by a fraction, students need to understand conceptually why the answer is reasonable.**

If I have 5 groups of 1/6, how many 1/6s are there? You can add 1/6 + 1/6 + 1/6 + 1/6 + 1/6 to equal 5/6. If you want to multiply 4 x 2/3, students need to think of this as 4 groups of 2/3 or 8 groups of 1/3. When added to show four groups of two-thirds, 2/3 + 2/3 + 2/3 + 2/3 = 8/3 or 2 2/3. Or this can be shown as 4 X 2/3 or 4/1 x 2/3= 8/3 or 2 2/3.

**Students need to understand what a reasonable answer looks like when multiplying fractions**.

Between what two whole numbers does your answer lie? If students have been led down the incorrect idea that when you multiply whole numbers the product is always greater. Giving students "rules" such as that sets them up for confusion as they continue to learn about rational numbers. This is not true when dealing with fractions. Students should focus on what an answer will look like prior to actually calculating the answer. Questions like: Does this answer make sense? How do you know it is going to be less than a certain number? Using word problems in a context helps students make sense (John SanGiovanni, Howard County Schools).

**Introductory Activity:**

**Math Solutions - Introducing Multiplication of Fractions**

http://www.mathsolutions.com/documents/0-941355-64-0_L.pdf

- This lesson builds on what students know about multiplying whole numbers to begin developing understanding of what occurs when we multiply fractions. There are examples that involve multiplying two fractions - do not use these.

**Teacher Notes for 4.MDA.4:**

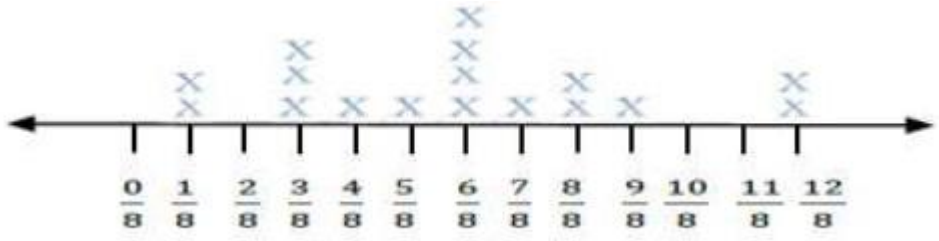
This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

Example:

Students measured objects in their desk to the nearest ½, ¼, or ⅛ inch. They displayed their data collected on a line plot. How many objects measured ¼ inch? ½ inch? If you put all the objects together end to end, what would be the total length of all the objects?

Data has been measured and represented on line plots in units of whole numbers, halves or quarters. Students have also represented fractions on number lines. Now students are using line plots to display measurement data in fraction units and using the data to solve problems involving addition or subtraction of fractions.

Have students create line plots with fractions of a unit (½, ¼, or ⅛) and plot data showing multiple data points for each fraction.




Pose questions that students may answer, such as:

• "How many one-eighths are shown on the line plot?" Expect "two one-eighths" as the answer.

• Then ask, "What is the total of these two one-eighths?" Encourage students to count the fractional numbers as they would with whole-number counting, but using the fraction name.

• "What is the total number of inches for insects measuring ⅜ inches?" Students can use skip counting with fraction names to find the total, such as, "three-eighths, six-eighths, nine-eighths. The last fraction names the total.

• Students should notice that the denominator did not change when they were saying the fraction name.

• Have them make a statement about the result of adding fractions with the same denominator.

• "What is the total number of insects measuring ⅛ inch or ⅝ inches?" Have students write number sentences to represent the problem and solution such as, ⅛ + ⅛ + ⅝ = ⅞ inches.

Use visual fraction strips and fraction bars to represent problems to solve problems involving addition and subtraction of fractions.

Common Misconceptions:

Students use whole-number names when counting fractional parts on a number line. The fraction name should be used instead. For example, if two-fourths is represented on the line plot three times, then there would be six-fourths.

Students also count the tick marks on the number line to determine the fraction, rather than looking at the "distance" or "space" between the marks. (Excerpt from KATM Grade 4 Flip Book)

**Introductory Activities:**

**Line Plots Practice**

http://www.glencoe.com/sites/common_assets/mathematics/mc2/cim/interactive_labs/M2_02/M2_02_dev_100.html

This interactive web tool provides a good introduction and practice for line plots.

**Bugs, Giraffes, Elephants, and More**

http://www.mathedleadership.org/docs/ccss/NCSM_GreatTasks_Bugs_Giraffes_Elephants_sample.pdf

Students will interpret line plots with scales written to the nearest quarter of a unit.

**Illustrative Mathematics - Button Diameters**

https://www.illustrativemathematics.org/content-standards/tasks/1039

The purpose of this task is for students to measure lengths to the nearest eighth and quarter-inch and to record that information in a line plot. While this task does not explicitly ask operations-based questions, it would be a strong task to introduce line plots with fractions. The teacher might consider prompting students to find the difference between the diameter of the biggest button and smallest button or to find the total length of the 5 biggest buttons placed right next to each other to address the other aspects of the standard. Note: Diameter is not a 4th grade standard so the emphasis should be on line plots, not measuring the diameter.

**Resources**

**Engage NY Grade 4 Mathematics Module 5**

https://www.engageny.org/resource/grade-4-mathematics-module-5

- This module addresses Fraction Equivalence, Ordering, and Operations. It includes math background, teacher notes, student activities, practice pages, and a variety of assessments.

**CCGPS Frameworks 4th Unit 4**

https://www.georgiastandards.org/Georgia-Standards/Frameworks/4th-Math-Unit-4.pdf

- This unit includes performance tasks and formative assessment lessons.

**Howard County Schools Resources for 4.NSF.3**

https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-3-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Learn Zillion - Grade 4, Unit 3 Decomposing and Composing Fractions for Addition and Subtraction**

https://hcpss.learnzillion.com/resources/64181-grade-4-unit-3

- This unit includes video lessons, practice resources for students, and a summative assessment. Students begin to integrate their understanding of fractions with their understanding of operations.

**Illustrative Mathematics - Cynthia's Perfect Punch**

https://www.illustrativemathematics.org/content-standards/tasks/874

- The purpose of this task is for students to estimate and compute sums of mixed numbers in a context.

**Illustrative Mathematics - Plastic Building Blocks**

https://www.illustrativemathematics.org/content-standards/tasks/856

The purpose of this task is to have students add mixed numbers with like denominators. This task illustrates the different kinds of solution approaches students might take to such a task. Two general approaches should be anticipated: one where students calculate exactly how many buckets of blocks the boys have to determine an answer, and one where students compare the given numbers to benchmark numbers. Ideally in a classroom situation, some students will approach it via calculation and others will approach it via comparison, which could generate a very good classroom discussion about fraction number sense.

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**Howard County Schools Resources for 4.NSF.4**

https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-4-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Learn Zillion - Grade 4, Unit 11 Multiplying Fractions by Whole Numbers**

https://hcpss.learnzillion.com/resources/64189-grade-4-unit-11

- This unit includes video lessons, practice resources for students, and a summative assessment. Students extend their understanding of fractions to see that a non-unit fraction can be thought of as a multiple of a unit fraction. Students also extend their fluency in multiplication as repeated addition and as a comparison. They apply these concepts to multiplying fractions by whole numbers.

**Illustrative Mathematics - Sugar in Six Cans of Soda**

https://www.illustrativemathematics.org/content-standards/tasks/857

- This task provides a familiar context allowing students to visualize multiplication of a fraction by a whole number, and is appropriate for instruction or assessment.

**Howard County Schools Resources for 4.NSF.5**
https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-5-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Mr. Nussbaun - Adding and Subtracting Fractions with Denominators of 10 and 100**
http://mrnussbaum.com/addfrac10/

- This is an interactive game for students to practice adding and subtracting fractions with denominators of ten and one hundred. Some simplifying is also required.

**Howard County Schools Resources for 4.MDA.4**
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-4-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Learn Zillion - Line Plots**
https://hcpss.learnzillion.com/resources/72571-display-data-in-a-line-plot-with-fractions-of-a-unit-and-use-line-plots-to-solve-problems-4-md-b-4

- This page includes video lessons and a performance task.

**K-5 Math Teaching Resources - Length of Ants Line Plot**
http://www.k-5mathteachingresources.com/support-files/length-of-ants-line-plot.pdf

- This print resource provides practice for students on using data to create a line plot.

**K-5 Math Teaching Resources - Objects in My Desk Line Plot**
http://www.k-5mathteachingresources.com/support-files/objects-in-my-desk-line-plot.pdf

- This print resource provides practice for students on measuring objects and then creating a line plot to represent the data.

**Sample Formative Assessment Tasks/Questions**

https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-3-assessment-tasks (Assessing 4.NSF.3)
https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-4-assessment-tasks (Assessing 4.NSF.4)
https://hcpss.instructure.com/courses/107/pages/4-dot-nf-dot-5-assessment-tasks (Assessing 4.NSF.5)
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-4-assessment-tasks (Assessing 4.MDA.4)

Return to Table of Contents

| **Grade 4 Unit 6** | Return to Table of Contents | Return to Fourth Grade Overview |
|---|---|---|

Decimal Concepts

**Content Standards with Clarifying Notes**
*(Open bullets indicate clarifying notes)*

- **4.NSF.6** Write a fraction with a denominator of 10 or 100 using decimal notation, and read and write a decimal number as a fraction.
- **4.NSF.7** Compare and order decimal numbers to hundredths, and justify using concrete and visual models.

**New Academic Vocabulary for This Unit**

- decimal notation
- tenth
- hundredth

**Prior Knowledge Required for This Unit**

This is the first time students have been exposed to decimals and their connection to fractions. In the previous 2 units in 4th grade (units 4 and 5), students have worked with fraction equivalents, as well as addition, subtraction, and multiplication of fractions with whole numbers.

**Subsequent Knowledge Related to This Unit**

This unit lays the foundation for performing operations with decimal numbers in fifth grade.

**Relationship Among Standards in This Unit**

Throughout this unit, students begin to work with decimal numbers, understanding their relationship to fractions and to whole numbers. Students write decimals as fractions and use concrete and visual models to compare and order decimal numbers.

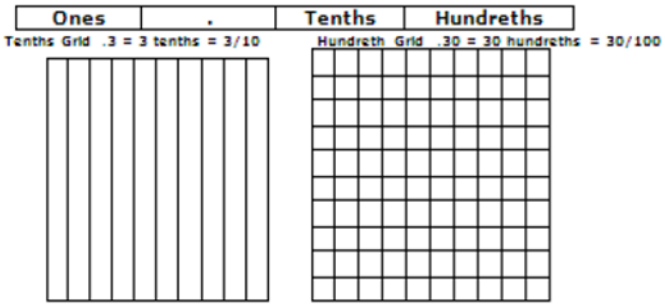
**Potential Instructional Strategies/Lessons**

**Teacher Notes: 4.NSF.6**

Students need to understand that decimals are an extension of our whole number base ten system. Decimals and fractions both represent parts of a whole. Students need to understand the value of the place and the relationship between place values so that 0.7 is ten times larger than 0.07. When reading decimals, the decimal point is read as AND which separates the whole number from the decimal. Having students read decimal names out loud helps them make connections to fractions. When we see 0.4 we say, "Four tenths" and 0.07 is read as "Seven Hundredths". (John SanGiovanni, Howard County Schools)

"Initially, student experiences should focus on working with grids. This helps students "see" the decimal numbers.

Students can use base ten blocks, graph paper, and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

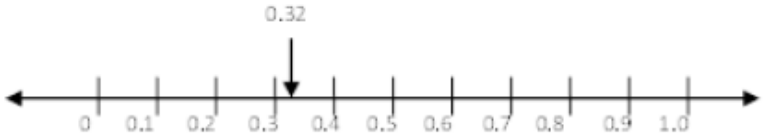



Base Ten Blocks: students may represent 3/10 with 3 longs and may also write the fraction as 30/100 with the whole in this case being the flat (the flat represents one hundred units with each unit equal to one hundredth). Students begin to make connections to the place value chart as shown in 4.NF.6. Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say 32/100 as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

| Hundreds | Tens | Ones | . | Tenths | Hundredths |
|---|---|---|---|---|---|
| | | | . | 3 | 2 |

Students use the representations explored in 4.NF.5 to understand 32/100 can be expanded to 3/10 and 2/100. Students represent values such as 0.32 or 32/100 on a number line. 32/100 is more than 30/100 (or 3/10) and less than 40/100 (or 4/10 ). It is closer to 30/100 so it would be placed on the

number line near that value." (excerpt from KATM Grade 4 Flip Book)




**Introductory Activity:**

**Decimal Designs (Georgia Department of Education)**

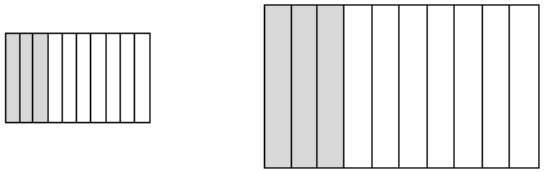
https://grade4commoncoremath.wikispaces.hcpss.org/file/view/Decimal+Designs+.pdf

- In this task, students will work with occurrences out of 10 and 100, translating them into decimal fractions and then decimals. Students will also explore and investigate the relationship between tenths and hundredths when in a visual model and in decimal notation. Students will also begin to rename tenths using hundredths.

**Teacher Notes for 4.NSF.7**

When comparing decimals, students should use models (such as hundredths grids) and number lines. When locating decimals on a number line the smaller numbers are farther to the left and the greater number is farther to the right. Often students are able to better understand comparing decimals if the problem is in context such as comparing scores or records of athletes. Students need to understand that some decimals are equivalent. Sharing examples with models to show that .4 = .40 will help students see the equivalency. Decimal numbers are rational numbers and so we can use them to indicate quantities that are less than one or between any two whole numbers. In between any two decimal numbers there is always another decimal number (John SanGiovanni, Howard County Schools).

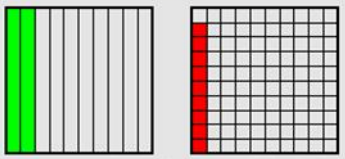
Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.



When the wholes are the same, the decimals or fractions can be compared (excerpt from KATM Grade 4 Flip Book).

Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator.
For example, to compare 0.2 and 0.09, students think of them as 0.20 and 0.09 and see that 0.20 > 0.09 (from Gr. 3-5 CCSS Fraction Progression, page 9).

Seeing that $0.2 > 0.09$ using a visual fraction model



*The shaded region on the left shows 0.2 of the unit square, since it is two parts when the square is divided int 10 parts of equal area. The shaded region on the right shows 0.09 of the unit square, since it is 9 parts when the unit is divided into 100 parts of equal area.*

$$\frac{20}{100} > \frac{9}{100}.$$

**Introductory Activities:**

**Learn Alberta - Comparing and Ordering Decimals**

http://www.learnalberta.ca/content/me5l/html/math5.html?goLesson=6

- This is an interactive lesson that scaffolds the learning for comparing and ordering decimals in a real-world context.

**Decimal Line Up (Georgia Department of Education)**

https://grade5commoncoremath.wikispaces.hcpss.org/file/view/5.NBT.3_DecimalLine-UP.pdf

- In this task, students will place decimal numbers (tenths and hundredths) on a number line and order them.

**Resources**

**EngageNY 4th Grade Module 6 Decimal Fractions**

https://www.engageny.org/resource/grade-4-mathematics-module-6

- This 20-day module gives students their first opportunity to explore decimal numbers via their relationship to decimal fractions, expressing a given quantity in both fraction and decimal forms. It includes math background, teacher notes, student activities, practice pages, and a variety of assessments.

**CCGPS Frameworks 4th Unit 5**
https://www.georgiastandards.org/Georgia-Standards/Frameworks/4th-Math-Unit-5.pdf

- This unit includes performance tasks and formative assessment lessons.

**Learn Zillion - Grade 4, Unit 12 Comparing Decimal Fractions and Understanding Notation**
https://hcpss.learnzillion.com/resources/64190-comparing-decimal-fractions-and-understanding-notation

- This unit includes video lessons and a summative assessment. Students will use their familiarity with money to make connections and understand decimals. Students will show decimals in different contexts, expressing them in tenths and hundredths. Using models to represent fractions, students will generate equivalent fractions and use decimal notation to express them. Since students can write decimal fractions as either a decimal or a fraction, we are still able to compare them. Students may struggle with understanding the size of numbers is not based on the number of digits, and understanding that as we increase the number of decimal places to the right of the decimal, the value of the digits in those places is getting smaller.

**Matific Decimals Episodes (free for teachers)**
https://www.matific.com/us/en-us/grades/4G/Fractions/Decimals

- This page offers multiple episodes and digital practice with decimals.

**Zearn Grade 4, Mission 6: Decimal Fractions, Topics A, B, and C**
https://www.zearn.org/curriculum/25

- This website, aligned to the EngageNY units, contains a variety of resources, including interactive manipulatives, exit tickets, homework, and assessments.

**Decimal Games (Sheppard Software)**
http://www.sheppardsoftware.com/mathgames/decimals.htm

- This site includes multiple games for students to practice their decimal skills.

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**Howard County Schools Resources for 4.NSF.6**
https://grade4commoncoremath.wikispaces.hcpss.org/4.NF.6

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**Show What You Know: Multiple Representations of Decimals and Fractions**
https://grade4commoncoremath.wikispaces.hcpss.org/file/view/4.NF.6_ShowWhatYouKnow-

MultipleRepresentationsofDecimalsandFractions.pdf/457307978/4.NF.6_ShowWhatYouKnow-MultipleRepresentationsofDecimalsandFractions.pdf

- In this task, students will represent a decimal in multiple ways: as a fraction, in expanded notation, on a number line, and on a hundredths grid.

**Howard County Schools Resources for 4.NSF.7**

https://grade4commoncoremath.wikispaces.hcpss.org/4.NF.7

- This page includes lesson plans, print resources, LearnZillion video links, and web resources.

**Illustrative Mathematics Project -How Many Tenths and Hundredths?**

https://www.illustrativemathematics.org/content-standards/tasks/103

- In this activity, students solve equations using place value of tenths and hundredths

**LearnZillion - Convert Between Fractions and Decimals**

https://learnzillion.com/resources/72780-convert-between-fractions-and-decimals

- 3 lessons that guide students to convert fractions into decimals (tenths and hundredths)

**The Teaching Channel - Games for Decimals**

https://www.teachingchannel.org/videos/elementary-math-lesson-plan

- In this Teaching Channel lesson, fourth grade students demonstrate their understanding of tenths and hundredths with the game "Fill Two"

**Sample Formative Assessment Tasks/Questions**

**Flag Fractions** (from Georgia Dept. of Ed.)

http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathFractions

- In this performance task, students create a flag by coloring fractional pieces of the flag and then name and write the fractional parts created on their flag. While exploring the fractional parts created, students add decimal fractions with like denominators, write decimal fractions as decimals, order two digit decimals, and add two digit decimals.

https://grade4commoncoremath.wikispaces.hcpss.org/Assessing+4.NF.6 (assesses 4.NSF.6)

https://grade4commoncoremath.wikispaces.hcpss.org/Assessing+4.NF.7 (assesses 4.NSF.7)

**Grade 4 Unit 7** Return to Table of Contents Return to Fourth Grade Overview

Conversions & Problem Solving with Measurement

**Content Standards with Clarifying Notes**
*(Open bullets indicate clarifying notes)*

- **4.MDA.1** Convert measurements within a single system of measurement, customary (i.e., in., ft., yd., oz., lb., sec., min., hr.) or metric (i.e., cm, m, km, g, kg, mL, L) from a larger to a smaller unit.
  - Students in 4th grade only convert from larger to smaller units, and only with the units named in the standards.
- **4.MDA.2** Solve real-world problems involving distance/length, intervals of time within 12 hours, liquid volume, mass, and money using the four operations.
- **4.MDA.3** Apply the area and perimeter formulas for rectangles.
  - Be sure students can work with/understand several different ways the formulas can be written. For example, students may see perimeter written as 2(L + W) or 2L + 2W or 4S (square).
- **4.MDA.8** Determine the value of a collection of coins and bills greater than $1.00.
  - 4th graders have not worked with money since 2nd grade.

**New Academic Vocabulary for This Unit**

- convert
- ounces
- pounds
- kilometer
- gram
- kilogram

**Prior Knowledge Required for This Unit**

In 2nd and 3rd grade, students had experiences in estimating and measuring liquid volume using milliliters, liters, cups, pints, quarts, and gallons (3.MDA.2) and estimating and measuring length/distance using inches, feet, yards, centimeters, and meters (2.MDA.3). In 2nd grade, students measured the same object using 2 different units of measure and explained how the measurements differed - the foundation of converting measurements (2.MDA.2). Students also had practice telling time to the nearest 5 minute (2.MDA.6) and the nearest minute (3.MDA.1), and solved problems involving time intervals within 60 minutes. In 3rd grade, students solved problems involving perimeters of polygons (3.MDA.6) and developed the concept of area (3.MDA.5) by building arrays and relating area to multiplication, and solved mathematical problems with rectangles that had the same area and different perimeters or the same perimeter and different areas (3.MDA.6). In 2nd grade, students solved word problems

using cents symbols and dollar signs (2.MDA.7), but did not evaluate a collection of coins and did not use decimals. 3rd graders did not do any work with money.

**Subsequent Knowledge Related to This Unit**

In 5th grade, students will continue to convert measurements, converting both from a larger unit to a smaller unit and from a smaller unit to a larger unit (5.MDA.1).

**Relationship Among Standards in This Unit**

The focus of this unit is problem solving. Students need many varied experiences with real world and mathematical problems, with a range of presentation styles and difficulty levels. Several standards can be blended together in these problems. For example, students may be asked to find the area of a rectangle, but be given one side length in feet and the other in inches.

**Potential Instructional Strategies/Lessons**

**Teacher Note 4.MDA.1**

"In order for students to have a better understanding of the relationships between units, they need to use measuring devices in class.
The number of units needs to relate to the size of the unit. They need to discover that there are 12 inches in 1 foot and 3 feet in 1 yard.
Allow students to use rulers or a yardstick to discover these relationships among these units of measurements. Using 12-inch rulers and yardsticks, students can see that three of the 12-inch rulers, is the same as 3 feet since each ruler is 1 foot in length, are equivalent to one yardstick.
Have students record the relationships in a two column table or t-chart. A similar strategy can be used with rulers marked with centimeters and a meter stick to discover the relationships between centimeters and meters. Present word problems as a source of students' understanding of the relationships among inches, feet and yards." *(*excerpt from KATM Grade 4 flip book)

**Common Misconception:** 4.MD.1 Students believe that larger units will give the larger measure.
Students should be given multiple opportunities to measure the same object with different measuring units. For example, have the students measure the length of a room with one-inch tiles, with one-foot rulers, and with yardsticks. Students should notice that it takes fewer yard sticks to measure the room than rulers or tiles and explain their reasoning." *(*excerpt from KATM Grade 4 flip book)

**Introductory Activities:**

**Study Jams - Units of Measurement**

http://studyjams.scholastic.com/studyjams/jams/math/measurement/units-of-measurement.htm

- This video could serve as a brief review of the two different measurement systems.

**Illustrative Mathematics - Who is the Tallest?**

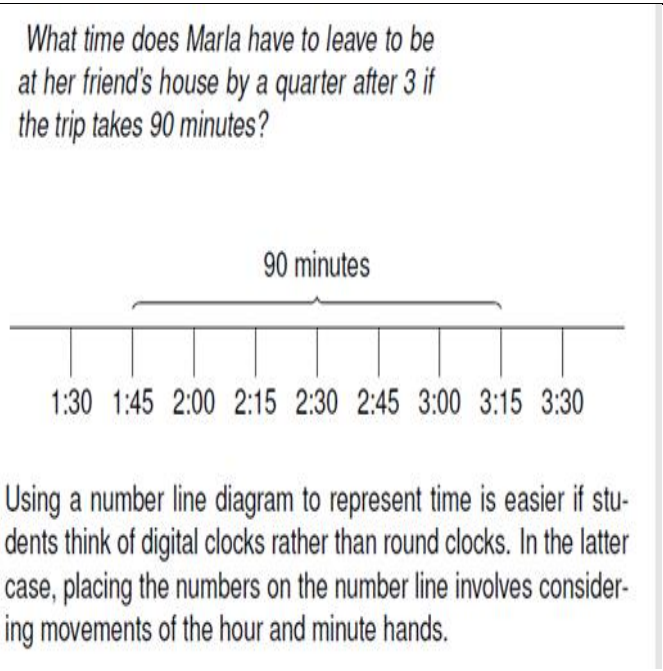
https://www.illustrativemathematics.org/content-standards/tasks/1508

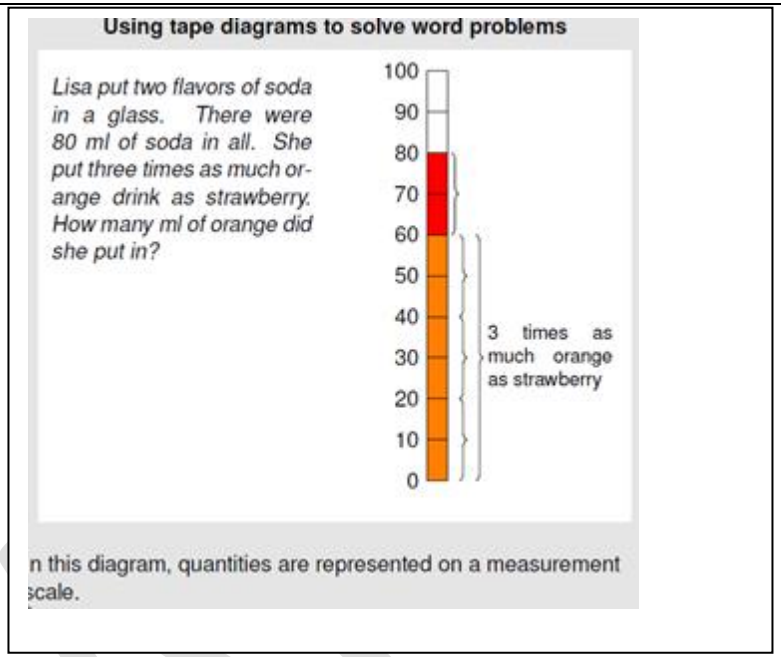
- This task provides a context for translating between different units. While there are in principle multiple approaches (using each of the different units for measurement), the 4.MDA.1 standard calls for converting a larger unit to a smaller unit. Students who convert all of the heights to inches are meeting the expectations of the standard, and students solving it in other ways are exceeding the expectations of the standard.

**Teacher Note 4.MDA.2**

"Students are to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Present problems that involve multiplication of a fraction by a whole number (denominators are 2, 3, 4, 5 6, 8, 10, 12, 25, and 100). Problems involving addition and subtraction of fractions should have the same denominators. Allow students to use strategies learned with these concepts. Students used models to find area and perimeter in Grade 3. They need to relate discoveries from the use of models to develop an understanding of the area and perimeter formulas to solve real-world and mathematical problems." (excerpt from KATM Grade 4 flip book)

Students also combine competencies from different domains as they solve measurement problems using all four arithmetic operations, addition, subtraction, multiplication, and division. For example, "How many liters of juice does the class need to have at least 35 cups if each cup takes 225 ml?" Students may use tape or number line diagrams for solving such problems (MP1). (excerpt from CCSS K-5 Geometric Measurement Progression Document)




**Introductory Activity:**

**K-5 Math Teaching Resources - Measurement Word Problems**

http://www.k-5mathteachingresources.com/support-files/measurement-word-problems.pdf

- This print resource provides students with a variety of real-world measurement problems to solve. It could be used to introduce students to the many types of measurement problems they could encounter and also serve as a springboard for discussions on types of strategies that are useful.

**Teacher Notes: 4.MDA.3**

"Students developed understanding of area and perimeter in 3rd grade by using visual models. While students are expected to use formulas to calculate area and perimeter of rectangles, they need to understand and be able to communicate their understanding of why the formulas work. The formula for area is $l \times w$ and the answer will always be in square units. The formula for perimeter can be $2l + 2w$ *or* $2(l + w)$ and the answer will be in linear units.

This standard calls for students to generalize their understanding of area and perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience not just memorization." *(*excerpt from KATM Grade 4 flip book)

"The area of a rectangle is the region inside the rectangle. The formula for area is A = l x w. Students can find a missing side if they know the length of one side and the area for the figure. Students need to use the relationship between multiplication and division to solve problems involving finding the missing side. The perimeter of a rectangle is the distance around it. The formula for finding perimeter is P = l + w + l + w or P = 2 l + 2 w. Students can find the missing side using the perimeter and the known side. Students need to remember that there are two sides that have the same length or width. Focus for instruction should be understanding the concepts of area and perimeter and not just knowing the formulas."
Examples for increasing rigor:

- A rectangular figure has a perimeter of 35 cm. What could the lengths of the sides be? Give two possibilities.
- A construction worker laid 54 square feet of hardwood in the rectangular family room of a new house. What could possible perimeters of the room be? Sketch and label the possibilities.
- The areas of two shapes are each 40 square inches, but the perimeters are very different. Sketch the two shapes and calculate the perimeters.
- Imagine a rectangle with an area of 28 square centimeters. If the length of the rectangle is 3 centimeters shorter than the width, find the dimensions of the rectangle.

(John SanGiovanni, Howard County Schools)

Perimeter problems often give only one length and one width, thus remembering the basic formula can help to prevent the usual error of only adding one length and one width. The formula P = 2($l$ + $w$) emphasizes the step of multiplying the total of the given lengths by 2. Students can make a transition from showing all length units along the sides of a rectangle or all area units within by drawing a rectangle showing just parts of these as a reminder of which kind of unit is being used. Writing all of the lengths around a rectangle can also be useful. Discussions of formulas such as P= 2$l$ + 2$w$, can note that unlike area formulas, perimeter formulas combine length measurements to yield a length measurement. Students learn to apply these understandings and formulas to the solution of real-world and mathematical problems. For example, they might be asked, "A rectangular garden has as an area real world and mathematical problems of 80 square feet. It is 5 feet wide. How long is the garden?" Here, specifying the area and the width, creates an unknown factor problem (see Table below). Similarly, students could solve perimeter problems that give the perimeter and the length of one side and ask the length of the adjacent side. Students could be challenged to solve multistep problems such as the following. "A plan for a house includes rectangular room with an area of 60 square meters and a perimeter of 32 meters. What are the length and the width of the room?" (excerpt from CCSS K-5 Geometric Measurement Progression Document)

Table 1: Multiplication and division situations for measurement

| | Unknown Product<br>A × B = ☐ | Group Size Unknown<br>A × ☐ = C and C ÷ A = ☐ | Number of Groups Unknown<br>☐ × B = C and C ÷ B = ☐ |
|---|---|---|---|
| **Grouped Objects (Units of Units)** | You need *A* lengths of string, each *B* inches long. How much string will you need altogether? | You have *C* inches of string, which you will cut into *A* equal pieces. How long will each piece of string be? | You have *C* inches of string, which you will cut into pieces that are *B* inches long. How many pieces of string will you have? |
| **Arrays of Objects (Spatial Structuring)** | What is the area of a *A* cm by *B* cm rectangle? | A rectangle has area *C* square centimeters. If one side is *A* cm long, how long is a side next to it? | A rectangle has area *C* square centimeters. If one side is *B* cm long, how long is a side next to it? |
| **Compare** | A rubber band is *B* cm long. How long will the rubber band be when it is stretched to be *A* times as long? | A rubber band is stretched to be *C* cm long and that is *A* times as long as it was at first. How long was the rubber band at first? | A rubber band was *B* cm long at first. Now it is stretched to be *C* cm long. How many times as long is the rubber band now as it was at first? |

Adapted from box 2-4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33. Note that Grade 3 work does not include Compare problems with "times as much," see the Operations and Algebraic Thinking Progression, Table 3, also p. 29.

**Introductory Activity:**

**Area and Perimeter (Georgia Department of Education)**

http://services.minecraftedu.com/worlds/sites/default/files/worlds/147/material/area_and_perimeter_147.pdf

- In this task, students create rectangles with different areas, but the same perimeter. Students then create formulas that help find the perimeters and areas of any given rectangle or square.

**Teacher Notes: 4.MDA.8**

When determining the value of a collection of bills and coins, students must first mentally give each bill or coin a value, order them, and then add up the values. Allow students to share their strategies with others, pointing out those that begin with the larger values first and those that utilize making combinations with 10s.

As a preparation for counting money, allow students to practice skip-counting by one number and then ask them to shift to count by a different number. For example, ask students to count by 25s and then at your signal they should continue skip counting by 10s from where they left off. Use any two of the following numbers: 100, 50, 25, 10, 5, and 1. Always start with the larger. As students gain experience, try giving them three numbers, still in descending order (taken from *Elementary and Middle School Mathematics: Teaching Developmentally*, John Van de Walle).

**Introductory Activity:**
**Coin and Bill Values**
https://s3.amazonaws.com/quantile-resources/resources/downloads/QuantileResource42110.pdf

- Start on page 12 for a lesson that helps students develop strategies to count combinations of coins and bills totaling more than $1.00. Several strategies are provided throughout the investigation, but in every example, it is acceptable for students to share other strategies.

**Resources**

**EngageNY Grade 4 Module 2: Unit Conversions and Problem Solving with Metric Measurement**
https://www.engageny.org/resource/grade-4-mathematics-module-2

- Module 2 uses length, mass and capacity in the metric system to convert between units using place value knowledge. Students recognize patterns of converting units on the place value chart. Conversions are recorded in two-column tables and number lines, and are applied in single- and multi-step word problems solved by the addition and subtraction algorithm or a special strategy.

**EngageNY Grade 4 Module 7: Exploring Measurement with Multiplication**
https://www.engageny.org/resource/grade-4-mathematics-module-7

- In this 20-day module, students build their competencies in measurement as they relate multiplication to the conversion of measurement units. Throughout the module, students will explore multiple strategies for solving measurement problems involving unit conversion. (Only covers 4.MDA.1 and 4.MDA.2)

**CCGPS Frameworks 4th Unit 7**
https://www.georgiastandards.org/Georgia-Standards/Frameworks/4th_Math-Unit-7.pdf

- This unit includes performance tasks and formative assessment lessons.

**Matific Measurement Episodes (free for teachers)**
https://www.matific.com/us/en-us/grades/4G/Measurements

- This page offers multiple episodes on measurement skills.

**Zearn Grade 4, Mission 2: Measure and Solve**
https://www.zearn.org/curriculum/14

- This online resource is a companion to EngageNY, adding extra resources and activities in an easy to use format. This module applies place value understanding to metric units of length, mass and capacity.

**Zearn Grade 4, Mission 7: Multiply and Measure**
https://www.zearn.org/curriculum/29

- This online resource is a companion to EngageNY, adding extra resources and activities in an easy to use format. This module provides students with an opportunity to synthesize what they've learned in Grade 4 and apply it in complex situations with conversions.

**Measurement Games (Sheppard Software)**
http://www.sheppardsoftware.com/mathgames/menus/measurement.htm

- This site includes multiple games for students to practice their measurement skills.

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**Howard County Schools Resources for 4.MDA.1**
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-1-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

***Hershey's Milk Chocolate Weights and Measure Book***

- This book uses assorted Hershey's candies to explain weights and measures. How long is a foot? What is the metric system? What weighs more, a ton of feathers or a ton of Hershey's candy? Author and educator Jerry Pallotta answers these and lots of other questions as only he can--using Hershey's Kisses, Twizzlers candies, Hershey's chocolate bars, and more to teach weights and measurements. Rob Bolster's comical clowns demonstrate the concepts.

***On the Scale, a Weighty Tale***

- In Cleary's trademark bouncy verse, *On the Scale* presents basic weights and measures through age-appropriate examples employing both English and metric units of measurement. Accompanying cartoons by Gable add energy and humor.

***Pastry School in Paris: An Adventure in Capacity***

- While in Paris, Matt and Bibi visit the pastry academy Les Jumelles Coccinelle, and are put in charge of liquids. Children will enjoy the slapstick culinary adventures and will also get a lesson on American standard and metric liquid measurement.

**K-5 Math Teaching Resources - Making a Kilogram/Pound**
http://www.k-5mathteachingresources.com/support-files/making-a-kilogram.pdf

- This print resource provides students with an opportunity to discover the weights of a kilogram and pound on their own.

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**Howard County Schools Resources for 4.MDA.2**
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-2-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Illustrative Mathematics - Margie Buys Apples**
https://www.illustrativemathematics.org/content-standards/tasks/873

- This task gives students an opportunity to work with familiar fractions and decimals in a context involving money.

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**Howard County Schools Resources for 4.MDA.3**
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-3-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**LearnZillion**
https://learnzillion.com/resources/72767-applying-formulas-for-rectangle-area-and-perimeter

- 5 lessons to help teach perimeter and area

**Math Playground.com - Design a Party**
http://www.mathplayground.com/PartyDesigner/PartyDesigner.html

- In this online game, students create rectangles that have the exact perimeter and area needed for the party.

**K-5 Math Teaching Resources - Fencing a Garden; How Many Tables; Designing a Zoo Enclosure**
http://www.k-5mathteachingresources.com/4th-grade-measurement-and-data.html

- In each of these 3 tasks, students apply their critical thinking skills and use many math skills to come up with possible solutions with justifications.

**Bridges in Mathematics - Measurement, Set D6**
http://catalog.mathlearningcenter.org/store/product-8798.htm

- 4 activities involving perimeter and area. Includes lesson plans and student sheets.

**Illustrative Mathematics - Karl's Garden**
https://www.illustrativemathematics.org/content-standards/4/MD/A/3/tasks/876

- The purpose of the task is for students to solve a multi-step multiplication problem in a context that involves area.

***Spaghetti and Meatballs for All***

- Mr. and Mrs. Comfort are having a family reunion! Mr. Comfort starts cooking up his famous spaghetti and meatballs, while Mrs. Comfort carefully arranges eight tables and thirty-two chairs so that everyone will have a seat. The tables look lovely, the food is ready, and here come the guests--with their own seating plans! This delightful Marilyn Burns Brainy Day Book uses wit and humor to draw children into thinking about area and perimeter.

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***Pigs will be Pigs***

- The pigs are very hungry, and there's no food in the house. Mr. Pig suggests eating out -- but oh, no! The Pigs are out of money! So the family goes on a money hunt. In beds, under the carpet, even in the washing machine the coins and bills add up, and soon it's off to the Enchanted Enchilada. How much money did the Pigs find? What can they afford to order from the menu? Join the fun and pig out on math and money concepts with the Pigs! If you do not have a copy of the book, it can be viewed on YouTube: (https://www.youtube.com/watch?v=g1PEYN9tEN8).

**Piggin' Out with Money - Learn NC**
http://www.learnnc.org/lp/pages/3505

- This lesson plan can be used with the *Pigs will be Pigs* literature connection listed above.

**IXL - Count Coins and Bills**
https://www.ixl.com/math/grade-4/count-coins-and-bills-up-to-5-dollar-bill

- This site provides students with practice in identifying the value of a set of coins and bills.

**Mr. Anker - Dollars and Cents**
http://www.henryanker.com/Math/Money/Dollars&Cents/Dollars&Cents_Set_01.swf

- This site provides students with practice in identifying the value of a set of coins and bills.

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**Sample Formative Assessment Tasks/Questions**

https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-1-assessment-tasks (assesses 4.MDA.1)
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-2-assessment-tasks (assesses 4.MDA.2)
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-3-assessment-tasks (assesses 4.MDA.3)

**Grade 4 Unit 8** Return to Table of Contents Return to Fourth Grade Overview

Geometric Classifications & Line Symmetry

**Content Standards with Clarifying Notes**
*(Open bullets indicate clarifying notes)*

- **4.G.1** Draw points, lines, line segments, rays, angles (i.e., right, acute, obtuse), and parallel and perpendicular lines. Identify these in two-dimensional figures.
- **4.G.2** Classify quadrilaterals based on the presence or absence of parallel or perpendicular lines.
  - This standard calls for students to sort quadrilaterals based on parallelism and perpendicularity.
- **4.G.3** Recognize right triangles as a category, and identify right triangles.
  - This standard calls for students to sort triangles based on angle types.
- **4.G.4** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
- **4.ATO.5** Generate a number or shape pattern that follows a given rule and determine a term that appears later in the sequence.
  - In this unit, the focus will be on shape patterns. Number patterns were addressed in Unit 2.

**New Academic Vocabulary for This Unit**

- point
- line
- line segment
- ray
- parallel
- perpendicular
- line of symmetry
- line symmetric figures
- right triangles

**Prior Knowledge Required for This Unit**

In 3rd grade, students built understanding that shapes in different categories (e.g., rhombus, rectangle, square, and other 4-sided shapes) may share attributes (e.g., 4-sided figures) and the shared attributes can define a larger category (e.g., quadrilateral). They recognized rhombuses, rectangles, and squares as examples of quadrilaterals, and drew examples of quadrilaterals that do not belong to any of these subcategories (3.G.1). Students also

used a right angle as a benchmark to identify and sketch acute and obtuse angles (3.G.3).

**Subsequent Knowledge Related to This Unit**

In Unit 9, students will build on their knowledge of angles by investigating how to measure angles using a protractor and recognizing the additive nature of angle measures. In 5th grade, students will understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category (5.G.3) and classify two-dimensional figures in a hierarchy based on their attributes (5.G.4).

**Relationship Among Standards in This Unit**

The primary focus of this unit is to build students' understanding of geometric concepts. First, they are introduced to points, lines, line segments, rays, and angles, and then they investigate the relationships between them. Students learn to construct, recognize, and define these geometric objects before using their new knowledge and understanding to classify figures and solve problems. Through their exploration of symmetry, students recognize specific attributes present in two-dimensional figures. They further develop their understanding of these attributes as they classify two-dimensional figures based on them.

**Potential Instructional Strategies/Lessons**

**Teacher Notes: 4.G.1**

Points, lines, segments, rays, and angles are the building blocks of geometry. Point and line are undefined terms because they do not have definitions. We can understand these terms by thinking of examples of what a point and line might look like. A point can be a tip of a pencil; it has position but no dimension. Euclid described a line by saying that through any two points there is always a line and every line contains at least two points. Line segment is part of a line and it contains two endpoints meaning it has a beginning and endpoints. A line contains an infinite number of points and has no endpoints and goes on and on forever. A ray is part of a line that has one endpoint and extends forever in only one direction. Parallel lines are lines that never cross and are the same distance apart. Perpendicular lines intersect to form right angles (John SanGiovanni, Howard County Schools).

Students can and should make geometric distinctions about angles without measuring or mentioning degrees. Angles should be classified in comparison to right angles, such as greater than, less than, or the same size as a right angle. Students can use the corner of a sheet of paper as a benchmark for a right angle. They can use a right angle to determine relationships of other angles (Excerpt from KATM Grade 4 Flip Book).

**Introductory Activities:**

**What Makes a Shape? (Georgia Department of Education)**

http://cloud.rpsar.net/edocs/Math/3rdGrade/CIResources/Q4/What%20Makes%20a%20Shape.pdf

- In this task, students begin the process of exploring shapes for their many attributes and use critical vocabulary to describe and compare those shapes through higher-level thinking skills.

**Illustrative Mathematics - What's the Point?**
https://www.illustrativemathematics.org/content-standards/tasks/1272

- The purpose of this task is to use what students intuitively understand about connecting points or "dots" with lines to generate a discussion about what points are and how they should be represented. This task is intended to lead into a class discussion about how we think about points vs. how we represent points. The summary conversation led by the teacher will determine the value of this instructional task.

**Teacher Notes: 4.G.2**
Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90∘). Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles (Excerpt from KATM Grade 4 Flip Book).

**Introductory Activities:**
**Illustrative Mathematics - Defining Attributes of Rectangles and Parallelograms**
https://www.illustrativemathematics.org/content-standards/tasks/1275

- The purpose of this task is for students to identify the defining attributes of rectangles and parallelograms.

**Illustrative Mathematics - What Shape am I?**
https://www.illustrativemathematics.org/content-standards/tasks/1274

- In this task, students ultimately use the definitions they are given for three types of quadrilaterals and what they know about parallel sides to identify that a square fits all the definitions and explain why. By drawing examples and non-examples for each kind of shape, students get an opportunity to explore their individual properties before relating their knowledge of all three to the defining attributes of a square. Students should be encouraged to work in small groups and use the correct vocabulary as they talk together about classifying shapes. Students should take care to draw their figures with a straight-edge so that they can draw precise shapes. It is a good idea to have a whole-group discussion about this task to be sure that students understand the relationship between these different quadrilaterals.

**Teacher Notes: 4.G.3**
Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides (Excerpt from KATM Grade 4 Flip Book).

**Introductory Activity:**

**Illustrative Mathematics - Are these Right?**

https://www.illustrativemathematics.org/content-standards/tasks/1273

- The purpose of this task is for students to observe angles and decide whether the triangles are right or not. Rather than using a protractor at this point, students can use the corner of a sheet of paper as a benchmark for a right angle.

**Teacher Notes: 4.G.4**

When introducing line of symmetry, provide examples of geometric shapes with and without lines of symmetry. Shapes can be classified by the existence of lines of symmetry in sorting activities. This can be done informally by folding paper, tracing, creating designs with tiles or investigating reflections in mirrors.

Students need experiences with figures which are symmetrical and nonsymmetrical. Figures include both regular and irregular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry.

This standard only includes line symmetry, not rotational symmetry.

Give student experience with many shapes that can be folded to determine if they have symmetry. Block letters of the alphabet is one set that students can explore. Students can search magazines to find shapes that are symmetrical and fold to show the line of symmetry.

The use of Miras help students see and draw line to show symmetry. The reflection from the Mira or a mirror helps students see symmetry. Pattern blocks and tangrams are also useful tools in discovering symmetry.

Common Misconception:

Some children may think that there can only be one line of symmetry for an object. Encourage them to try folding shapes in more than one way. Giving students multiple copies of the same shapes could help avoid confusion. Coloring one side of the line one color and the other side of the line a different color may aid in seeing multiple lines. In essence the student is seeing if the shape can be folded into ½ halves (Excerpt from KATM Grade 4 Flip Book).

**Introductory Activities:**

**Symmetry Introduction Lesson**

http://www.asset.asu.edu/new/mathactive/lessons/116/look.swf

- This interactive lesson could be used as a quick introduction to symmetry.

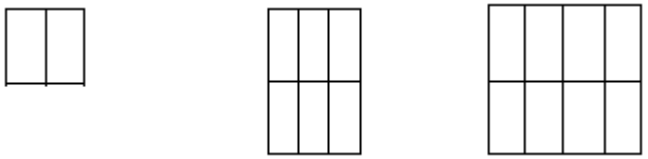
**E-Lab Symmetry (HMH School Publishers)**

http://www.harcourtschool.com/activity/elab2004/gr3/21.html

- This interactive tool gives students practice with dragging lines of symmetry onto various shapes.

**Teacher Notes: 4.ATO.5**

Patterns involving shapes either repeat or grow. Students need multiple opportunities creating and extending shape patterns.
Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like.
Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.
After students have identified rules and features from patterns, they need to generate a shape pattern from a given rule.
Provide patterns that involve shapes so that students can determine the rule for the pattern. For example, students may state that the rule is to multiply the previous number of squares by 3 (Excerpt from KATM Grade 4 Flip Book).



**Introductory Activities:**

**K-5 Math Teaching Resources - Square Numbers**
http://www.k-5mathteachingresources.com/support-files/square-numbers.pdf

- In this activity, students use square tiles to construct a geometric shape pattern and then analyze the pattern.

**K-5 Math Teaching Resources - Triangular Numbers**
http://www.k-5mathteachingresources.com/support-files/triangular-numbers.pdf

- In this activity, students use pattern blocks to construct a geometric shape pattern and then analyze the pattern.

**Resources**

**EngageNY - Grade 4, Module 4: Angle Measures and Plane Figures**
https://www.engageny.org/resource/grade-4-mathematics-module-4

- Topics A and D specifically match this unit, while Topics B and C match the Angle Measurement unit (Unit 9)

**CCGPS Frameworks 4th Unit 6**
https://www.georgiastandards.org/Georgia-Standards/Frameworks/4th-Math-Unit-6.pdf

- This unit includes performance tasks and formative assessment lessons.

**Learn Zillion - Grade 4, Unit 10 Angle Measurement**
https://hcpss.learnzillion.com/resources/64188-angle-measurement

- This unit includes video lessons, practice resources for students, and a summative assessment. Students start this unit drawing points, lines, segments, rays and angles. The remaining portions of the unit go along with the next unit on Angle Measurement (Unit 9).

**Zearn Grade 4, Mission 4: Construct Lines, Angles, and Shapes, Topics A and D**
https://www.zearn.org/curriculum/16

- This website, aligned to the EngageNY units, contains a variety of resources, including interactive manipulatives, exit tickets, homework, and assessments.

**Matific Geometry Episodes (free for teachers)**
https://www.matific.com/us/en-us/grades/4G/Geometry

- This page offers multiple episodes on 4th grade geometry concepts.

**Geometry Games (Sheppard Software)**
http://www.sheppardsoftware.com/mathgames/menus/geometry.htm

- This site includes multiple games for students to practice their geometry skills.

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**Howard County School Resources for 4.G.1**
https://hcpss.instructure.com/courses/107/pages/4-dot-g-1-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Be an Expert! (Georgia Department of Education)**
http://cloud.rpsar.net/edocs/Math/4thGrade/CIResources/Q4/Be_An_Expert.pdf

- In this task, students will construct graphic organizers about the main geometric objects in this unit.

**Illustrative Mathematics - The Geometry of Letters**
https://www.illustrativemathematics.org/content-standards/tasks/1263

- The purpose of this task is for students to analyze the geometry of letters. Letters provide a good opportunity for students to broaden their understanding of what constitutes a 2-dimensional geometric figure. This task has students composing and decomposing figures, which is an important way of looking at geometric figures and a skill that students start working on in kindergarten and continue building throughout elementary school and beyond.

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**Howard County School Resources for 4.G.2 and 4.G.3**
https://hcpss.instructure.com/courses/107/pages/4-dot-g-2-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas. While South Carolina's standard is split into two parts - one for quadrilaterals and one for triangles, this page covers the Common Core Standard, which includes both quadrilaterals and triangles.

**Quadrilateral Quest**
http://teams.lacoe.edu/documentation/classrooms/amy/geometry/6-8/activities/quad_quest/quad_quest.html

- This interactive site allows students to examine the properties of quadrilaterals.

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**Howard County School Resources for 4.G.4**
https://hcpss.instructure.com/courses/107/pages/4-dot-g-3-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Illustrative Mathematics - Lines of Symmetry for Triangles**
https://www.illustrativemathematics.org/content-standards/tasks/1058

- This task is intended for instruction, providing the students with a chance to experiment with physical models of triangles, gaining spatial intuition by executing reflections.

**Illustrative Mathematics - Lines of Symmetry for Quadrilaterals**
https://www.illustrativemathematics.org/content-standards/tasks/1059

- This task is best suited for instruction although it could be adapted for assessment. If students have not yet learned the terminology for trapezoids and parallelograms, the teacher can begin by explaining the meaning of those terms. 4.G.2 states that students should classify figures based on the presence or absence of parallel and perpendicular lines, so this task would work well in a unit that is addressing all the standards in cluster 4.G.A. The students should try to visualize the lines of symmetry first, and then they can make or be provided with cutouts of the four quadrilaterals or trace them on tracing paper. It is useful for students to experiment and see what goes wrong, for example, when reflecting a rectangle (which is not a square) about a diagonal. This activity helps develop visualization skills as well as experience with different shapes and how they behave when reflected.

**Illustrative Mathematics - Lines of Symmetry for Circles**
https://www.illustrativemathematics.org/content-standards/tasks/1060

- This is an instructional task that gives students a chance to reason about lines of symmetry and discover that a circle has an an infinite number of lines of symmetry.

**Illustrative Mathematics - Finding Lines of Symmetry**
https://www.illustrativemathematics.org/content-standards/tasks/676

- The purpose of this task is for students to identify figures that have line symmetry and draw appropriate lines of symmetry. If students are first learning about symmetry, it would be good for them to create their own line-symmetric shapes by folding a piece of paper in half and cutting a shape out. Then they can darken the line represented by the fold to reinforce that it is a line of symmetry for their shape.

**Howard County School Resources for 4.ATO.5**
https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-5-about-the-math-learning-targets-and-rigor

**Math Wire - Fir Tree**
http://mathwire.com/problemsolving/4firtree.pdf

- In this activity, students use pattern blocks to construct a geometric shape pattern and then make predictions about continuing the pattern.

**Math Wire - Hexagon Dragons**
http://mathwire.com/problemsolving/4hexagondragons.pdf

- In this activity, students use pattern blocks to construct a geometric shape pattern and then make predictions about continuing the pattern.

**Sample Formative Assessment Tasks/Questions**

https://hcpss.instructure.com/courses/107/pages/4-dot-g-1-assessment-tasks (Assessing 4.G.1)
https://hcpss.instructure.com/courses/107/pages/4-dot-g-2-assessment-tasks (Assessing 4.G.2 and 4.G.3)
https://hcpss.instructure.com/courses/107/pages/4-dot-g-3-assessment-tasks (Assessing 4.G.4)
https://hcpss.instructure.com/courses/107/pages/4-dot-oa-dot-5-assessment-tasks (Assessing 4.ATO.5 - Use Quarter 3 Tasks for shapes)

Return to Table of Contents

| **Grade 4 Unit 9** | Return to Table of Contents | Return to Fourth Grade Overview |
|---|---|---|

Angle Measurement

**Content Standards with Clarifying Notes**
*(Open bullets indicate clarifying notes)*

- **4.MDA.5** Understand the relationship of an angle measurement to a circle.
  - This standard calls for students to explore the connection between angles (measure of rotation) and circular measurement (360 degrees).
- **4.MDA.6** Measure and draw angles in whole number degrees using a protractor.
- **4.MDA.7** Solve addition and subtraction problems to find unknown angles in real-world and mathematical problems.
  - This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.

**New Academic Vocabulary for This Unit**

- angle measurement
- degrees
- protractor

**Prior Knowledge Required for This Unit**

In 3rd grade, students used a right angle as a benchmark to identify and sketch acute and obtuse angles (3.MDA.3). In 4th grade Unit 8, students drew angles (right, acute, and obtuse) and identified them in two-dimensional figures. This is the first experience students have had with measuring angles and using a protractor.

**Subsequent Knowledge Related to This Unit**

In 7th grade, students will write equations to solve problems involving the relationships between angles formed by two intersecting lines, including supplementary, complementary, vertical, and adjacent.

**Relationship Among Standards in This Unit**

The primary focus of this unit is on students' understanding of the degree as a unit of measure. First, students explore the relationship between an angle's measure and the degrees in a circle. From here, they move on to measuring and drawing angles using a protractor. Finally, they explore the additive nature of angles in order to find the measure of unknown angles.

**Potential Instructional Strategies/Lessons**

Angle measure is a "turning point" in the study of geometry. Students often find angles and angle measure to be difficult concepts to learn, but that learning allows them to engage in interesting and important mathematics. An angle is the union of two rays, *a* and *b*, with the same initial point P. The rays can be made to coincide by rotating one to the other about P; this rotation determines the size of the angle between *a* and *b*. The rays are sometimes called the sides of the angles.

Angles are measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and degrees are the unit used to measure angles in elementary school. A full rotation is thus 360°.
Like length, area, and volume, angle measure is additive: The sum of the measurements of adjacent angles is the measurement of the angle formed by their union (excerpt from CCSS K-5 Geometric Measurement Progression Document).

Diagram referred to in "Teacher Notes"

**Teacher Note:** 4.MDA.5
The diagram above will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles, yet the angle measure is the same.

Angles are geometric shapes composed of two rays that are infinite in length. Students can understand this concept by using two rulers held together near the ends. The rulers can represent the rays of an angle. As one ruler is rotated, the size of the angle is seen to get larger. Ask questions about the types of angles created. Responses may be in terms of the relationship to right angles.

Introduce angles as acute (less than the measure of a right angle) and obtuse (greater than the measure of a right angle). Have students draw representations of each type of angle. They also need to be able to identify angles in two-dimensional figures.
Students can also create an angle explorer (two strips of cardboard attached with a brass fastener) to learn about angles. They can use the angle explorer to get a feel of the relative size of angles as they rotate the cardboard strips around.

Students can compare angles to determine whether an angle is acute or obtuse. This will allow them to have a benchmark reference for what an angle measure should be when using a tool such as a protractor or an angle ruler.

Provide students with four pieces of straw, two pieces of the same length to make one angle and another two pieces of the same length to make an angle with longer rays. Each set of straws can be attached with two jointed paper clips.
Another way to compare angles is to place one angle over the other angle. Provide students with a transparency to compare two angles to help them conceptualize the spread of the rays of an angle. Students can make this comparison by tracing one angle and placing it over another angle. The side lengths of the angles to be compared need to be different.
(Excerpt from KATM Grade 4 Flip Book)

**Introductory Activity:**
**Angle Tangle (Georgia Department of Education)**
http://cloud.rpsar.net/edocs/Math/4thGrade/CIResources/Q4/Angle_Tangle.pdf

- In this task, students will explore angles and their measurements using a 360 degree circle. Students will also begin to understand the measures of benchmark angles.

**Teacher Note:** 4.MDA.6
Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles.
They transfer their understanding that a 360 degree rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90 degrees and 180 degrees.

They extend this understanding and recognize and sketch angles that measure approximately 45 degrees and 30 degrees. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

Students should estimate angles, measure angles and sketch angles. They need to experience measuring angles using an angle ruler as well as a protractor. (The angle ruler allows them to "see" the turns or rotations).

**Misconceptions:**
Students are often confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers.

Students should have multiple experiences estimating and comparing angles to the Benchmark 90° or right angle.

They should explain their reasoning by deciding first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to 89°.

If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°.

Ask questions about the appearance of the angle to help students in deciding which number to use.

Some protractors have a protective edge along the bottom. Zero degrees begins about ¼ of an inch above the bottom edge. Students often to not take this into account and therefore will have inaccurate measures of angles. (Excerpt from KATM Grade 4 Flip Book)

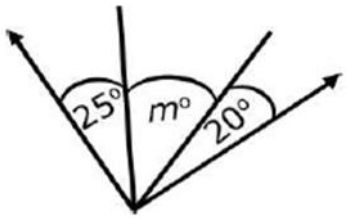
**Introductory Activity:**
**What's My Angle?**
http://www.amblesideprimary.com/ambleweb/mentalmaths/protractor.html

- This interactive website contains activities to support the teaching and investigation of angles. It contains an introduction and ten scaffolding activities to move students toward an understanding of measuring angles. Note: Students do not need to know a reflex angle.

**Teacher Note:** 4.MDA.7
Angle measure is additive. If you draw a third array and divide an angle into two smaller angles, the measures of the two smaller angles equal the original angle. Remember to keep in mind, that the three angles all have the same vertex. The two smaller angles share a common ray. The smaller angles do not overlap. You can find an unknown angle measure, if you know the measure of the original angle and one of the smaller angles (John SanGiovanni, Howard County Schools).

Example:
A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved?
If the water sprinkler rotates a total of 25 degrees then pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

Example:
If the two rays are perpendicular, what is the value of m?
(Excerpt from KATM Grade 4 Flip Book)




**Resources**

**Literature Connection**
***Sir Cumference and the Great Knight of Angleland***: In this story, young Radius, son of Sir Cumference and Lady Di of Ameter, undertakes a quest. With the help of a family heirloom that functions similar to a protractor, he is able to locate the elusive King Lell and restore him to the throne. In gratitude, the king bestows knighthood on Sir Radius.

**EngageNY - Grade 4, Module 4: Angle Measures and Plane Figures**
https://www.engageny.org/resource/grade-4-mathematics-module-4

- Topics B and C specifically match this unit, while Topics A and D match the Geometry unit (Unit 8)T

**Learn Zillion - Grade 4, Unit 10 Angle Measurement**
https://hcpss.learnzillion.com/resources/64188-angle-measurement

- This unit includes video lessons, practice resources for students, and a summative assessment. It is an introduction to angles and angle measurement. Students start this unit drawing points, lines, segments, rays and angles since it is foundational to the other standards in this unit. Students use their understanding of equal partitioning and unit measurement to understand angle and turn measure.

**Matific Angle Measurement Episodes (free for teachers)**
https://www.matific.com/us/en-us/search?q=angle%20measurement&grades=4

- This page offers multiple episodes on measuring angles.

**Zearn Grade 4, Mission 4: Construct Lines, Angles, and Shapes, Topics B and C**
https://www.zearn.org/curriculum/16

- This website, aligned to the EngageNY units, contains a variety of resources, including interactive manipulatives, exit tickets, homework, and assessments.

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**Howard County School Resources for 4.MDA.5**
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-5-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Banana Hunt**
http://www.oswego.org/ocsd-web/games/bananahunt/bhunt.html

- This interactive game gives students practice with estimating angle measurements within a circle.

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**Howard County School Resources for 4.MDA.6**
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-6-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Illustrative Mathematics - Measuring Angles**
https://www.illustrativemathematics.org/content-standards/tasks/909

- The purpose of this task is to gain experience drawing and measuring angles, developing an understanding of the additive structure of angles (can be used as an introduction to 4.MDA.7).

**Alien Angles**
http://www.mathplayground.com/alienangles.html

- This interactive game gives students practice with estimating angle measurements between 0 and 180 degrees.

**Age of the Angles**
http://mrnussbaum.com/age-of-the-angles-ipad.html

- This interactive tool gives students practice with using a protractor to measure assorted angles.

**ABCYA.com - Measuring Angles**
http://www.abcya.com/measuring_angles.htm

- This interactive game gives students practice using an online protractor to measure angles. It also reviews terms (acute, obtuse, and right).

**Howard County School Resources for 4.MDA.7**
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-7-about-the-math-learning-targets-and-rigor

- This page includes lesson plans, print resources, LearnZillion video links, and ideas for prompting rigor, web resources, and math center ideas.

**Illustrative Mathematics - Finding an Unknown Angle**
https://www.illustrativemathematics.org/content-standards/tasks/1168

- The purpose of this task is to give 4th grade students a problem involving an unknown quantity that has a clear visual representation. Students must understand that the four interior angles of a rectangle are all right angles (4.G.2) and that right angles have a measure of 90∘and that angle measure is additive (4.MD.7). In a teaching scenario, students may be allowed to verify the computations using a protractor to measure the angles. However, care should be taken beforehand to ensure that the measurements of the printed figure match the stated measurements.

**IXL - Adjacent Angles**
https://www.ixl.com/math/grade-4/adjacent-angles

- This site provides students with practice in identifying unknown angles by using the additive nature of angle measurement.

**K-5 Math Teaching Resources - Additive Angles**
http://www.k-5mathteachingresources.com/support-files/additive-angles.pdf

- This print resource provides students practice with identifying the measures of angles within a larger angle.

**K-5 Math Teaching Resources - Unknown Angle Word Problems**
http://www.k-5mathteachingresources.com/support-files/unknown-angle-word-problems.pdf

- This print resource provides students practice with real-world problems involving unknown angles.

**Sample Formative Assessment Tasks/Questions**

https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-5-assessment-tasks (Assessing 4.MDA.5)
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-6-assessment-tasks (Assessing 4.MDA.6)
https://hcpss.instructure.com/courses/107/pages/4-dot-md-dot-7-assessment-tasks (Assessing 4.MDA.7)

Return to Table of Contents